Thirada 6304641662

(1)

a.) $Y_i = \beta_1 + \beta_2 X_i + u_i$

$u_i = Y_i - \beta_1 - \beta_2 x_i$

$u_i^2 = (Y_i - \beta_1 - \beta_2 x_i)^2$

$$\min_{\beta_1, \beta_2} \sum \hat{u}_i^2 = \sum (Y_i - \beta_1 - \beta_2 x_i)^2$$

β_1 ;

$$\frac{\partial \sum \hat{u}_i^2}{\partial \hat{\beta}_1} = 0$$

$$\frac{\partial \sum (y_i - \hat{\beta}_1 - \hat{\beta}_2 x_i)^2}{\partial \hat{\beta}_1} = 0$$

$$2 \sum (y_i - \hat{\beta}_1 - \hat{\beta}_2 x_i)(-1) = 0$$

$$\sum (y_i - \hat{\beta}_1 - \hat{\beta}_2 x_i) = 0$$

$$\sum y_i - \sum \hat{\beta}_1 - \sum \hat{\beta}_2 x_i = 0$$

$$\frac{\sum y_i}{n} - \frac{\sum \hat{\beta}_1}{n} - \frac{\sum \hat{\beta}_2 x_i}{n} = 0$$

$$\bar{y} - \hat{\beta}_1 - \hat{\beta}_2 \bar{x} = 0$$

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x} \quad - (1)$$

β_2 ;

$$\frac{\partial \sum \hat{y}_i^2}{\partial \hat{\beta}_2} = 0$$

$$\frac{\partial \sum (y_i - \hat{\beta}_1 - \hat{\beta}_2 x_i)^2}{\partial \hat{\beta}_2} = 0$$

$$2 \sum (y_i - \hat{\beta}_1 - \hat{\beta}_2 x_i)(-x_i) = 0$$

$$\sum \left[(y_i - \hat{\beta}_1 - \hat{\beta}_2 x_i)(x_i) \right] = 0 \quad - (2)$$

Thirada 6304647662

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2\bar{x} \quad -(1)$$

$$\Sigma((y_i - \hat{\beta}_1 - \hat{\beta}_2 x_i)(x_i)) = 0 \quad -(2)$$

plug (1) into (2),

$$\Sigma((y_i - (\bar{y} - \hat{\beta}_2\bar{x}) - \hat{\beta}_2 x_i)(x_i)) = 0$$

$$\Sigma\left[(y-\bar{y})x_i - \hat{\beta}_2(\bar{x}-x_i)x_i\right] = 0$$

$$\Sigma(y-\bar{y})x_i = \Sigma\hat{\beta}_2(\bar{x}-x_i)x_i$$

$$\frac{\Sigma(y_i-\bar{y})x_i}{\Sigma(\bar{x}-x_i)x_i} = \hat{\beta}_2$$

$$\frac{\Sigma(x_i-\bar{x})(y_i-\bar{y})}{\Sigma(\bar{x}-x_i)^2} = \hat{\beta}_2$$

$$\therefore \hat{\beta}_2 = \frac{20.58}{211} = \frac{1029}{10550} = 0.0975 \#$$

sub $\hat{\beta}_2 = \frac{1029}{10550}$ into $\Sigma(y_i - \hat{\beta}_1 - \hat{\beta}_2 x_i) = 0$

$$\Sigma(y_i - \hat{\beta}_2 x_i) = \Sigma\hat{\beta}_1$$

$$\frac{\Sigma(y_i - \hat{\beta}_2 x_i)}{n} = \hat{\beta}_1$$

$$\frac{50.9 - \left(\frac{1029}{10550}\right)(388)}{18} = \hat{\beta}_1$$

$$\hat{\beta}_1 = 0.7253 \#$$

Thirada 6304647662

① a. The estimator of $\beta_1 = \hat{\beta}_1 = 0.7253$
The estimator of $\beta_2 = \hat{\beta}_2 = 0.0975$

$$Y_i = \beta_1 + \beta_2 X_i + u_i$$

$$Y_i = 0.7253 + 0.0975 X_i + \hat{u}_i$$

The interception is 0.7253 and slope is 0.0975 #

1 Thirada 6304647662

b.) $R^2 = 1 - \dfrac{\sum \hat{u}_i^2}{\sum (Y_i - \bar{Y})^2}$

$R^2 = 1 - \dfrac{0.5781}{2.5844}$

$R^2 = 0.7763$

The r^2 is determined by how much the r^2 is described by the SRF, or the measurement of goodness of fit of the fitted regression line comparing to an estimator $\bar{Y}$.

This mean $r^2 = 0.7763$ has high cofidential because it has small range and the data closes to SRF line.

c.) $X_i = 30$ $\hat{Y}_i = ?$

$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$

$\hat{Y}_i = 0.7253 + (0.0975)\,30 = 3.6503$

When $X_i = 30$ the expect value is 3.6503

d.) Variance u_i : $\sigma^2_{u_i} = \dfrac{\sum \hat{u}_i^2}{n-k} = \dfrac{0.5781}{(18-2)} = 0.0361$

Variance $\hat{\beta}_1$: $\sigma^2_{\hat{\beta}_1} = \dfrac{\sum x_i^2}{n\sum (x_i - \bar{x})^2}\sigma^2 = \dfrac{9{,}620}{18(211)} \times 0.0361 = 0.0914$

Variance $\hat{\beta}_2$: $\sigma^2_{\hat{\beta}_2} = \dfrac{\sigma^2}{\sum (x_i - \bar{x})^2} = \dfrac{0.0361}{211} = 1.7109 \times 10^{-4}$

Thirada 6304641662

1 e.) $\alpha = 0.1$

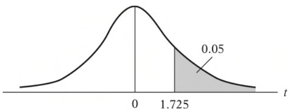
$$P\left[\hat{\beta}_2 - \left(t_{\frac{\alpha}{2}} \cdot se_{\hat{\beta}_2}\right) \leq \beta_2 \leq \hat{\beta}_2 + \left(t_{\frac{\alpha}{2}} \cdot se_{\hat{\beta}_2}\right)\right] = 0.9$$

Example

Pr(t > 2.086) = 0.025

Pr(t > 1.725) = 0.05 for df = 20

Pr(|t| > 1.725) = 0.10



Pr / df	0.25 0.50	0.10 0.20	0.05 0.10	0.025 0.05	0.01 0.02	0.005 0.010	0.001 0.002
1	1.000	3.078	6.314	12.706	31.821	63.657	318.31
2	0.816	1.886	2.920	4.303	6.965	9.925	22.327
3	0.765	1.638	2.353	3.182	4.541	5.841	10.214
4	0.741	1.533	2.132	2.776	3.747	4.604	7.173
5	0.727	1.476	2.015	2.571	3.365	4.032	5.893
6	0.718	1.440	1.943	2.447	3.143	3.707	5.208
7	0.711	1.415	1.895	2.365	2.998	3.499	4.785
8	0.706	1.397	1.860	2.306	2.896	3.355	4.501
9	0.703	1.383	1.833	2.262	2.821	3.250	4.297
10	0.700	1.372	1.812	2.228	2.764	3.169	4.144
11	0.697	1.363	1.796	2.201	2.718	3.106	4.025
12	0.695	1.356	1.782	2.179	2.681	3.055	3.930
13	0.694	1.350	1.771	2.160	2.650	3.012	3.852
14	0.692	1.345	1.761	2.145	2.624	2.977	3.787
15	0.691	1.341	1.753	2.131	2.602	2.947	3.733
16	0.690	1.337	1.746	2.120	2.583	2.921	3.686
17	0.689	1.333	1.740	2.110	2.567	2.898	3.646
18	0.688	1.330	1.734	2.101	2.552	2.878	3.610
19	0.688	1.328	1.729	2.093	2.539	2.861	3.579
20	0.687	1.325	1.725	2.086	2.528	2.845	3.552
21	0.686	1.323	1.721	2.080	2.518	2.831	3.527
22	0.686	1.321	1.717	2.074	2.508	2.819	3.505
23	0.685	1.319	1.714	2.069	2.500	2.807	3.485
24	0.685	1.318	1.711	2.064	2.492	2.797	3.467
25	0.684	1.316	1.708	2.060	2.485	2.787	3.450
26	0.684	1.315	1.706	2.056	2.479	2.779	3.435
27	0.684	1.314	1.703	2.052	2.473	2.771	3.421
28	0.683	1.313	1.701	2.048	2.467	2.763	3.408
29	0.683	1.311	1.699	2.045	2.462	2.756	3.396
30	0.683	1.310	1.697	2.042	2.457	2.750	3.385
40	0.681	1.303	1.684	2.021	2.423	2.704	3.307
60	0.679	1.296	1.671	2.000	2.390	2.660	3.232
120	0.677	1.289	1.658	1.980	2.358	2.617	3.160
∞	0.674	1.282	1.645	1.960	2.326	2.576	3.090

Note: The smaller probability shown at the head of each column is the area in one tail; the larger probability is the area in both tails.

$t_{\frac{\alpha}{2}} = t_{0.05} = 1.746$ when df = 16

degree of freedom = n-k = 18-2 = 16

$se^2 = \text{variance}$

$se = \sqrt{\text{variance}}$

$se_{\hat{\beta}_2} = \sqrt{1.7109 \times 10^{-4}} = 0.0131$

$$P\left[0.0975 - \left(1.746 \times 0.0131\right) \leq \beta_2 \leq 0.0975 + \left(1.746 \times 0.0131\right)\right] = 0.9$$

$$P\left[0.0975 - 0.0228726 \leq \beta_2 \leq 0.0975 + 0.0228726\right]$$

$$P\left[0.0746 \leq \beta_2 \leq 0.1204\right] = 0.9$$

From 90 out of 100 times β_2 is between 0.0746 and 0.1204. #

Thirada 6304647662

① f.) step 1 : $H_0 : \beta_2 = 0$ – Null hypothesis

$H_a : \beta_2 \neq 0$ – Alternative hypothesis

step 2 : $\alpha = 0.05$

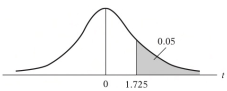
step 3 : $t_{cal} = \dfrac{\hat{\beta}_2 - \beta_2}{se_{\hat{\beta}_2}} = \dfrac{0.0975 - 0}{0.0131} = 7.4427$

step 4 : The upper boundary $= t_{\frac{\alpha}{2}} = t_{\frac{0.05}{2}} = t_{0.0025}$ df = 16 $= 2.120$

The lower boundary $= t_{\frac{\alpha}{2}} = -2.120$

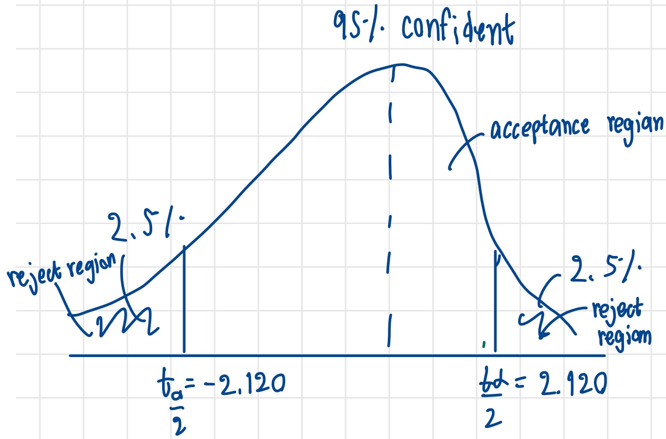
; of
on and
etrika
ol. 1,
ge
rk,
and

Example
Pr(t > 2.086) = 0.025
Pr(t > 1.725) = 0.05   for df = 20
Pr(|t| > 1.725) = 0.10



Pr / df	0.25 / 0.50	0.10 / 0.20	0.05 / 0.10	0.025 / 0.05	0.01 / 0.02	0.005 / 0.010	0.001 / 0.002
1	1.000	3.078	6.314	12.706	31.821	63.657	318.31
2	0.816	1.886	2.920	4.303	6.965	9.925	22.327
3	0.765	1.638	2.353	3.182	4.541	5.841	10.214
4	0.741	1.533	2.132	2.776	3.747	4.604	7.173
5	0.727	1.476	2.015	2.571	3.365	4.032	5.893
6	0.718	1.440	1.943	2.447	3.143	3.707	5.208
7	0.711	1.415	1.895	2.365	2.998	3.499	4.785
8	0.706	1.397	1.860	2.306	2.896	3.355	4.501
9	0.703	1.383	1.833	2.262	2.821	3.250	4.297
10	0.700	1.372	1.812	2.228	2.764	3.169	4.144
11	0.697	1.363	1.796	2.201	2.718	3.106	4.025
12	0.695	1.356	1.782	2.179	2.681	3.055	3.930
13	0.694	1.350	1.771	2.160	2.650	3.012	3.852
14	0.692	1.345	1.761	2.145	2.624	2.977	3.787
15	0.691	1.341	1.753	2.131	2.602	2.947	3.733
16	0.690	1.337	1.746	2.120	2.583	2.921	3.686
17	0.689	1.333	1.740	2.110	2.567	2.898	3.646
18	0.688	1.330	1.734	2.101	2.552	2.878	3.610
19	0.688	1.328	1.729	2.093	2.539	2.861	3.579
20	0.687	1.325	1.725	2.086	2.528	2.845	3.552
21	0.686	1.323	1.721	2.080	2.518	2.831	3.527
22	0.686	1.321	1.717	2.074	2.508	2.819	3.505
23	0.685	1.319	1.714	2.069	2.500	2.807	3.485
24	0.685	1.318	1.711	2.064	2.492	2.797	3.467
25	0.684	1.316	1.708	2.060	2.485	2.787	3.450
26	0.684	1.315	1.706	2.056	2.479	2.779	3.435
27	0.684	1.314	1.703	2.052	2.473	2.771	3.421
28	0.683	1.313	1.701	2.048	2.467	2.763	3.408
29	0.683	1.311	1.699	2.045	2.462	2.756	3.396
30	0.683	1.310	1.697	2.042	2.457	2.750	3.385
40	0.681	1.303	1.684	2.021	2.423	2.704	3.307
60	0.679	1.296	1.671	2.000	2.390	2.660	3.232
120	0.677	1.289	1.658	1.980	2.358	2.617	3.160
∞	0.674	1.282	1.645	1.960	2.326	2.576	3.090

Note: The smaller probability shown at the head of each column is the area in one tail; the larger probability is the area in both tails.



Step 5 : t_{cal} lies beyond any boundary of test value critical region, we can reject null hypothesis at the significance level of 95%.
We are sure that β_2 is not zero from sample 95 out of 100 times.

Thirada 6304647662

② a.) Setp 1: $H_0 : \beta_1 = 0$ - null hypothesis

$H_a : \hat{\beta}_1 \neq 0$ - alternative hypothesis

step 2 : $\alpha = 0.05$

step 3 : $t_{cal_{\hat{\beta}_1}} = \dfrac{\hat{\beta}_1 - \beta_1}{se_{\hat{\beta}_1}} = \dfrac{0.4279898 - 0}{0.0140339} = 30.4969$

step 4 : upper boundary $t_{\frac{\alpha}{2}} = t_{0.025}$ df 29,885 $= 1.960$

lower boundary $= -1.960$

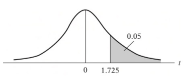
ts of
n

son and
netrika
vol. 1,
idge
York,

s and

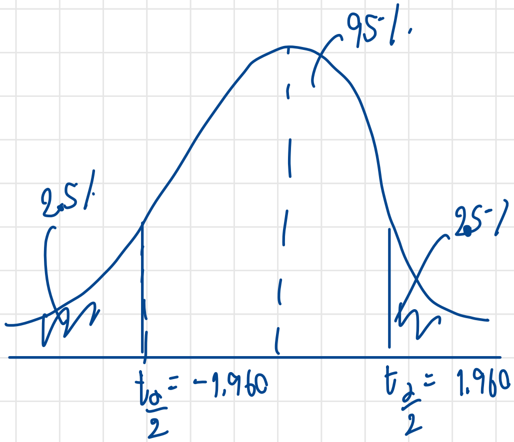
Example

Pr(t > 2.086) = 0.025
Pr(t > 1.725) = 0.05 for df = 20
Pr($|t|$ > 1.725) = 0.10



Pr / df	0.25 0.50	0.10 0.20	0.05 0.10	0.025 0.05	0.01 0.02	0.005 0.010	0.001 0.002
1	1.000	3.078	6.314	12.706	31.821	63.657	318.31
2	0.816	1.886	2.920	4.303	6.965	9.925	22.327
3	0.765	1.638	2.353	3.182	4.541	5.841	10.214
4	0.741	1.533	2.132	2.776	3.747	4.604	7.173
5	0.727	1.476	2.015	2.571	3.365	4.032	5.893
6	0.718	1.440	1.943	2.447	3.143	3.707	5.208
7	0.711	1.415	1.895	2.365	2.998	3.499	4.785
8	0.706	1.397	1.860	2.306	2.896	3.355	4.501
9	0.703	1.383	1.833	2.262	2.821	3.250	4.297
10	0.700	1.372	1.812	2.228	2.764	3.169	4.144
11	0.697	1.363	1.796	2.201	2.718	3.106	4.025
12	0.695	1.356	1.782	2.179	2.681	3.055	3.930
13	0.694	1.350	1.771	2.160	2.650	3.012	3.852
14	0.692	1.345	1.761	2.145	2.624	2.977	3.787
15	0.691	1.341	1.753	2.131	2.602	2.947	3.733
16	0.690	1.337	1.746	2.120	2.583	2.921	3.686
17	0.689	1.333	1.740	2.110	2.567	2.898	3.646
18	0.688	1.330	1.734	2.101	2.552	2.878	3.610
19	0.688	1.328	1.729	2.093	2.539	2.861	3.579
20	0.687	1.325	1.725	2.086	2.528	2.845	3.552
21	0.686	1.323	1.721	2.080	2.518	2.831	3.527
22	0.686	1.321	1.717	2.074	2.508	2.819	3.505
23	0.685	1.319	1.714	2.069	2.500	2.807	3.485
24	0.685	1.318	1.711	2.064	2.492	2.797	3.467
25	0.684	1.316	1.708	2.060	2.485	2.787	3.450
26	0.684	1.315	1.706	2.056	2.479	2.779	3.435
27	0.684	1.314	1.703	2.052	2.473	2.771	3.421
28	0.683	1.313	1.701	2.048	2.467	2.763	3.408
29	0.683	1.311	1.699	2.045	2.462	2.756	3.396
30	0.683	1.310	1.697	2.042	2.457	2.750	3.385
40	0.681	1.303	1.684	2.021	2.423	2.704	3.307
60	0.679	1.296	1.671	2.000	2.390	2.660	3.232
120	0.677	1.289	1.658	1.980	2.358	2.617	3.160
∞	0.674	1.282	1.645	1.960	2.326	2.576	3.090

Note: The smaller probability shown at the head of each column is the area in one tail; the larger probability is the area in both tails.



Steps : t_{cal} lies beyond any boundary of test value critical region, we can reject null hypothesis at the significance level of 95%.
we are sure that β_1 is not zero from sampling 95 out of 100 times.

Thirada 630464762

2.) a.) step1. $H_0 : \beta_2 = 0$ - null hypotheris

$H_a : \hat{\beta}_2 \neq 0$ - alternative hypothesis

step2 $\alpha = 0.05$

step3 $t_{cal_{\beta_2}} = \dfrac{\hat{\beta}_2 - \beta_2}{se_{\hat{\beta}_2}} = \dfrac{0.0031338 - 0}{0.0002292} = 13.6728$

step4 upper boundary $t_{\frac{\alpha}{2}} = t_{0.025}$ df = 27,885 = 1.960

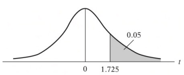
ts of
n

son and
netrika
vol. 1,
idge
York,

s and

Example

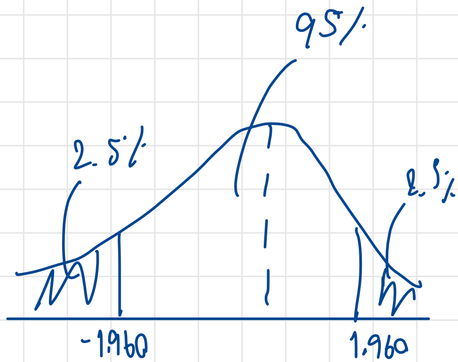
$Pr(t > 2.086) = 0.025$
$Pr(t > 1.725) = 0.05$ for df = 20
$Pr(|t| > 1.725) = 0.10$



df \ Pr	0.25 0.50	0.10 0.20	0.05 0.10	0.025 0.05	0.01 0.02	0.005 0.010	0.001 0.002
1	1.000	3.078	6.314	12.706	31.821	63.657	318.31
2	0.816	1.886	2.920	4.303	6.965	9.925	22.327
3	0.765	1.638	2.353	3.182	4.541	5.841	10.214
4	0.741	1.533	2.132	2.776	3.747	4.604	7.173
5	0.727	1.476	2.015	2.571	3.365	4.032	5.893
6	0.718	1.440	1.943	2.447	3.143	3.707	5.208
7	0.711	1.415	1.895	2.365	2.998	3.499	4.785
8	0.706	1.397	1.860	2.306	2.896	3.355	4.501
9	0.703	1.383	1.833	2.262	2.821	3.250	4.297
10	0.700	1.372	1.812	2.228	2.764	3.169	4.144
11	0.697	1.363	1.796	2.201	2.718	3.106	4.025
12	0.695	1.356	1.782	2.179	2.681	3.055	3.930
13	0.694	1.350	1.771	2.160	2.650	3.012	3.852
14	0.692	1.345	1.761	2.145	2.624	2.977	3.787
15	0.691	1.341	1.753	2.131	2.602	2.947	3.733
16	0.690	1.337	1.746	2.120	2.583	2.921	3.686
17	0.689	1.333	1.740	2.110	2.567	2.898	3.646
18	0.688	1.330	1.734	2.101	2.552	2.878	3.610
19	0.688	1.328	1.729	2.093	2.539	2.861	3.579
20	0.687	1.325	1.725	2.086	2.528	2.845	3.552
21	0.686	1.323	1.721	2.080	2.518	2.831	3.527
22	0.686	1.321	1.717	2.074	2.508	2.819	3.505
23	0.685	1.319	1.714	2.069	2.500	2.807	3.485
24	0.685	1.318	1.711	2.064	2.492	2.797	3.467
25	0.684	1.316	1.708	2.060	2.485	2.787	3.450
26	0.684	1.315	1.706	2.056	2.479	2.779	3.435
27	0.684	1.314	1.703	2.052	2.473	2.771	3.421
28	0.683	1.313	1.701	2.048	2.467	2.763	3.408
29	0.683	1.311	1.699	2.045	2.462	2.756	3.396
30	0.683	1.310	1.697	2.042	2.457	2.750	3.385
40	0.681	1.303	1.684	2.021	2.423	2.704	3.307
60	0.679	1.296	1.671	2.000	2.390	2.660	3.232
120	0.677	1.289	1.658	1.980	2.358	2.617	3.160
∞	0.674	1.282	1.645	1.960	2.326	2.576	3.090

Note: The smaller probability shown at the head of each column is the area in one tail; the larger probability is the area in both tails.

lower boundary = -1.960



step 5 : t_{cal} lies beyond any boundary of test value critical regian, we can reject null hypothesis at the significance level of 95%.
we are sure that β_2 is not zero from sampling 95 out of 100 times.

Thirada 6304641662

2. b) $\hat{\beta}_2$ mean when age increase 1-year-old the average to visit to hosiptal will increase by 0.0031338 time. $\hat{\beta}_2$ make economic sense because when age increase the appotunity to visit harpital also higher.

c.) $\widehat{outp_i} = \hat{\beta}_1 + \hat{\beta}_2 age_i$

$$\widehat{outp_i} = 0.4279898 + 0.031338 \, age_i$$

$$\ln \widehat{outp_i} = 0.427898 + 0.031338 \, age_i$$

$$\frac{\ln \widehat{outp_i}}{\partial \, age_i} = 0.031338$$

An increase of age by 1 year is associated with apptunity to visit hospital in 2015 incease of $100 \times 0.031338 = 3.1338$ percent

d.) If age_i variable is divided by 10, the coefficients, standard errors, and confidence intervals of age are change. the constant is 0.4279898 time and slope (coefficient of age) is 0.00031338 year.

Thirada 6304647662

2. e)

$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 x_i \qquad x_0 = 50 \qquad \alpha = 0.01 \qquad var(\hat{Y}_0) = 0.00002$

$\hat{Y}_i = 0.4279898 + 0.031338\, x_i$

$\hat{Y}_0 = 0.4279898 + 0.0031338 \times 50$

$\hat{Y}_0 = 0.5847$

$\hat{se}_{\hat{Y}_0} = \sqrt{Var(\hat{Y}_0)} = \sqrt{0.00002} = 4.4721 \times 10^{-3}$

$t_{\alpha/2} = t_{0.005} \qquad df = 27{,}885 \qquad = 2.576$

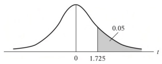
Appendix D *Statistical Tables* 879

Example

Pr(t > 2.086) = 0.025

Pr(t > 1.725) = 0.05 for df = 20

Pr(|t| > 1.725) = 0.10



Pr / df	0.25 0.50	0.10 0.20	0.05 0.10	0.025 0.05	0.01 0.02	0.005 0.010	0.001 0.002
1	1.000	3.078	6.314	12.706	31.821	63.657	318.31
2	0.816	1.886	2.920	4.303	6.965	9.925	22.327
3	0.765	1.638	2.353	3.182	4.541	5.841	10.214
4	0.741	1.533	2.132	2.776	3.747	4.604	7.173
5	0.727	1.476	2.015	2.571	3.365	4.032	5.893
6	0.718	1.440	1.943	2.447	3.143	3.707	5.208
7	0.711	1.415	1.895	2.365	2.998	3.499	4.785
8	0.706	1.397	1.860	2.306	2.896	3.355	4.501
9	0.703	1.383	1.833	2.262	2.821	3.250	4.297
10	0.700	1.372	1.812	2.228	2.764	3.169	4.144
11	0.697	1.363	1.796	2.201	2.718	3.106	4.025
12	0.695	1.356	1.782	2.179	2.681	3.055	3.930
13	0.694	1.350	1.771	2.160	2.650	3.012	3.852
14	0.692	1.345	1.761	2.145	2.624	2.977	3.787
15	0.691	1.341	1.753	2.131	2.602	2.947	3.733
16	0.690	1.337	1.746	2.120	2.583	2.921	3.686
17	0.689	1.333	1.740	2.110	2.567	2.898	3.646
18	0.688	1.330	1.734	2.101	2.552	2.878	3.610
19	0.688	1.328	1.729	2.093	2.539	2.861	3.579
20	0.687	1.325	1.725	2.086	2.528	2.845	3.552
21	0.686	1.323	1.721	2.080	2.518	2.831	3.527
22	0.686	1.321	1.717	2.074	2.508	2.819	3.505
23	0.685	1.319	1.714	2.069	2.500	2.807	3.485
24	0.685	1.318	1.711	2.064	2.492	2.797	3.467
25	0.684	1.316	1.708	2.060	2.485	2.787	3.450
26	0.684	1.315	1.706	2.056	2.479	2.779	3.435
27	0.684	1.314	1.703	2.052	2.473	2.771	3.421
28	0.683	1.313	1.701	2.048	2.467	2.763	3.408
29	0.683	1.311	1.699	2.045	2.462	2.756	3.396
30	0.683	1.310	1.697	2.042	2.457	2.750	3.385
40	0.681	1.303	1.684	2.021	2.423	2.704	3.307
60	0.679	1.296	1.671	2.000	2.390	2.660	3.232
120	0.677	1.289	1.658	1.980	2.358	2.617	3.160
∞	0.674	1.282	1.645	1.960	2.326	$\boxed{2.576}$	3.090

Note: The smaller probability shown at the head of each column is the area in one tail; the larger probability is the area in both tails.

$Pr\left(\hat{Y}_0 - \left(t_{\frac{\alpha}{2}} \cdot se_{\hat{Y}_0}\right) \leq Y_0 \leq \hat{Y}_0 + \left(t_{\frac{\alpha}{2}} \cdot se_{\hat{Y}_0}\right)\right) = 1-\alpha$

$Pr\left(0.5847 - \left(2.576 \times 4.4721 \times 10^{-3}\right) \leq Y_0 \leq 0.5847 + \left(2.576 \times 4.4721 \times 10^{-3}\right)\right) = 0.99$

$Pr\left(0.5732 \leq Y_0 \leq 0.5962\right) = 0.99$

at 99% confidence Y_0 is between 0.5732 and 0.5962

Thirada 6304647662

3. Mean prediction is providing that mean estimation follows this equation $\hat{Y}_0 = \hat{\beta}_1 + \hat{\beta}_2 X_0$ when $\hat{Y}_0$ is an estimator of $E(Y|X_0)$ while X_0 represents a value of interest.

Individual prediction is contrast to the mean prediction, which estimates the variance around Y_0, individual prediction focuses on forecasting error (fe). $fe = \hat{Y}_0 - Y_0$ Therefore, we define the variance of this fe as $var(fe) = var(\hat{Y}_0 - Y) = \sigma^2 \left[1 + \frac{1}{n} + \frac{(X_0 - X)^2}{\Sigma (X_i - \bar{X})^2} \right]$

Similarly, replacing the unknown σ^2 with the unbiased estimator $\hat{\sigma}^2$, we can derive the CI for Y_0 corresponding to X_0. $Pr\left[\hat{Y}_0 - (t_{\frac{\alpha}{2}} \cdot se_{fe}) \leq Y_0 \leq \hat{Y}_0 + (t_{\frac{\alpha}{2}} \cdot se_{fe}) \right] = 1 - \alpha$

if $X_0 - \bar{X}$ is largers so the variance is larger and $se(\hat{Y}_0)$ also increase ($se(\hat{Y}_0) = \sqrt{var(\hat{Y}_0)}$).

The big variance make the data lower confidence.