EE431: Economics of financial market and institutions

Semester 1/2017

Assignment# 1

Instructions:

1. Attempt all.
2. You have to write your answer with your handwriting.
3. Due date: Feb 15th 2017 at the BE office (before 15.00).
4. Late homework will not be accepted.

Question 1: Mean-Variance and project selection

Consider two projects with the required amount of initial investment of $100 million. Both projects are assumed to be indivisible. Financial cash flows of each project are summarized in the two tables below.

Project A

Probability	Receivable cash flow (million dollars)	Rate of return (%)
¼	80	**-20%**
¼	100	**0%**
¼	120	**20%**
¼	140	**40%**

Project B

Probability	Receivable cash flow (million dollars)	Rate of return (%)
½	90	-10%
½	130	30%

1.1) Define the rate of return as the net operating profit over the investment. Calculate the rate of under all possible outcomes.

See the table above.

1.2) Calculate the expected rate of return on each project.

Both projects equally earn an expected return on 10%

1.3) Calculate the standard deviation of the rate of return on each project. (Hint: you must convert the figures for rate of return into decimal units.)

Project A has a SD of 0.22 while Project B has as SD of 0.2.

1.4) Which one of the project would you select? What kind of decision criteria do you use? Explain.

Depends on your attitude toward risk. If you are risk averse, you would prefer B over A. Both A and B provides the same rate of return, but project B is less risky.

On the other hand, if you are risk lover, you might tempt to give the high value to the upside gain that project A may give you (the 40% return that A may give out), and therefore betting against the risk and choose for A as the project of investment.

Question 2: *Risk aversion coefficient in greater detail.*

In class, we discussed about the two types of risk aversion coefficient, i.e. CARA and CRRA. We argue that the two coefficients serve their role as the proxy for the ranking of the degree of risk aversion. This is because the two coefficients are used as the input for *approximating* the level of risk premium required by the individual. For the sake of an illustration, suppose that an individual, with his initial wealth set equal to "w", is facing with a risky option that generates the final outcome equal to $w + \varepsilon$, where ε is a random variable with $E(\varepsilon) = 0$ and $E(\varepsilon^2) = \sigma_\varepsilon^2$.

Arrow-Pratt showed that the level of risk premium can be approximate by the following formula;

$$R_P(w) = \frac{1}{2}\sigma_\varepsilon^2 R_A(w) = \frac{1}{2}\sigma^2\left[-\frac{u''(w)}{u'(w)}\right]$$

As one can see from the formula above, the level of risk premium ($R_P(w)$) is an increasing function in the coefficient of absolute risk aversion ($R_A(w)$), i.e. $\left[-\frac{u''(w)}{u'(w)}\right]$. Given the expression, the agent with bigger value of the CARA would require greater amount of risk premium, and hence associating with the greater degree of risk aversion.

Assume that you have a logarithm utility function for wealth U(W) = $\sqrt{W}$ and that you are faced with a 50/50 chance of winning or losing $1,000. Consider the following questions.

2.1) Calculate the mean and variance of the random pay-off. (Hint: ε can take two possible values, and hence represents the discrete random variable.)

Mean of the random component of the pay-off is

½ (1000) + ½(-1000) = 0

Variance of the random component of the pay-off is

$$\tfrac{1}{2}(1000-0)^2 + \tfrac{1}{2}(-1000-0)^2 = (1000)^2$$

SD of the random component of the pay-off is 1000.

2.2) How much will you pay to avoid this risk if your current level of wealth is $10,000?

First, calculate expected utility when the initial wealth is $10,000. This is equal to

$$\frac{1}{2}\sqrt{10{,}000+1000} + \frac{1}{2}\sqrt{10{,}000-1000} = \frac{1}{2}(104.88) + \frac{1}{2}(94.86) = 99.87$$

Second, calculate the certainty equivalence. That is, we solve for CE such that

$$\sqrt{CE} = 99.87 \rightarrow CE = (99.87)^2 = 9{,}974.84$$

Risk premium = Expected Wealth - CE = 10,000 – 9,974.84 = 25.151 #

2.3) Use the approximate formula given above to calculate the level of risk premium. Compare your solution with the exact solution obtained from the question 2.2

$$R_A(W) = -\frac{u''(W)}{u'(W)} = -\frac{-\frac{1}{4}(W)^{-\frac{3}{2}}}{\frac{1}{2} * (W)^{-\frac{1}{2}}} = \frac{1}{2}W^{-1}$$

Note first that the risk aversion is decreasing in W. Hence, we expect that the degree of risk aversion should be decreasing as people get rich. When W is equal to 10,000, the coefficient is then equal to;

$$R_A(10{,}000) = \frac{1}{2}(10{,}000)^{-1} = \frac{1}{20{,}000}.$$

From the approximate formula, we know that

$$R_P(w) = \frac{1}{2}\sigma_\varepsilon^2 R_A(w) = \frac{1}{2}\sigma^2\left[-\frac{u''(w)}{u'(w)}\right]$$

As a result, $R_P(10{,}000) = \frac{1}{2}(1000)^2 * \left[\frac{1}{20{,}000}\right] = \frac{(1000)^2}{40{,}000} = \frac{1{,}000{,}000}{40{,}000} = 25.$

2.4) How much would you pay to avoid the risk if you level of wealth were instead $1,000,000?

First, calculate the expected utility when the initial wealth is $10,000. This is equal to

$$\frac{1}{2}\sqrt{1{,}000{,}000 + 1{,}000} + \frac{1}{2}\sqrt{1{,}000{,}000 - 1{,}000} = \frac{1}{2}(1{,}000.49) + \frac{1}{2}(999.49) = 999.99$$

Second, calculate the certainty equivalence. That is, we solve for CE such that

$$\sqrt{CE} = 999.99 \rightarrow CE = (999.99)^2 = 999{,}989.87$$

Risk premium = Expected Wealth - CE = 1,000,000 – 999,989.87 = 10.13 #

$$R_A(1{,}000{,}000) = \frac{1}{2{,}000{,}000}$$

From the approximate formula, we know that

$$R_P(w) = \frac{1}{2}\sigma_\varepsilon^2 R_A(w) = \frac{1}{2}\sigma^2\left[-\frac{u''(w)}{u'(w)}\right]$$

Hence, $R_P(1{,}000{,}000) = \frac{1}{2}(1000)^2 * \left[\frac{1}{2{,}000{,}000}\right] = \frac{(1{,}000{,}000)}{4{,}000{,}000} = 0.25$

Note: *The approximate error is high because the random component part of the pay-off is large.* The formula would produce a more accurate result if variance of the pay-off is small; this approximate formulae works well under a small risk. However, as one can see, the degree of risk tolerance is still preserved under two different level of the initial wealth. That is, because the risk aversion coefficient is decreasing in W, one expects that risk premium will decrease in wealth. Our prediction is confirmed by the two figures of risk premium that we calculate using the derivation that is based on the definition.

Question 3: Diversification and Risks

Consider the following investment strategy of two investors, namely John and Robert.

John: He has $100 million in cash, and invested in a real estate in London. The property has its current value of $80 million. Future value of the estate depends on the possibility of fire incident occurred in London. Under the unfortunate event with the occurrence of a fire accident, his property will lose its full value. If not, his property remains at the current value of $80 million.

Robert: He also has $100 million in cash. However, he owns two properties located in London and Paris. The current value of both properties is $40 million. The value of his property could be changed depending upon whether there is any fire accident any of these two places. If a fire accident occurs at any location, the value of the property in that location will be all gone, i.e. losing its full value

A study has found that the chance of having a fire incident in each location is 10% equally, and the occurrence of the incidence is statistically independent. Suppose that there is no insurance market that can provide the coverage to this fire risk, consider the following problem.

3.1) Calculate the level of wealth for John when fire occurs in London, and when everything goes as normal without any fire incidents.

Case	Probability	Wealth	Return
Fire	0.1	100	-44.44%
normal	0.9	180	0 %

3.2) Given the *independence* assumption, what is the probability that both London and Paris would have a fire incident at the same time? **Prob = (0.1)*(0.1) = 0.01**

3.3) What about the chance that neither places would have a fire incident?

Prob = (0.9)*(0.9) = 0.81

3.4) How about the chance that fire accident would occur at most one particular location, i.e. either London or Paris. **Prob = 2*(0.9)*(0.1) = 0.18**

3.5) Calculate the level of contingent wealth of Robert under the four possible scenarios described in (3.2) – (3.4)

case	probability	Wealth	Return
Fires at both place	0.01	100	-44.44%
Fires at one single location	0.18	140	-22.22%
Both are safe	0.81	180	0 %

3.6) Calculate the expected value of wealth and the standard deviation of wealth for John and Robert.

- Expected value of wealth A = 0.9*180 + 0.1*100 = 172 ; expected return = -4.44%
- Expected value of wealth B= 0.81*180 + 0.18*140 + 0.01*100 = 172 ; expected return = -4.44%

3.7) Given the investment decision chosen by John and Robert, do you think which one of them would likely be bearing upon more risk? What do we learn about the risk mitigation method adopted by John and Robert?

- John's investment contains greater degree of risk exposure comparing to the investment of Robert while both approaches to the investment yields the same expected return.
- They both start with the same amount of money, but Robert chooses to invest into two different housing assets. Financially, Robert is attempting to gain some benefit from the diversification technique; at a certain targeted level of return, his risk exposure can be reduced.
- Note one thing that the success of diversification depends on the correlation of the risky outcomes. Having said so, because the fire incidence in both locations are independent, this implies that both assets have zero correlation. The attempt to diversify risk would be less

desirable (effective) if asset returns are highly correlated. In our context, if the fire incident in both locations always occur at the same time, having diversified not diversified would not make any changes to the portfolio performance of Robert who decided to diversify his portfolio.

Question 4:

Suppose that Mao is an expected utility maximizer, with the VNM utility function $u(w) = 3w^2$

4.1) What type of attitude toward risk does Mao's preference exhibit? Why?

Notice that the utility function is convex. Then, Mao's preference is risk lover

4.2) What is Mao's certainty equivalent of the following lottery:

Probability	Money
.4	30
.5	100
.1	500

Expected utility $= 3[0.4 * (30)^2 + 0.5 * (100)^2 + 0.1 * (500)^2] = 91080$

$3(CE)^2 = 91080 ==> CE = 174.24$

4.3) Calculate the amount of risk premium.

Risk premium = expected value of wealth – CE = 67 - 174.24 = - 107.24

4.4) What is the sign of the coefficient of absolute risk aversion?

The sign of the coefficient of absolute risk aversion is negative, which implies that the level of risk premium will be negative. This makes sense because the utility function is convex and Mao's preference toward risk is classified as risk-lover.

Question 5: Certainty equivalence and risk premium

An investor with his initial wealth of $10 is weighing the two investment options between risky and riskless asset. Risky asset offers the net return of 200% if the economy enters into the boom state. But if the economy gets into the stagnate condition, net return of the risky asset would be -100%. We suppose that the chance that economy would be entering into a booming and stagnating stage are equal.

Suppose that the investment has VNM expected utility given by,

$$u(W) = \frac{(W^{1-\gamma}) - 1}{1-\gamma}$$

where W is the level of wealth at the end of the investment horizon.

Consider the following problems

5.1) Calculate the expected utility that can be derived under the full investment on risky asset. Show you calculate under two cases, i.e. $\gamma = 0$ and $\gamma = \frac{1}{2}$.

W1/G = 10*(1+2) = 30
W1/B = 10*(1-1) = 0
Average pay-off (expected wealth) = 15

Expected utility:

$$Eu(W) = \frac{1}{2}\left[\frac{(30^{1-\gamma}) - 1}{1-\gamma}\right] + \frac{1}{2}\left[\frac{(0^{1-\gamma}) - 1}{1-\gamma}\right]$$

$\gamma = 0 ==> \; Eu(W) = 14$

$\gamma = \frac{1}{2} ==> \; Eu(W) = 3.47$

5.2) Using the information in (5.1), calculate the certainty equivalence and the risk premium under both cases of coefficient values.

Following the definition where U(CE) = E(U(W)), we yield that

$\gamma = 0;$

$$\left[\frac{(CE^{1})-1}{1}\right] = 14 \implies \text{CE} = 15$$

Risk premium = expected wealth – CE = 15 – 15 = 0.

$$\gamma = \frac{1}{2};$$

$$\left[\frac{\left(CE^{1-\frac{1}{2}}\right)-1}{1-\frac{1}{2}}\right] = 3.47 \implies \text{CE} = 7.5$$

Risk premium = expected wealth – CE = 15 – 7.5 = 7.5

5.3) Following (5.2), how can we relate the size of risk premium to the value of γ. What is the value of risk premium when γ is equal to 0. Explain the reason why you obtain the number from your calculation.

When $\gamma = 0$, the utility function is linear; agent is risk neutral. As a result, risk-premium is equal to zero because the agent is indifferent to risk.

5.4) Calculate the coefficient of risk aversion, both Absolute and Relative. Describe the relationship between the two coefficients of risk aversion and γ.

$$CARA = \frac{\gamma}{W}; \quad \text{CRRA} = \gamma$$

Evidently, both coefficient increases with respect to γ. Thus, higher γ implies higher degree of risk aversion.

Question 6: Risk and insurance

Assume that the preference of Ken is determined by the amount of net outstanding value of his asset, and can be represented by the following utility function,

$$U = 24w - 2w^2$$

where w is the net asset value, measuring in terms of million dollar.

Suppose that Ken currently owns a house whose current value is $8 million. Housing value would change over the course of event. Under the normal situation, his house will remain at $8 million. Under the unfortunate event when flooding occurs, his house would be partially damaged by $6 million.

6.1) Suppose that p is the probability that flooding will not occur. Calculate the expected utility under the risk of flooding incident.

w/no-flood = 8

w/flood = 8 – damage = 8 – 6 = 2

U(w/flood) = 40

U(w/no-flood) = 64

E(U(w)) = p(40) + (1-p)(64) = 64 – 24p

6.2) What is the attitude of Ken toward risk?

Utility is concave; his preference toward risk is classified as risk averse.

6.3) Calculate the certainty equivalence if we assume that p = ½.

E(U(w)) = 64 – 24(1/2) = 52

Following the definition, we must yield that

$24CE - 2(CE)^2 = 52 \rightarrow CE = 2.13, 9.16$

Now, an insurance company is offering Ken an insurance contract that covers the loss of flooding. The contract says the followings. For every single dollar of the insured amount, Ken would need to pay the company $y. That is, if the insured amount under the coverage is equal to $x, Ken would need to pay for the premium of $xy. This will ensure that he can receive $y if the flooding event occurs. (We call $y as premium per each dollar of insured amount.)

6.4) Suppose that "p" is the probability of flooding. What is the maximum amount of $y that Ken is willing to pay? How does the amount vary with respect to "p"? Explain your answer.

Without insurance, your wealth will be contingent on the occurrence of flooding
w/flooding = 8 – x
w/no-flooding = 8
Suppose that x = 6, we yield that E(U(w)) = p(40) + (1-p)(64) = 64 – 24p

With the insurance that provides the full coverage of loss of 6 million dollars, your utility will be $u(8-6y) = 24(8-6y) - (8-6y)^2$.

The maximum "y" that you will be paying must satisfy the following condition

$24(8-6y)-(8-6y)^2$ = 64 – 24p.

If "y" exceeds the threshold that makes the above equation satisfied, you will not buy the insurance because it is too expensive.

Question 7: Optimal portfolio selection

Suppose that an investor has an initial wealth (W_0) of $100. The investor faces with an investment decision problem with two choices of assets to be chosen. One is the risky asset while the other is risk-free asset. Assume that in the good state, risky asset offer 30% of net return to the investor. The return earned under the bad state for the risky asset is 5%. Suppose that the return on risk-free asset is equal to 10%. Consider the following problem with the utility function of the said investor given by,

$$u(W_1) = k_1 * ln(W_1) + k_2 W_1$$

where W_1 is the terminal-period wealth after the investment. k_1 and k_2 are two positive constants.

7.1) Derive CARA and CRRA.

CARA = $\frac{\frac{k_1}{W^2}}{\frac{k_1}{W}+k_2}$ and CRRA = $W\frac{\frac{k_1}{W^2}}{\frac{k_1}{W}+k_2}$

7.2) Suppose that "a" is the amount of investment on risky asset (in dollar). Derive the condition that characterizes the optimal mixture of investment on risky asset and risk free asset. (Hint: At this stage, you only need to provide the optimality conditions. No need to solve out the explicit solution.)

$$W_1 = (1+r) * a + 1.1 * (W_0 - a)$$

$$W_1|G = (1.3) * a + 1.1 * (W_0 - a) = 1.1W_0 + 0.2a$$

$$W_1|B = (1.05) * a + 1.1 * (W_0 - a) = 1.1W_0 - 0.05a$$

Our problem is to solve of "a" that maximizes the VNM expected utility.

$$p_G u(W_1|G) + (1-p_G)u(W_1|B)$$

$$p_G \frac{du(W_1|G)}{da} + (1 - p_G)\frac{u(W_1\ |B)}{da} = 0$$

$$p_G\left(\frac{k_1}{W_1|G} + k_2\right)(0.2) + (1 - p_G)\left(\frac{k_1}{W_1|B} + k_2\right)(-0.05) = 0$$

7.3) Calculate the value of "a" when $k_1 = 1$ and $k_2 = 0$. Does the investor use the leverage in his investment strategy?

$$p_G\left(\frac{1}{W_1|G}\right)(0.2) + (1 - p_G)\left(\frac{1}{W_1|B}\right)(-0.05) = 0$$

$$p_G\left(\frac{1}{1.1W_0 + 0.2a}\right)(0.2) = (1 - p_G)\left(\frac{1}{1.1W_0 - 0.05a}\right)(0.05)$$

$$W_0 = 100; \quad we\ know\ that\ p_G\left(\frac{1}{110+0.2a}\right)(4) = (1 - p_G)\left(\frac{1}{110-0.05a}\right)$$

With some algebraic manipulations, we yield that

$$a^* = \frac{\left[440 - \left(\frac{1 - p_G}{p_G}\right)(110)\right]}{2 + 0.2\left(\frac{1 - p_G}{p_G}\right)}$$

Leverage would occur if $\frac{a^*}{100}$ is positive and greater than on. This depends on the range of p_G. Intuitively, as p_G increases, investor will be more likely to adopt the leverage investment.

Question 8:

Suppose that an investor has the VNM preference given by $= -\frac{e^{-aW}}{a}, a > 0$. Consider the following problems.

8.1) What is the attitude toward risk of this investor? Explain your answer.

$u' = e^{-aW} > 0$

$u'' = -ae^{-aW} < 0$

Agent is risk averse as the utility function is concave.

8.2) Does the preference exhibit an increasing/decreasing/constant relative risk aversion?

CARA = a

CRRA = aW

8.2) Following the investment theory, do you think that the investor would invest in the risky asset if the expected return of the risky asset is lower than that of risk-free asset. Justify your answer with economic reasoning.

No, you will not invest in risky asset because you would have to bear upon risk without getting any upside gain compensated.

8.3) Suppose that expected return of risky-asset is higher than that of risk-free asset. Do you think the investor would change the fraction of investment on risky asset if the level of his initial wealth increases?

Note that CRRA is increase. Agent becomes more risk averse in the relative sense. Thus, fraction of investment on risky asset would decrease as wealth increases.