

Chapter 17 Long-Run Production

Long-Run Production: Production in the time frame long enough to change every input \Rightarrow No Fixed Cost

- In Long-Run, when output $Q = 0$, Total Cost = 0.

A firm in Long-Run faces two possible problems:

subject to

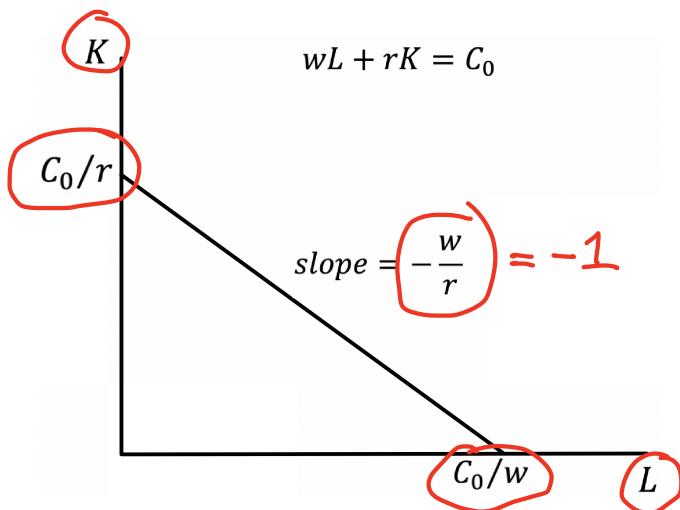
1. $\begin{cases} \max Q = f(L, K) \\ \text{St. } wL + rK = C_0 \end{cases}$ *output.*
2. $\begin{cases} \min C = wL + rK \\ \text{St. } f(L, K) = Q_0 \end{cases}$ *cost*

where $f(L, K)$ is the production function
 w is wage, the price of labor L
 r is interest, the price of capital K

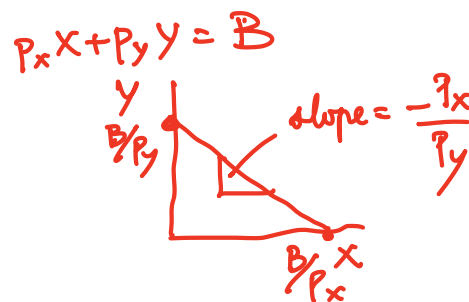
Just like Budget Line and Indifference Curve in the consumption problem, we have Isocost and Isoquant in production problem.

Isocost Given cost level C_0 , $wL + rK = C_0$ is a line called Isocost where every point on this line is the amount of L and K that cost the firm the same cost C_0 .

- Isocost is similar to the Budget Line as given below:

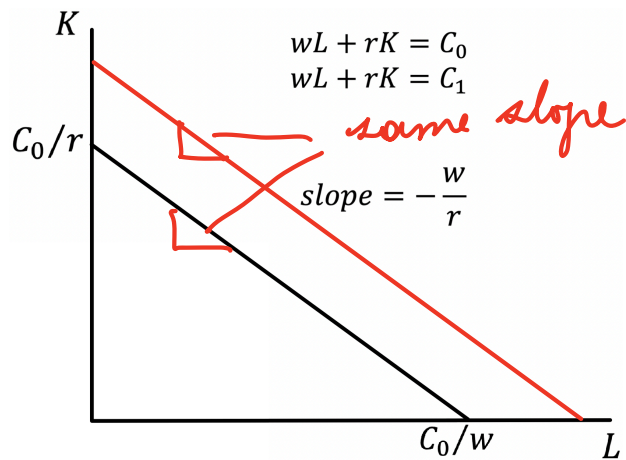


$\text{slope} = -\frac{w}{r} = \text{relative price of } L \text{ in terms of } K$
 $= \text{exchange rate between } L \text{ and } K \text{ in the market}$

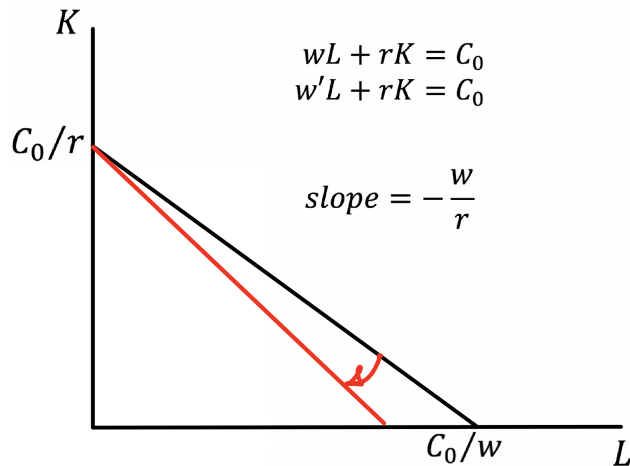


Changes in Isocost

1. C_0 increases to C_1 $C_1 > C_0$

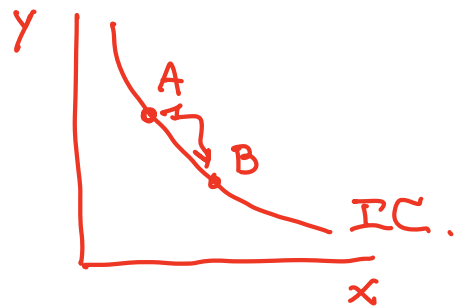
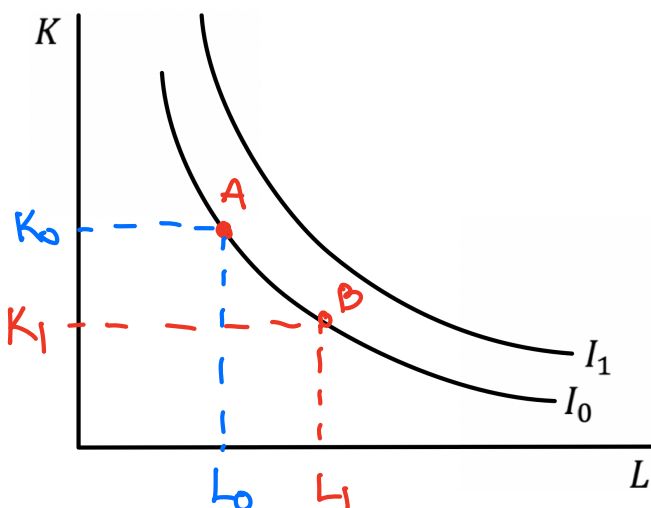


2. w increases to w' $w' > w$



Isoquant Given output level Q_0 , $Q_0 = f(L, K)$ is a line called Isoquant where every point on this line is the amounts of L and K that produce the same output Q_0 .

- Isoquant is similar to Indifference Curve



$f(L_0, K_0) = Q_0$
 $f(L_1, K_1) = Q_0$

Properties of Isoquant

1. For each point of $A = (L_0, K_0)$, there is exactly one Isoquant passing through it. That is, no two Isoquants can intersect nor be tangent to each other.
2. There are infinite number of Isoquants, each never intersects nor is tangent to another.
3. Higher Isoquant means higher output Q .
4. Each Isoquant always has negative slope—called Marginal Rate of Technical Substitution ($MRTS$). At point $A = (L_0, K_0)$, slope of the Isoquant is

$$MRTS = -\frac{MP_L(L_0, K_0)}{MP_K(L_0, K_0)}$$

$MRTS$ is the **instantaneous exchange rate** between L and K **inside the firm** such that output Q is unchanged.

5. **Diminishing $MRTS$** On a given Isoquant, as the use of x increases, the value of $|MRTS|$ decrease.

Production Equilibrium As mentioned in Chapter 15 Production: How a firm employs inputs (labor L and capital K) to

1. Maximize output Q for a given cost level C_0 , or
2. Minimize cost C to produce a give level of output Q_0 .

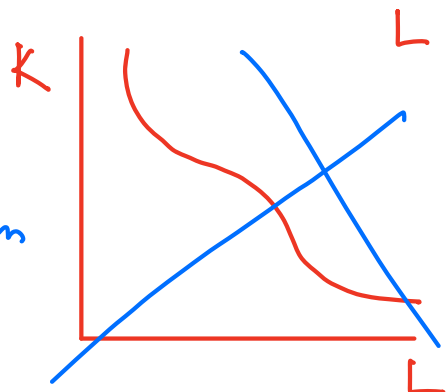
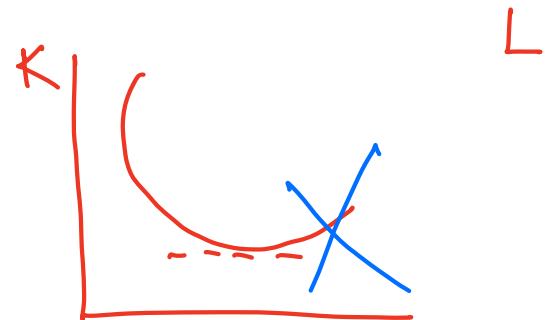
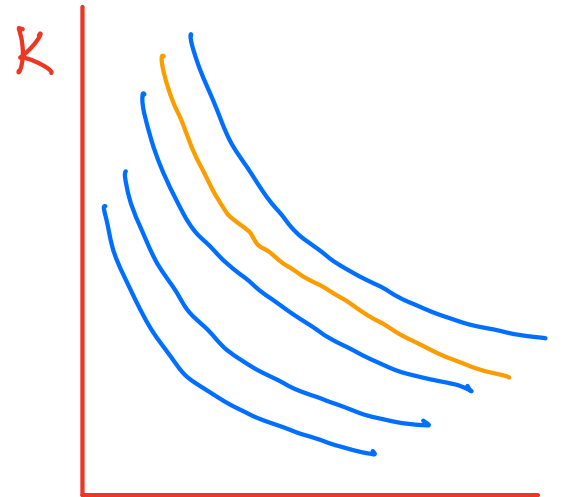
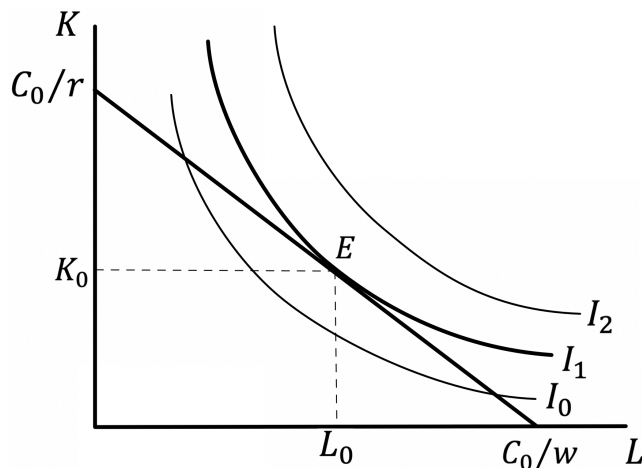
Thus, there are two possible cases of productions equilibrium.

1. **Maximization of Output Q :** *max output, given the cost.*

$$\begin{aligned} \max Q &= f(L, K) \\ \text{st. } wL + rK &= C_0 \end{aligned}$$

The equilibrium is at $E = (L_0, K_0)$ because

- 1) E is on the Isocost $wL + rK = C_0$ line— E is feasible.
- 2) E is on the highest Isoquant (I_1) the producer can reach.



At the equilibrium point E , the Isoquant curve I_1 is tangent to the Isocost line $wL + rK = C_0$.

Equilibrium Conditions: Maximization Output The firm has production equilibrium of maximizing Q at $E = (L_0, K_0)$ with the following conditions:

- 1) $wL_0 + rK_0 = C_0 \Leftrightarrow E$ is feasible, *- because E is on the Isocost!*
- 2) $MRTS_{(at E)} = \frac{MP_L(L_0, K_0)}{MP_K(L_0, K_0)} = \frac{w}{r}$, or equivalently
 $\frac{MP_L(L_0, K_0)}{w} = \frac{MP_K(L_0, K_0)}{r}$.

The condition (2) means that

$$\left. \begin{array}{l} \text{Exchange rate between} \\ L \text{ and } K \\ \text{in the firm} \end{array} \right\} = \left\{ \begin{array}{l} \text{Exchange rate between} \\ L \text{ and } K \\ \text{in the market} \end{array} \right.$$

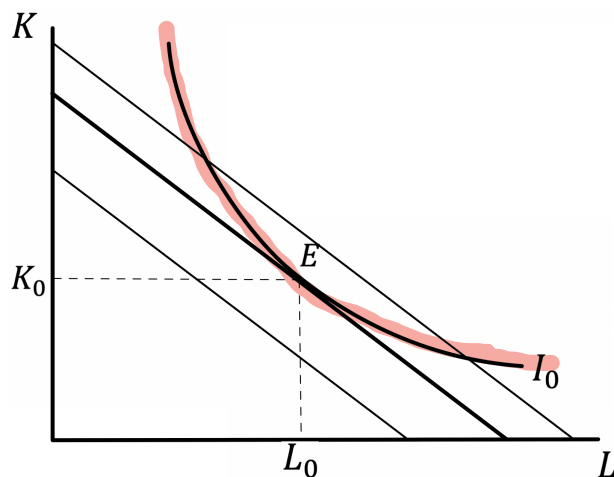
Or equivalently, the output from the last baht spent on L is equal to that spent on K .

2. Minimization of Cost C :

$$\begin{aligned} \min C &= wl + rK \\ \text{st. } f(L, K) &= Q_0 \end{aligned}$$

The equilibrium is at $E = (L_0, K_0)$ because

- 1) E is on the Isoquant $f(L, K) = Q_0$ — E is feasible.
- 2) E is on the lowest Isocost ($Isocost_1$).



At the equilibrium point E , the Isoquant curve IC_0 is tangent to the Isocost line $wL + rK = C_0$.

Equilibrium Conditions: Minimizing Cost The firm has production equilibrium of maximizing Q at $E = (L_0, K_0)$ with the following conditions:

$f(L_0, K_0) = Q_0$

- 1) ~~$wL_0 + rK_0 = C_0$~~ $\Leftrightarrow E$ is feasible, - E is on the given Isoquant I_0
- 2) $MRTS_{(at E)} = \frac{MP_L(L_0, K_0)}{MP_K(L_0, K_0)} = \frac{w}{r}$, or equivalently
- $$\frac{MP_L(L_0, K_0)}{w} = \frac{MP_K(L_0, K_0)}{r}$$

The condition (2) means that

$$\left. \begin{array}{l} \text{Exchange rate between} \\ L \text{ and } K \\ \text{in the firm} \end{array} \right\} = \left\{ \begin{array}{l} \text{Exchange rate between} \\ L \text{ and } K \\ \text{in the market} \end{array} \right.$$

Or equivalently, the output from the last baht spent on L is equal to that spent on K .

Comparison of Consumption and Production Problems

| Consumption | Production |
|---|--|
| <p><u>Budget Line:</u> $p_x x + p_y y = B$</p> <p>Slope = $-\frac{p_x}{p_y}$</p> | <p><u>Isocost:</u> $wL + rK = C_0$</p> <p>Slope = $-\frac{w}{r}$</p> |
| <p><u>Indifference Curve:</u></p> <p>Slope = $-\frac{MU_x}{MU_y}$</p> | <p>Indifference Curve: Isoquant.</p> <p>Slope = $-\frac{MP_L}{MP_K}$</p> |
| <p><u>Equilibrium Conditions at $E = (x_0, y_0)$</u></p> <p>1) $p_x x_0 + p_y y_0 = B$</p> <p>2) $MRS_{(at E)} = \frac{MU_x(x_0, y_0)}{MU_y(x_0, y_0)} = \frac{p_x}{p_y}$</p> <p>To maximize Utility given $p_x x + p_y y = B$.</p> | <p><u>Equilibrium Conditions at $E = (L_0, K_0)$</u></p> <p><u>Maximization of Output:</u></p> <p>1) $wL_0 + rK_0 = C_0$ - spend the given cost C_0</p> <p>2) $MRTS_{(at E)} = \frac{MP_L(L_0, K_0)}{MP_K(L_0, K_0)} = \frac{w}{r}$ ✓</p> <p><u>Minimization of Cost:</u></p> <p>1) $f(L_0, K_0) = Q_0$ - produce the given output</p> <p>2) $MRTS_{(at E)} = \frac{MP_L(L_0, K_0)}{MP_K(L_0, K_0)} = \frac{w}{r}$ ✓</p> |

Consumption.

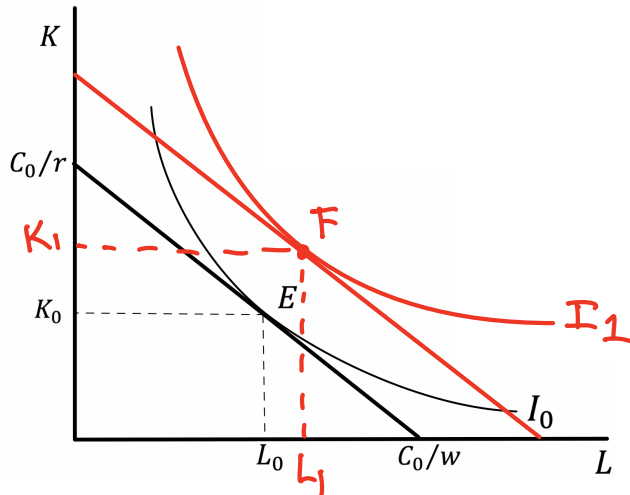
$\max u(x, y)$
 st. $p_x x + p_y y = B$.

Production.

- ① $\max Q = f(L, K)$
 st. $wL + rK = C_0$.
- ② $\min \text{Cost} = wL + rK$
 st. $f(L, K) = Q_0$

Change in Equilibrium: Maximization of Output

1) Cost C_0 increases to C_1

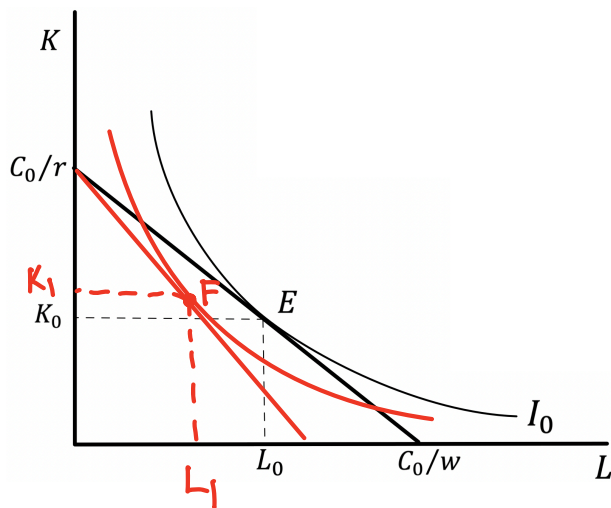


- Same as higher income in consumption case
- No discussion of Necessary/Luxury goods
- Equilibrium conditions: *at new equilibrium point $F = (L_1, K_1)$*

$$1) \omega L_1 + r K_1 = C_1$$

$$2) MRTS_F = \frac{MP_L(L_1, K_1)}{MP_K(L_1, K_1)} = \frac{\omega}{r}$$

2. Wage w increases to w'



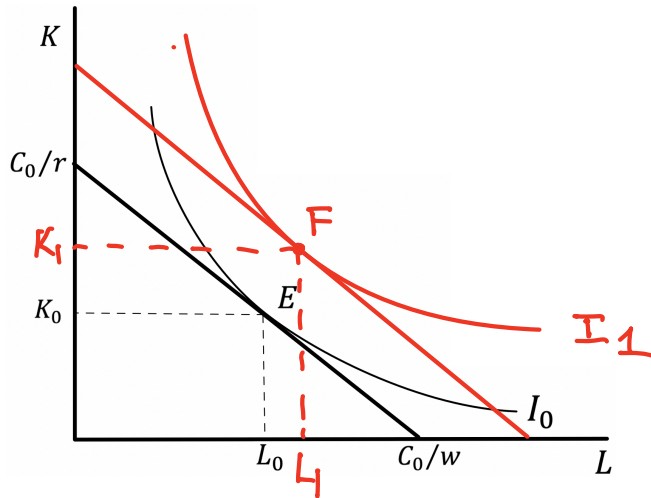
- Same as when p_x increases in consumption case
- No discussion of Substitution and Income Effects
- Equilibrium conditions: *at $F = (L_1, K_1)$*

$$1) \omega' L_1 + r K_1 = C_0$$

$$2) MRTS_F = \frac{MP_L(L_1, K_1)}{MP_K(L_1, K_1)} = \frac{\omega'}{r}$$

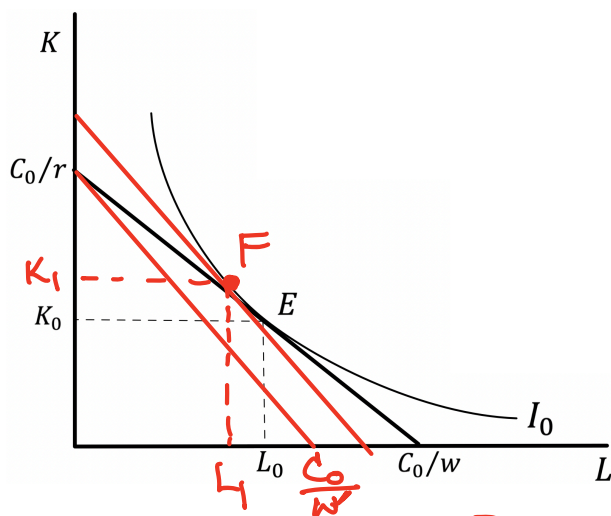
Change in Equilibrium: Minimization of Cost

1) Output Q_0 increases to Q_1



- Equilibrium conditions: **at F**
 - 1) $f(L_1, K_1) = Q_1$
 - 2) $MRTS_F = \frac{MP_L(L_1, K_1)}{MP_K(L_1, K_1)} = \frac{w}{r}$

2. Wage w increases to w'



- Equilibrium conditions: **at F.**
 - 1) $f(L_1, K_1) = Q_0$
 - 2) $MRTS_F = \frac{MP_L(L_1, K_1)}{MP_K(L_1, K_1)} = \frac{w'}{r}$

Production Expansion Path: a line whose every point is a (least cost) production equilibrium for a given output level at given fixed input prices of w and r .

Suppose $w = 30, r = 100$, the Isocost is given by

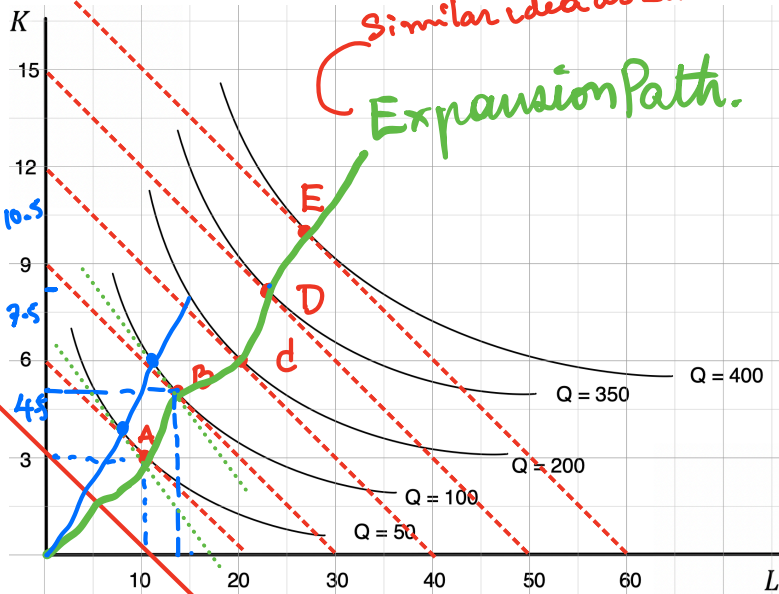
$$30L + 100K = C = 300.$$

$L=0, K=3$
 $K=0, L=10$

- We have the following equilibria at various levels of output $Q = 50, 100, 200, 350, 450, \text{etc.}$

Similar idea as Income Consumption Curve in Consumer's Problem

Expansion Path.



| | Q | $w=30$ L | $r=100$ K | Least. Cost |
|---|-----|---------------|----------------|----------------|
| A | 50 | 10 ✓ | 3 ✓ | 600 |
| B | 100 | 14 ? | 5 ? | 900 |
| C | 200 | 20 | 6 | 1200 |
| D | 350 | 23 ? | 8 ? | 1500 |
| E | 400 | 22 ? | 10 ? | 1800 |

- An alternative definition of Expansion Path: a line whose every point is a (maximum output) production equilibrium for a given cost level at given fixed input prices of w and r .

Expansion Path and Long-Run Total Cost

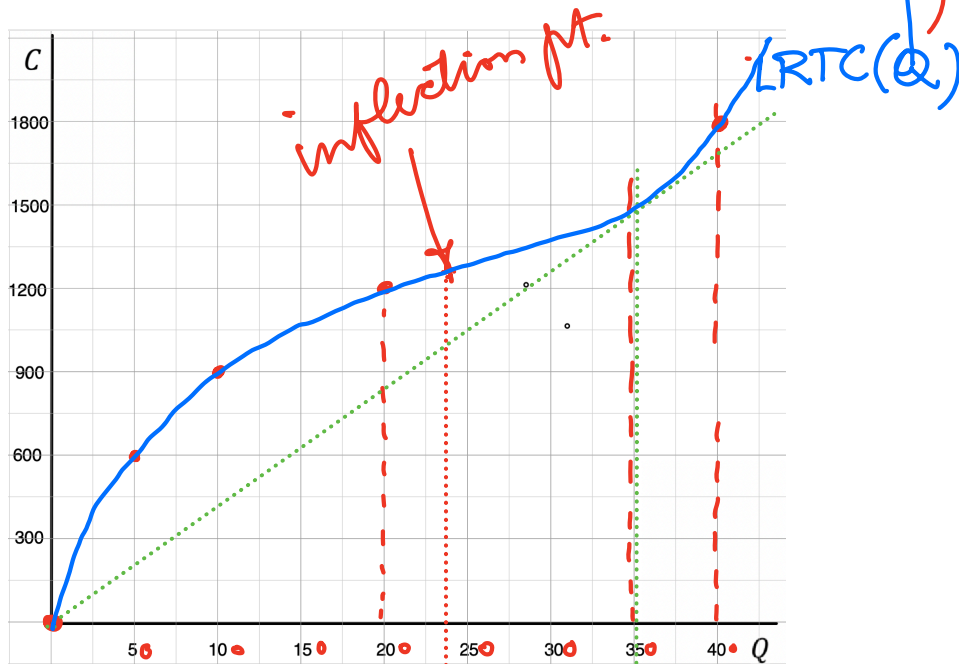
Long-Run Total Cost (LRTC) the least cost of producing a given output Q by the use of inputs (labor L and capital K) in the Long-Run.

- with fixed input prices (wage w and interest r)
- in the most efficient way (no unnecessary wastes)
- with the available best technology

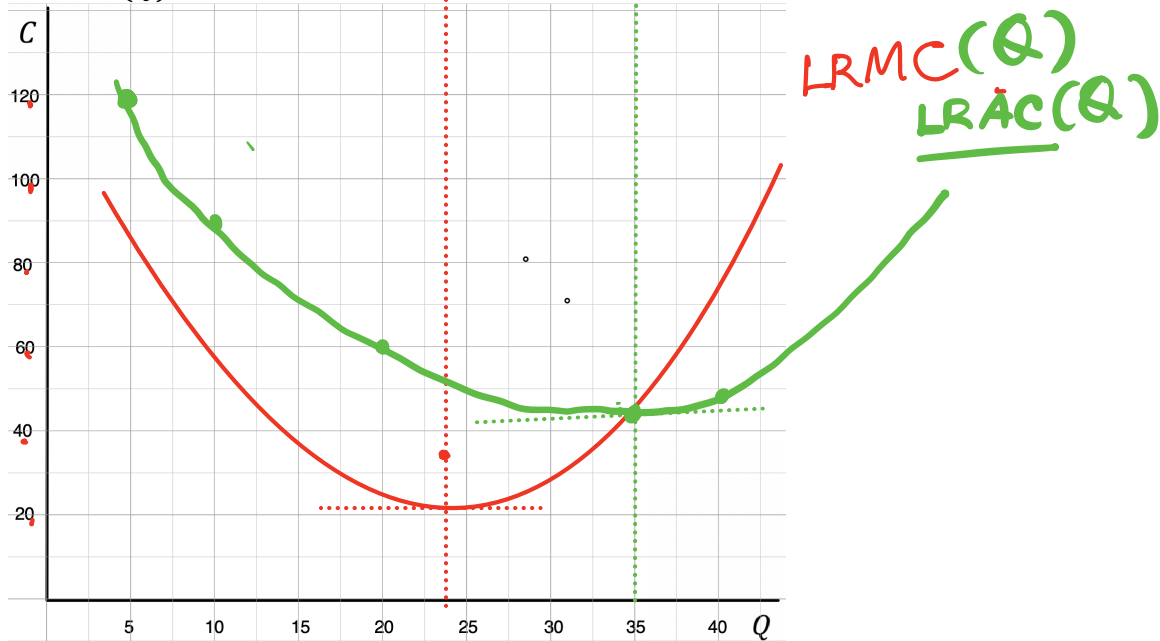
From the Expansion Path above, we have the following relationship between the output and the least cost in the Long Run,

| Q | $LRTC(Q)$ | $LRAC(Q)$ |
|-----|-----------|-----------|
| 0 | 0 | |
| 50 | 600 | 120 |
| 100 | 900 | 90 |
| 200 | 1200 | 60 ✓ |
| 350 | 1500 | 42.9 |
| 400 | 1800 | 45 |

| | Q | L | K | Least Cost |
|---|-----|------|-----|------------|
| A | 50 | 10 ✓ | 3 ✓ | 600 |
| B | 100 | 14? | 5? | 900 |
| C | 200 | 20 | 6 | 1200 |
| D | 350 | 23? | 8? | 1500 |
| E | 400 | 24? | 10? | 1800 |



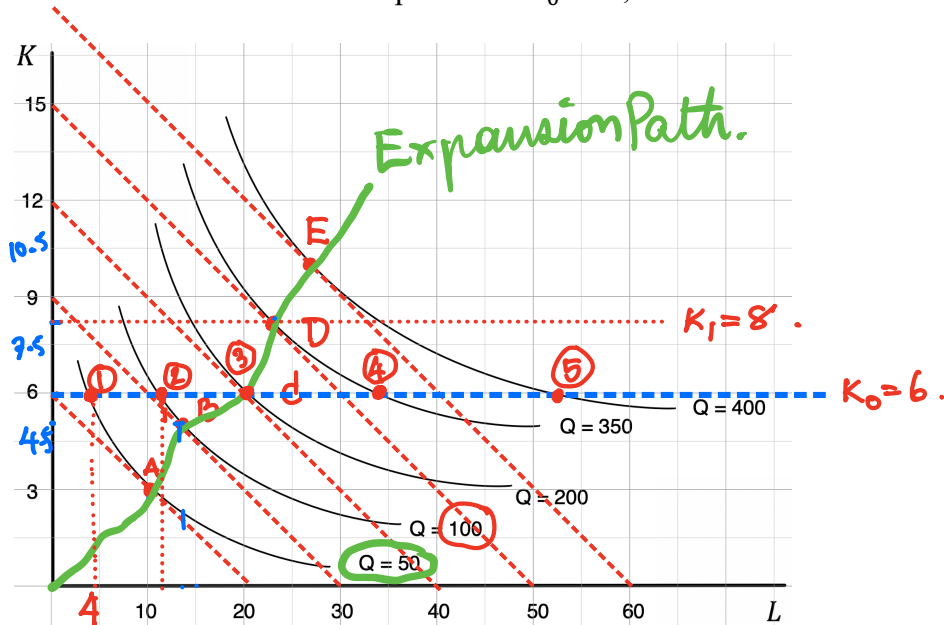
Given the $LRTC(Q)$, we can draw the $LRAC(Q)$ and $LRMC(Q)$



- Economies of scale— $LRAC$ is decreasing
- Diseconomies of scale— $LRAC$ is increasing

Relationship between $LRTC$ and $SRTC$

- Given the fixed factor is capital K at $K_0 = 6$,



- We have the following relationship of output Q and $SRTC(Q)$

$r=100$ $w=30$ 120
 600

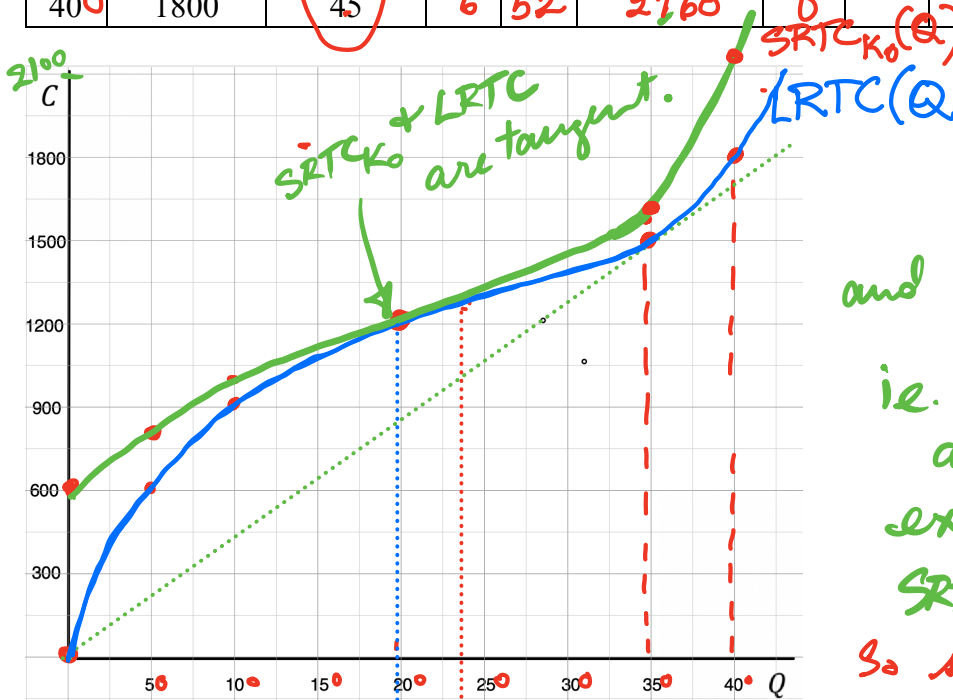
| Q | LRTC(Q) | LRAC(Q) | SR with $K_0 = 6$ | | | SR with $K_1 = 8$ | | |
|-----|---------|---------|-------------------|----|-----------------|-------------------|---|-----------------|
| | | | K | L | $SRTC_{K_0}(Q)$ | K | L | $SRTC_{K_1}(Q)$ |
| 0 | 0 | | 6 | 0 | 600 | | | |
| 50 | 600 | 120 | 6 | 4 | 720 | | | |
| 100 | 900 | 90 | 6 | 12 | 960 | | | |
| 200 | 1200 | 60 | 6 | 20 | 1200 | | | |
| 350 | 1500 | 42.9 | 6 | 34 | 1620 | | | |
| 400 | 1800 | 45 | 6 | 52 | 2160 | | | |

$600 + 34 \times 30 = 1620$
1020

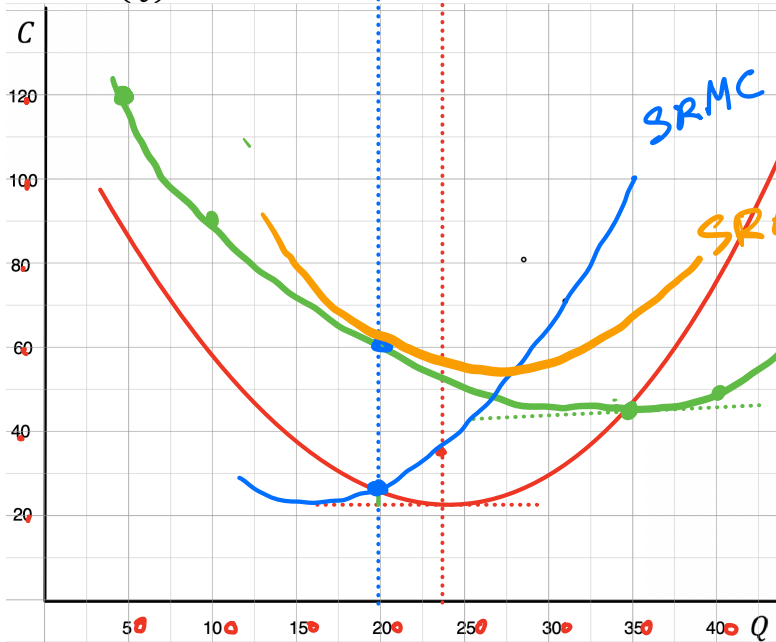
$6 \times 100 + 52 \times 30 = 2160$
1560

$r=100$
 $w=90$
120
600

| Q | LRTC(Q) | LRAC(Q) | SR with $K_0 = 6$ | | | SR with $K_1 = 8$ | | |
|-----|---------|---------|-------------------|----|-----------------|-------------------|---|-----------------|
| | | | K | L | $SRTC_{K_0}(Q)$ | K | L | $SRTC_{K_1}(Q)$ |
| 0 | 0 | | 6 | 0 | 600 | 8 | | |
| 50 | 600 | 120 | 6 | 4 | 720 | 8 | | |
| 100 | 900 | 90 | 6 | 12 | 960 | 8 | | |
| 200 | 1200 | 60 | 6 | 20 | 1200 | 8 | | |
| 350 | 1500 | 42.9 | 6 | 34 | 1620 | 8 | | 1500 ✓ |
| 400 | 1800 | 45 | 6 | 52 | 2160 | 8 | | |



Given the $SRTC(Q)$, we can draw the $SRAC(Q)$ and $SRMC(Q)$



$$\frac{SRTC_{K_0}(Q)}{Q} \geq \frac{LRTC(Q)}{Q}$$

$$\therefore SRAC_{K_0}(Q) \geq LRAC(Q)$$

at $Q=200$

- When $K_0 = 6$, $SRTC_{K_0}(Q) \geq LRTC(Q)$ at all output Q , and $Q = 20$ where $SRTC_{K_0}(20) = LRTC(20)$.
- Thus, $SRTC_{K_0}$ and $LRTC$ are tangent at $Q = 20$
 - \Rightarrow 1) $SRMC_{K_0}(20) = LRMC(20)$
 - 2) $SRAC_{K_0}(Q) \geq LRAC(Q)$
 - 3) $SRAC_{K_0}(20) = LRAC(20)$
- When $K_1 = 8$, $SRTC_{K_0}(Q) \geq LRTC(Q)$ at all output Q , and $Q = 35$ where $SRTC_{K_0}() = LRTC()$.
- Thus, $SRTC_{K_0}$ and $LRTC$ are tangent at $Q = 20$
 - \Rightarrow 1) $SRMC_{K_0}() = LRMC()$
 - 2) $SRAC_{K_0}(Q) \geq LRAC(Q)$
 - 3) $SRAC_{K_0}() = LRAC()$

- We can think of $LRTC$ as the lower envelope of all the possible $SRTC$, each at a given level of fixed factor K .
- At any point on $LRTC$, there is a $SRTC$ that is tangent to that point on $LRTC$.
- We can demonstrate the relationship of LR and SR cost curves in the following three cases.

