Assignment 3
Solution Guideline

From the data set `assign3.dta`:

The study of production function assumes the model follows CES production function:

$$y = \gamma\left[\delta K^{-\rho} + (1-\delta)L^{-\rho}\right]^{-\frac{\nu}{\rho}} + \varepsilon \qquad (1)$$

where:

y = Output
K = Capital Input
L = Labor Input
γ = Efficiency Parameter
δ = Distribution Parameter
ν = Parameter
ρ = Substitution Parameter
ε = Disturbance Term

Elasticity of Substitution can be computed from: $\sigma = \dfrac{1}{1-\rho}$

The model can then be transformed as:

$$\ln y = \ln\gamma - \frac{\nu}{\rho}\ln\left[\delta K^{-\rho} + (1-\delta)L^{-\rho}\right] + \varepsilon \qquad (2)$$

a. Estimate the model (1) using NLS estimation method using initial values of γ=4, δ=0, ρ= –1, ν=1. Determine the estimated value of efficiency parameter (γ), distribution parameter (δ), parameter (ν), and substitution parameter (ρ), and elasticity of substitution (σ). Perform F-test to test whether δ=0, ρ=0, and ν=0.

```
. nl (y = {gamma}*(({delta}*k^(-{rho})+(1-{delta})*l^(-{rho})))^(-({nu}/{rho}))),
init(gamma 4 delta 0 rho -1 nu 1)
(obs = 800)

Iteration 0:  residual SS =  2.35e+20
...
Iteration 357:  residual SS =  9.27e+18

      Source |       SS           df       MS
-------------+----------------------------------    Number of obs =        800
       Model |  2.363e+20          4  5.9080e+19    R-squared     =     0.9622
    Residual |  9.271e+18        796  1.1647e+16    Adj R-squared =     0.9621
-------------+----------------------------------    Root MSE      =   1.08e+08
       Total |  2.456e+20        800  3.0699e+17    Res. dev.     =   31861.36

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      /gamma |   2.89e-17   6.45e-17     0.45   0.654    -9.77e-17    1.55e-16
      /delta |    .801761   .0319332    25.11   0.000     .7390776    .8644443
        /rho |   -1.88296   .2889716    -6.52   0.000    -2.450197   -1.315724
         /nu |    3.39589   .1261607    26.92   0.000     3.148243    3.643537
------------------------------------------------------------------------------
```

```
. est store nls1

. sca sigma=1/(1-(_b[/rho]))

. sca list sigma
     sigma =  .34686566

. test (_b[/nu]=0) (_b[/rho]=0) (_b[/delta]=0)

 ( 1)  [nu]_cons = 0
 ( 2)  [rho]_cons = 0
 ( 3)  [delta]_cons = 0

       F(  3,   796) = 7383.35
            Prob > F =    0.0000
```

According to the above estimated results, estimated value of γ=2.89e-17, δ=0.8018, ρ= –1.8830, ν=3.3959. F-test whether δ=0, ρ=0, and ν=0 with p-value<0.05, the null hypothesis is rejected.

b. Estimate the model (2) using NLS estimation method initial values of $\ln\gamma$=4, δ=0, ρ= –1, ν=1. Determine the estimated value of efficiency parameter (γ), distribution parameter (δ), parameter (ν), and substitution parameter (ρ), and elasticity of substitution (σ). Perform F-test to test whether δ=0, ρ=0, and ν=0.

```
. nl (lny = {lngamma}-({nu}/{rho})*ln({delta}*k^(-{rho})+(1-{delta})*l^(-
{rho}))), init(lngamma 4 delta 0 rho -1 nu 1)
(obs = 800)

Iteration 0:  residual SS =  1143.768
...
Iteration 16:  residual SS =  819.9925

      Source |       SS           df       MS
-------------+----------------------------------    Number of obs =        800
       Model |  976.20485          3  325.401616    R-squared     =     0.5435
    Residual |  819.99246        796  1.03014129    Adj R-squared =     0.5418
-------------+----------------------------------    Root MSE      =   1.014959
       Total |  1796.1973        799  2.24805671    Res. dev.     =   2290.048

------------------------------------------------------------------------------
         lny |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    /lngamma |   3.798801   .4234183     8.97   0.000     2.967652    4.629949
         /nu |   .9727292   .0317774    30.61   0.000     .9103518    1.035107
        /rho |  -.8079827   .2186238    -3.70   0.000     -1.23713   -.3788354
      /delta |   .2351025   .0328464     7.16   0.000     .1706266    .2995784
------------------------------------------------------------------------------
Parameter lngamma taken as constant term in model & ANOVA table

. est store lognls1

. sca sigma=1/(1-(_b[/rho]))

. sca list sigma
     sigma =  .55310264

. test (_b[/nu]=0) (_b[/rho]=0) (_b[/delta]=0)

 ( 1)  [nu]_cons = 0
 ( 2)  [rho]_cons = 0
 ( 3)  [delta]_cons = 0
```

```
F(  3,    796) =  366.55
     Prob > F =    0.0000
```

According to the above estimated results, estimated value of lnγ=3.7988, δ=0.2351, ρ= –0.8080, ν=0.9727. F-test whether δ=0, ρ=0, and ν=0 with p-value<0.05, the null hypothesis is rejected.

c. From (a), if we change initial values to be γ=55, δ=0, ρ= –1, ν=1, what will happen to the estimated result? Interpret the estimated result and why do we get this kind of result? (Make comparison between previous result and the new result)

```
. nl (y = {gamma}*(({delta}*k^(-{rho})+(1-{delta})*l^(-{rho})))^(-({nu}/{rho}))),
init(gamma 55 delta 0 rho -1 nu 1)
(obs = 800)

Iteration 0:  residual SS =  1.65e+20
...
Iteration 19:  residual SS =  1.35e+20

      Source |       SS            df       MS
-------------+----------------------------------    Number of obs =        800
       Model |  1.105e+20           4  2.7613e+19    R-squared     =     0.4497
    Residual |  1.351e+20         796  1.6977e+17    Adj R-squared =     0.4470
-------------+----------------------------------    Root MSE      =   4.12e+08
       Total |  2.456e+20         800  3.0699e+17    Res. dev.     =   34004.87

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      /gamma |   15.20637   9.357021     1.63   0.105    -3.160981    33.57372
      /delta |   .0547786   2.018277     0.03   0.978    -3.906995    4.016552
        /rho |  -38.85704   533.1722    -0.07   0.942    -1085.447    1007.733
         /nu |   1.063633   .0348958    30.48   0.000     .9951345    1.132132
------------------------------------------------------------------------------

. est store nls2

. est table nls1 nls2, star(.1 .05 .01) stat(N rss r2 r2_a)

----------------------------------------------
    Variable |     nls1             nls2
-------------+--------------------------------
gamma        |
       _cons |  2.889e-17       15.206371
-------------+--------------------------------
delta        |
       _cons |  .80176096***    .05477862
-------------+--------------------------------
rho          |
       _cons | -1.8829605***   -38.857043
-------------+--------------------------------
nu           |
       _cons |  3.3958902***    1.0636331***
-------------+--------------------------------
Statistics   |
           N |        800             800
         rss |  9.271e+18       1.351e+20
          r2 |  .96224959       .44974701
        r2_a |  .96205989       .44698192
----------------------------------------------
           legend: * p<.1; ** p<.05; *** p<.01
```

According to the above estimated results, it indicated that using different sets of initial values can give the different estimated results.

d. From (a), if we change convergence value from default of 0.00001 or (1e-5) to (i) 0.1 or (1e-1) and (ii) (1e-15) with maximum iteration of 100, what will happen to the estimated result? Interpret the estimated result and why do we get this kind of result? (Make comparison between previous result and the new result)

(i) Convergence value of 0.1 or (1e-1)

```
. nl (y = {gamma}*(({delta}*k^(-{rho})+(1-{delta})*l^(-{rho})))^(-
({nu}/{rho}))), init(gamma 4 delta 0 rho -1 nu 1) eps(1e-1)
(obs = 800)

Iteration 0:  residual SS =  2.35e+20
Iteration 1:  residual SS =  2.35e+20

      Source |       SS           df       MS
-------------+----------------------------------    Number of obs =        800
       Model |  1.043e+19          4  2.6070e+18    R-squared     =     0.0425
    Residual |  2.352e+20        796  2.9543e+17    Adj R-squared =     0.0376
-------------+----------------------------------    Root MSE      =   5.44e+08
       Total |  2.456e+20        800  3.0699e+17    Res. dev.     =   34448.06

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      /gamma |   4.000008   15.18432     0.26   0.792    -25.80603    33.80605
      /delta |   1.29e-07   .0001603     0.00   0.999    -.0003145    .0003148
        /rho |   -29.0199   2406.782    -0.01   0.990     -4753.41    4695.371
         /nu |          1   .2148918     4.65   0.000     .5781783    1.421822
------------------------------------------------------------------------------

. est store nls3
```

(ii) Convergence value of 1e-15

```
. nl (y = {gamma}*(({delta}*k^(-{rho})+(1-{delta})*l^(-{rho})))^(-
({nu}/{rho}))), init(gamma 4 delta 0 rho -1 nu 1) eps(1e-15) iter(100)
(obs = 800)

Iteration 0:  residual SS =  2.35e+20
...
Iteration 99:  residual SS =  2.35e+20


      Source |       SS           df       MS
-------------+----------------------------------    Number of obs =        800
       Model |  1.044e+19          4  2.6109e+18    R-squared     =     0.0425
    Residual |  2.351e+20        796  2.9541e+17    Adj R-squared =     0.0377
-------------+----------------------------------    Root MSE      =   5.44e+08
       Total |  2.456e+20        800  3.0699e+17    Res. dev.     =   34448.01

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      /gamma |   4.002508    14.7236     0.27   0.786    -24.89917    32.90419
      /delta |   .0000247    .007176     0.00   0.997    -.0140615    .0141109
        /rho |   -19.1599   594.6246    -0.03   0.974    -1186.378    1148.058
         /nu |   .9999954   .2082142     4.80   0.000     .5912816    1.408709
------------------------------------------------------------------------------
convergence not achieved
```

```
r(430);

. est store nls4

. est table nls1 nls3 nls4, star(.1 .05 .01) stat(N rss r2 r2_a)

-------------------------------------------------------------
    Variable |     nls1            nls3            nls4
-------------+-----------------------------------------------
gamma        |
       _cons |  2.889e-17       4.0000078       4.0025082
-------------+-----------------------------------------------
delta        |
       _cons |  .80176096***    1.286e-07       .00002472
-------------+-----------------------------------------------
rho          |
       _cons | -1.8829605***   -29.019904        -19.1599
-------------+-----------------------------------------------
nu           |
       _cons |  3.3958902***            1***    .99999535***
-------------+-----------------------------------------------
Statistics   |
           N |        800             800             800
         rss |  9.271e+18       2.352e+20       2.351e+20
          r2 |  .96224959       .04246041       .04252446
        r2_a |  .96205989       .03764866       .03771302
-------------------------------------------------------------
                           legend: * p<.1; ** p<.05; *** p<.01
```

According to the above estimated results, it indicated that using different convergence values can give the different estimated results.

e. From (b), if we change initial values to be lnγ=55, δ=0, ρ= –1, ν=1, what will happen to the estimated result? Interpret the estimated result and why do we get this kind of result? (Make comparison between previous result and the new result)

```
. nl (lny = {lngamma}-({nu}/{rho})*ln({delta}*k^(-{rho})+(1-{delta})*l^(-
{rho}))), init(lngamma 55 delta 0 rho -1 nu 1)
(obs = 800)

Iteration 0:  residual SS =   2124801
...
Iteration 22:  residual SS =  819.9925

      Source |       SS           df       MS
-------------+----------------------------------    Number of obs =        800
       Model |  976.20485          3  325.401616    R-squared     =     0.5435
    Residual |  819.99246        796  1.03014129    Adj R-squared =     0.5418
-------------+----------------------------------    Root MSE      =   1.014959
       Total |  1796.1973        799  2.24805671    Res. dev.     =   2290.048

------------------------------------------------------------------------------
         lny |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    /lngamma |     3.7988   .4234183     8.97   0.000     2.967652    4.629949
         /nu |   .9727292   .0317774    30.61   0.000     .9103518    1.035107
        /rho |  -.8079832   .2186233    -3.70   0.000    -1.237129   -.3788369
      /delta |   .2351025   .0328464     7.16   0.000     .1706267    .2995783
------------------------------------------------------------------------------
  Parameter lngamma taken as constant term in model & ANOVA table

. est store lognls2
```

```
. est table lognls1 lognls2, star(.1 .05 .01) stat(N rss r2 r2_a)

----------------------------------------------
    Variable |    lognls1         lognls2     
-------------+--------------------------------
lngamma      |
       _cons |  3.7988005***    3.7988004***  
-------------+--------------------------------
nu           |
       _cons |  .97272923***    .97272923***  
-------------+--------------------------------
rho          |
       _cons | -.80798268***   -.80798318***  
-------------+--------------------------------
delta        |
       _cons |  .23510249***    .23510246***  
-------------+--------------------------------
Statistics   |
           N |        800             800     
         rss |  819.99246       819.99246     
          r2 |  .54348419       .54348419     
        r2_a |  .54176366       .54176366     
----------------------------------------------
           legend: * p<.1; ** p<.05; *** p<.01
```

According to the above estimated results, it indicated that using log-form can lead to more robust estimated results.

f. From (b), if we change convergence value from default of 0.00001 or (1e-5) to (i) 0.1 or (1e-1) and (ii) (1e-15) with maximum iteration of 100, what will happen to the estimated result? Interpret the estimated result and why do we get this kind of result? (Make comparison between previous result and the new result)

(i) Convergence value of 0.1 or (1e-1)

```
. nl (lny = {lngamma}-({nu}/{rho})*ln({delta}*k^(-{rho})+(1-{delta})*l^(-
{rho}))), init(lngamma 4 delta 0 rho -1 nu 1) eps(1e-1) iter(100)
(obs = 800)

Iteration 0:  residual SS =  1143.768
...
Iteration 10:  residual SS =   819.993


      Source |       SS           df       MS
-------------+----------------------------------    Number of obs =        800
       Model |   976.2043          3  325.401435    R-squared     =     0.5435
    Residual |  819.99301        796  1.03014197    Adj R-squared =     0.5418
-------------+----------------------------------    Root MSE      =   1.014959
       Total |  1796.1973        799  2.24805671    Res. dev.     =   2290.049

------------------------------------------------------------------------------
         lny |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    /lngamma |   3.799766    .423419     8.97   0.000     2.968616    4.630915
         /nu |   .9727009   .0317765    30.61   0.000     .9103252    1.035077
        /rho |  -.8036163   .2223873    -3.61   0.000    -1.240151   -.3670814
      /delta |   .2354804   .0332066     7.09   0.000     .1702977    .3006632
------------------------------------------------------------------------------
  Parameter nu taken as constant term in model & ANOVA table

. est store lognls3
```

(ii) Convergence value of 1e-15

```
. nl (lny = {lngamma}-({nu}/{rho})*ln({delta}*k^(-{rho})+(1-{delta})*l^(-
{rho}))), init(lngamma 4 delta 0 rho -1 nu 1) eps(1e-15) iter(100)
(obs = 800)

Iteration 0:  residual SS =  1143.768
...
Iteration 23:  residual SS =  819.9925


      Source |       SS           df       MS
-------------+----------------------------------    Number of obs =        800
       Model |  222930.73          4  55732.6813    R-squared     =     0.9963
    Residual |  819.99246        796  1.03014129    Adj R-squared =     0.9963
-------------+----------------------------------    Root MSE      =   1.014959
       Total |  223750.72        800  279.688397    Res. dev.     =   2290.048

------------------------------------------------------------------------------
         lny |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    /lngamma |     3.7988   .4234183     8.97   0.000     2.967652    4.629949
         /nu |   .9727292   .0317774    30.61   0.000     .9103518    1.035107
        /rho |   -.807984   .2186225    -3.70   0.000    -1.237129   -.3788392
      /delta |   .2351024   .0328463     7.16   0.000     .1706267    .2995781
------------------------------------------------------------------------------

. est store lognls4

. est table lognls1 lognls3 lognls4, star(.1 .05 .01) stat(N rss r2 r2_a)

--------------------------------------------------------------
    Variable |    lognls1          lognls3          lognls4
-------------+------------------------------------------------
lngamma      |
       _cons |  3.7988005***    3.7997656***    3.7988003***
-------------+------------------------------------------------
nu           |
       _cons |  .97272923***    .97270091***    .97272924***
-------------+------------------------------------------------
rho          |
       _cons | -.80798268***   -.80361634***   -.80798395***
-------------+------------------------------------------------
delta        |
       _cons |  .23510249***    .23548043***     .2351024***
-------------+------------------------------------------------
Statistics   |
           N |        800             800             800
         rss |  819.99246       819.99301       819.99246
          r2 |  .54348419       .54348389       .99633524
        r2_a |  .54176366       .54176335       .99631683
--------------------------------------------------------------
                          legend: * p<.1; ** p<.05; *** p<.01
```

According to the above estimated results, it indicated that using log-form can lead to more robust estimated results.

g. Make comparison of the estimated results of model (1) and model (2). Which estimated result should be used in this case? Why?

```
. est table nls1 nls2 nls3 nls4, star(.1 .05 .01) stat(N rss r2 r2_a)

------------------------------------------------------------------------------
    Variable |     nls1             nls2             nls3             nls4
```

```
-------------+----------------------------------------------------------------
gamma        |
       _cons |  2.889e-17       15.206371       4.0000078       4.0025082
-------------+----------------------------------------------------------------
delta        |
       _cons |  .80176096***    .05477862       1.286e-07       .00002472
-------------+----------------------------------------------------------------
rho          |
       _cons | -1.8829605***   -38.857043      -29.019904        -19.1599
-------------+----------------------------------------------------------------
nu           |
       _cons |  3.3958902***    1.0636331***            1***    .99999535***
-------------+----------------------------------------------------------------
Statistics   |
           N |        800             800             800             800
         rss |  9.271e+18       1.351e+20       2.352e+20       2.351e+20
          r2 |  .96224959       .44974701       .04246041       .04252446
        r2_a |  .96205989       .44698192       .03764866       .03771302
------------------------------------------------------------------------------
                                           legend: * p<.1; ** p<.05; *** p<.01

. est table lognls1 lognls2 lognls3 lognls4, star(.1 .05 .01) stat(N rss r2
r2_a)

------------------------------------------------------------------------------
    Variable |   lognls1         lognls2         lognls3         lognls4
-------------+----------------------------------------------------------------
lngamma      |
       _cons |  3.7988005***    3.7988004***    3.7997656***    3.7988003***
-------------+----------------------------------------------------------------
nu           |
       _cons |  .97272923***    .97272923***    .97270091***    .97272924***
-------------+----------------------------------------------------------------
rho          |
       _cons | -.80798268***   -.80798318***   -.80361634***   -.80798395***
-------------+----------------------------------------------------------------
delta        |
       _cons |  .23510249***    .23510246***    .23548043***     .2351024***
-------------+----------------------------------------------------------------
Statistics   |
           N |        800             800             800             800
         rss |  819.99246       819.99246       819.99301       819.99246
          r2 |  .54348419       .54348419       .54348389       .99633524
        r2_a |  .54176366       .54176366       .54176335       .99631683
------------------------------------------------------------------------------
                                           legend: * p<.1; ** p<.05; *** p<.01
```

According to the above estimated results, it indicated that log-form (model 2) should be employed since it leads to more robust estimated results.