Question 1 (20 points)

Consider the following Treasury bill quote. For the Treasury bill that pays the $100 face value on 5 October 2000, answer the following questions:

TREASURY BILLS

Maturity	Days to Mat.	Bid	Asked	Chg.	Ask Yld.
Sep 14 '00	6	6.28	6.20	+0.02	6.29
Sep 21 '00	13	6.38	6.30	+0.11	6.40
Sep 28 '00	20	5.91	5.83	+0.03	5.93
Oct 05 '00	27	5.96	5.88	−0.01	5.99
Oct 12 '00	34	6.00	5.96	−0.04	6.08

a) (5 points) Calculate the Treasury bill bid and ask price. If you want to buy the Treasury bill from the dealer, which price would you face?

The bid price is the price at which you can sell while the ask price is the price at which you can buy. Therefore you will face the ask price.

the bid and ask price is calculated from the bank discount yield formula.

Bid Price: $P = 100 \times \left(1 - \frac{0.0596 \times 27}{360}\right) = 99.553$

Ask Price $P = 100 \times \left(1 - \frac{0.0588 \times 27}{360}\right) = 99.559$

b) (3 points) Calculate the bond equivalent yield based on the ask price and confirm that it is equal to the AskYld rate.

The bond equivalent yield is $r_{BEY} = \left(\frac{100 - 99.559}{99.559}\right) \times \frac{365}{27}$

$= 0.05988$

$= 5.99\%$ which is equal to the Ask Yld.

c) (5 points) What is the actual rate of return you will earn over the next 27 days on the Treasury bill? What is the effective annual interest rate?

The interest rate over the next 27 days is

$99.559(1 + r_{27day}) = 100$

$r_{27day} = \frac{100}{99.559} - 1 = 0.443\%$

$(1 + EAR) = (1 + 0.00443)^{365/27} = 1.06157$

$\Rightarrow EAR = 6.16\%$

which is slightly higher than the quoted discount rate.

d) (4 points) If the inflation rate over this period is 3% annually, what is your annual *real* rate of return? If you were to purchase a T-bill would you prefer low or high inflation rates? Why?

The Fisher relation states that

Real = Nominal − expected inflation

We assume that expected inflation is equal to the actual inflation rate.

⇒ Real Rate of Return = 6.16 − 3 = 3.16%

Since you will be receiving a fixed face value amount at maturity, the higher the inflation rate, the lower the real rate of return. You would prefer a lower inflation rate.

e) (3 points) What is the difference between a primary and secondary market? Are the above Treasury bills listed on the primary or secondary market?

— The primary market is where initial sales of securities are made to the public by firms. (in this case, it would be by the government)

— The secondary market is where previously issued securities are traded among individuals and institutions.

— The above treasury bills are sold by a dealer on the secondary market

Question 2 (20 points)

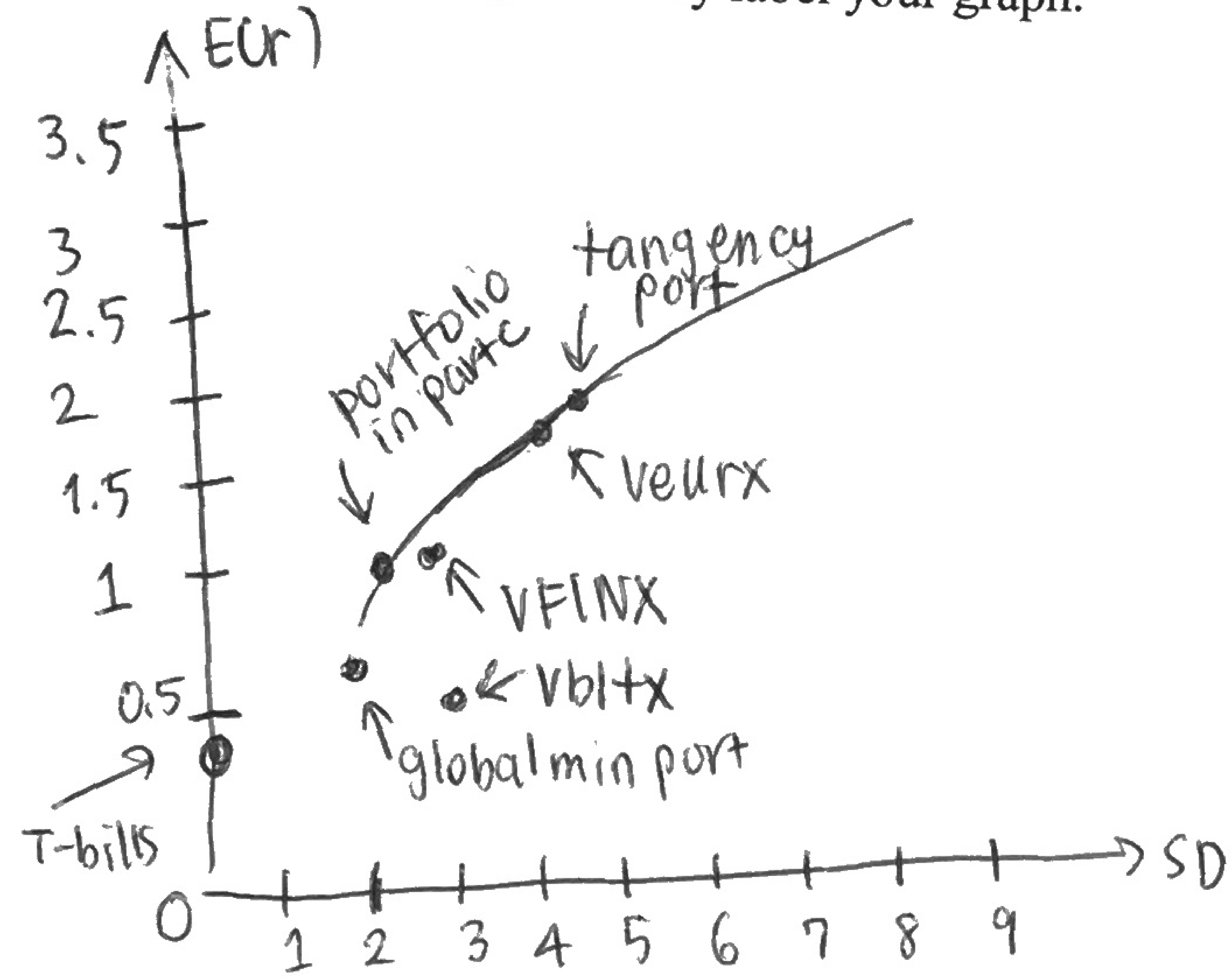
The expected return and standard deviation values for three Vanguard mutual funds: S&P 500 index (VFINX), European stock index (VEURX) and the long-term bond index (VBLTX) are as shown below.

Asset	Mean E(r)	Standard Deviation	Weight in global minimum portfolio	Weight in tangency portfolio
VFINX	1.11%	2.60%	83%	-44%
VEURX	1.68%	3.70%	-27%	139%
VBLTX	0.51%	2.85%	44%	5%
T-bills	0.42%	0%		
Global minimum Portfolio	0.69%	1.83%		
Tangency Portfolio	1.88%	4.23%		

The covariance terms of the three risky assets are according to the table below.

	VFINX	VEURX	VBLTX
VFINX	1	0.8544	-0.0163
VEURX	0.8544	1	0.0479
VBLTX	-0.0163	0.0479	1

a) (5 points) Plot the efficient frontier and denote all assets and portfolios on your graph. Don't forget to fully label your graph.



b) (5 points) Based on the variance-covariance matrix, do you think there will be diversification benefits from forming portfolios from the three mutual funds? If yes, which asset should provide the largest diversification benefit? Briefly justify your answer.

Diversification works best when assets are not positively correlated. Here the 3 assets are imperfectly correlated so some diversification should be possible.

Since vbltx is essentially uncorrelated with vfinx and veurx, the biggest diversification benefit should come from vbltx.

c) (5 points) How much should be invested in T-bills and the tangency portfolio to create an efficient portfolio with the same expected return as VFINX? What is the standard deviation of this efficient portfolio? Locate this portfolio on your graph in part a).

$0.0188y + (1-y)\,0.0042 = 0.0111$

$y = \frac{0.0111 - 0.0042}{0.0188 - 0.0042} = \boxed{0.47}$ (Share in the tangency portfolio)

Share ~~amount~~ invested in T-bills $\Rightarrow 1 - 0.47 = 0.53$

$\sigma_p = y\,\sigma_{tan} = 0.47 \times 0.0423 = \boxed{0.02}$

$E(r_p) = 0.0111$

d) (5 points) The global minimum portfolio has a negative weight on VEURX. What is the interpretation of a negative weight? Is this position associated with any additional downside risk?

A negative weight represents a short-sale of the asset. That is, you borrow the asset, sell it, use the proceeds to purchase more of other assets.

A shortsale position is associated with additional downside risk. A long position has finite amount of downside risk because the most the investor can lose is the entire investment. A short sale position, on the other hand, has unlimited downside risk as the price of the security could continue rising indefinitely.

Question 3 (20 points)

Two companies have stock for which the expected returns and standard deviations are:

Company	A	B
Expected Return	10%	15%
Standard Deviation	3%	5%

a) (5 points) Consider a portfolio with 60% in asset A and 40% in asset B. Assume that there is a risk-free asset available with 5% return and your degree of risk aversion is A=5. If the covariance between the two assets is zero and if you have $100,000 to invest, how much would you invest in stock A, B and the risk-free asset?

Risky Portfolio: $E(r_p) = 0.6 * 10 + 0.4 * 15 = 12\%$

$\sigma(r_p) = \sqrt{0.6^2 * 0.09 + 0.4^2 * 0.25}$

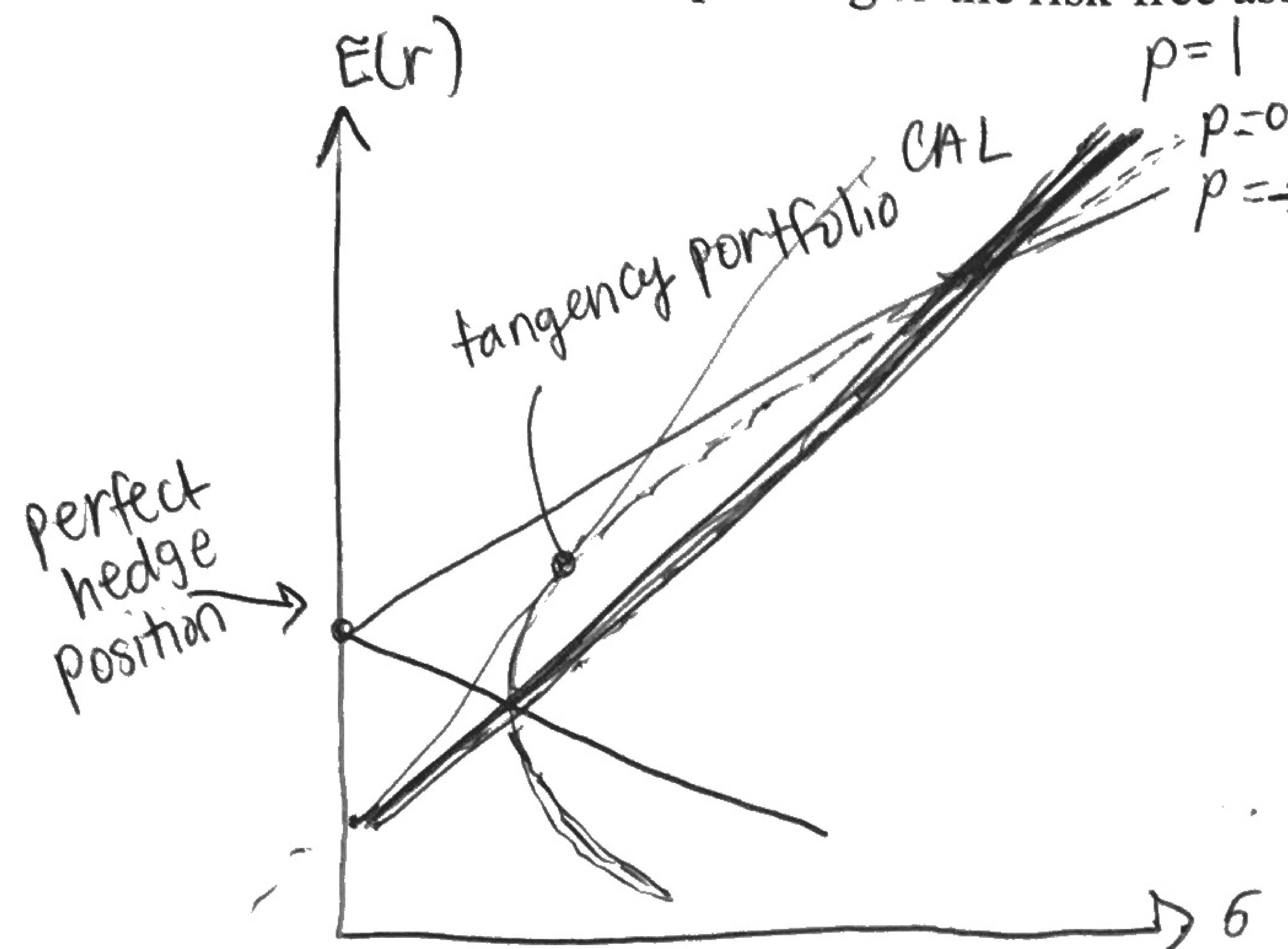
$= 0.269$

Position in risky portfolio $y^* = \frac{E(r_p) - 5\%}{A(0.269)^2} = \frac{0.07}{5(0.269)^2} = 0.193$

Investor would put 19,347 in the risky portfolio where 60% or 11,608 would be put in asset A and 7,738 would be put in asset B.

The remaining 80,652 would be put in the risk free asset

b) (5 points) Draw the minimum variance frontier for the cases when the correlations between the returns on asset A and B are 1, 0, and -1. Also draw the capital allocation line(s) corresponding to the risk-free asset with 5% return.



$$\phi = w * 0.03 + (1-w) * 0.05$$
$$= -0.02\,w + 0.05$$
$$\Rightarrow w = -2.5$$
$$E(r_p) = -2.5 * 0.1 + 3.5 * 0.15$$
$$= 0.275 = 27.5\%$$

c) (5 points) Explain what is a perfect hedge position and when it can be achieved. Label the perfect hedge position on your graph in part b). What are the weights for asset A and B in this scenario? What is the expected return on the portfolio?

A perfect hedge is a hedge that gives a portfolio with zero risk. This can occur when $\rho = -1$

$$\sigma_p = w\sigma_A - (1-w)\sigma_B$$

$$\phi = w * 0.03 - (1-w) * 0.05$$
$$= 0.03\,w - 0.05 + 0.05\,w$$
$$0.05 = 0.08\,w$$

$$\Rightarrow w = 0.625 \quad \text{in asset A}$$
$$0.375 \quad \text{in asset B}$$

$$E(r_p) = 0.625 * 0.1 + 0.375 * 0.15$$
$$= 0.118 \Rightarrow 11.87\%$$

d) (5 points) Would you prefer the portfolio in parts a) or c) if your degree of risk aversion remained equal to A=5? Would your answer change if you were a risk-loving investor? Explain briefly.

$U = E(r) - 0.5 A \sigma^2$

In part a) $U = 0.12 - 0.5(5)(0.269)^2 < 0$

part c) $U = 0.118$

⇒ The portfolio in part c) is preferred!! The investor is extremely risk-averse

For a riskloving investor $A < 0$ and the utility function would add on more utility for the investor for taking on more risk.

Therefore, the investor would probably prefer the portfolio in part a)

Question 4 (20 points)

a) (5 points) List five main assumptions underlying CAPM.

1. Perfect competition
2. No tax, transaction costs
3. Rational, mean-variance optimizing agents
4. Homogenous expectations
5. Unlimited shortsale and borrowing

b) (3 points) In a CAPM equilibrium, can you have a risky asset that has a lower expected return than the risk-free rate?

Yes. A risky asset whose return is negatively correlated with the market's return will have a negative beta. According to the expected return-beta representation of CAPM, this risky asset will have a lower expected return than the risk free rate.

c) (3 points) If a portfolio has beta=1 does this mean that it is perfectly correlated with the market?

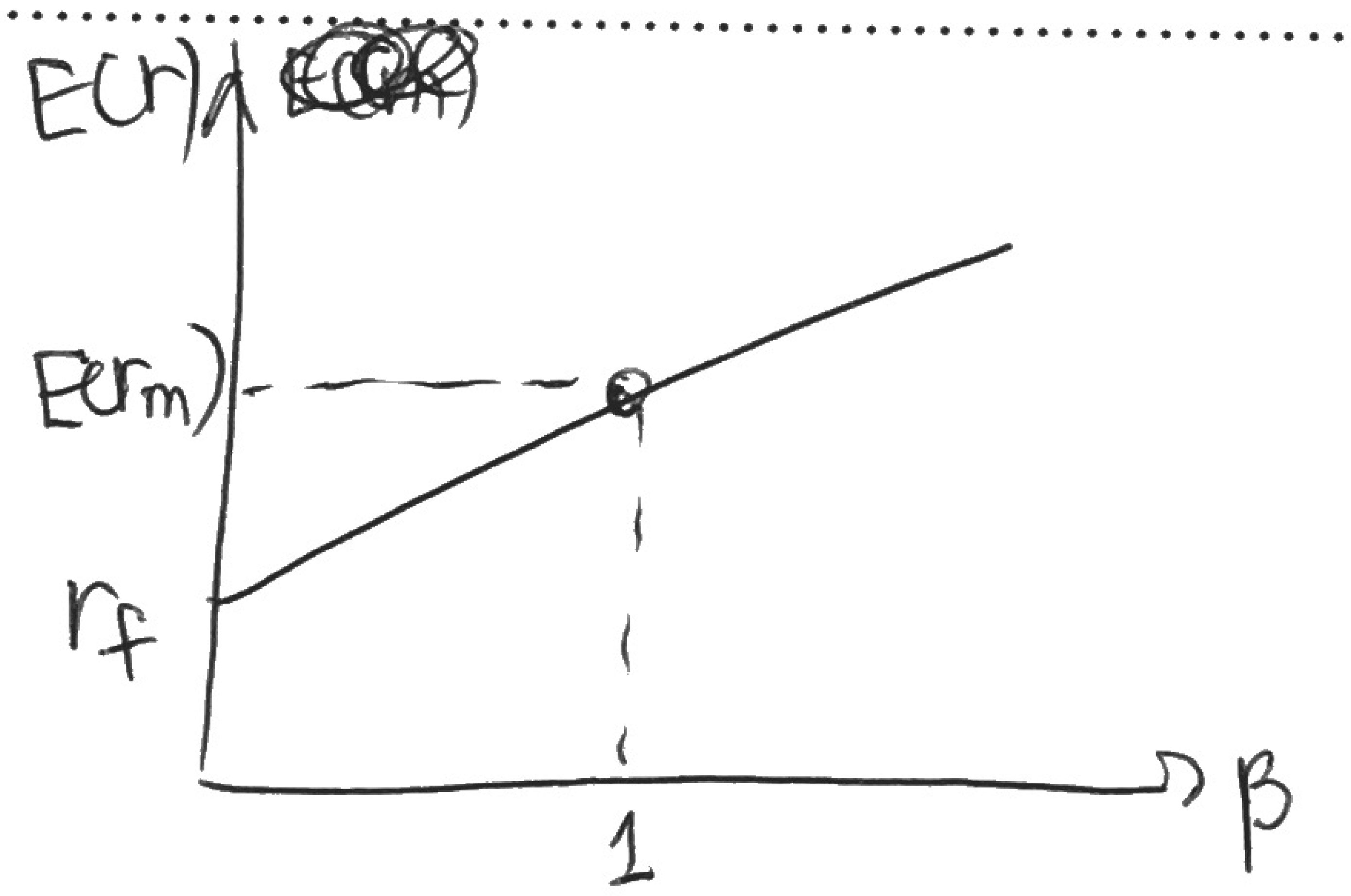
Not necessarily. When $\beta = 1$ this implies $cov(r_i, r_m) = var(r_m)$.

Recall that correlation $\rho(r_i, r_m) = \frac{cov(r_i, r_m)}{\sigma(r_i)\sigma(r_m)}$

Thus, correlation is is not one unless $\sigma(r_m) = \sigma(r_i)$

d) (4 points) What is the security market line (SML) pricing relationship? Draw a graph showing this relationship and explain what the intercept and slope correspond to.

The SML shows the expected return of efficiently "fairly priced" assets given their beta risk



"fairly priced" assets have E(r) according to CAPM ie

$$E(r_i) = \underbrace{r_f}_{\text{intercept}} + \beta(\underbrace{E(r_m) - r_f}_{\text{slope}})$$

e) (5 points) Explain why we tend to find that small stocks tend to lie above the SML and why large firms tend to lie below. How would you modify the CAPM relationship to account for this real-world anomaly?

This is because small stocks are more illiquid and may therefore command a higher premium.

We can ~~estim~~ use a multi factor model with size as an additional factor to help explain expected returns of assets

Question 5 (15 points)

Consider the following single index model for asset returns

$$r_{it} - r_{ft} = \alpha_i + \beta_i(r_{Mt} - r_{ft}) + \varepsilon_{it}$$

where i=1,…,n assets and t=1,…,T time periods.

a) (2 points) What is the interpretation of the residual term in the single index model?

Firm-specific or idiosyncratic risk. This is the portion of risk that cannot be diversified away

b) (5 points) What is an important assumption that pertains to the residual term in the single index model? Do you think it is realistic? Discuss how you think this assumption may affect Markowitz portfolio optimization results.

$Cov(\varepsilon_{it}, \varepsilon_{jt}) = \emptyset$ ie. no industry effects

This is not realistic as in the real world there could be shocks that effect many firms in a particular industry but not the broad market.

If industry effects exists, then Markowitz may give biased weights to the assets as it is not able to take into account diversification benefits that could come from mutual covariance.

Suppose you use the above equation to test CAPM for the Vanguard Utilities Index ETF and the S&P 500 index. You obtain the following results from the regression:

Asset	α_i	$SE(\alpha_i)$	β_i	$SE(\beta_i)$
Vanguard Utilities Index ETF	0.0053	0.0130	0.0579	0.1882
S&P 500 Index	0.0033	0.0035	1.2155	0.1268

c) (5 points) Based on the estimated values of beta, what can you say about the risk characteristics of these two assets relative to the market index? Based on the characteristics of the two assets, do the magnitudes of beta for both assets make sense? Explain your answer.

Beta measures the extent to which returns on an asset moves with the market.
It makes sense that the utilities ETF has lower beta as utilities are rather stable companies, less effected by business cycle conditions

The S&P 500 index is comprised of many stocks and should move more or less in line with the market portfolio so its magnitude makes sense

the S&P 500 is more risky than the utilities ETF

d) (3 points) Does the CAPM hold for both assets? Explain why or why not.

Yes. For both assets, α is not statistically different from zero.

$tstat_{ETF} = 0.0053 / 0.013 = 0.407$

$tstat_{S\&P500} = 0.0033 / 0.0035 = 0.942$

Question 6 (5 points)

What is the two-fund separation theorem and describe a scenario where it may not hold in practice. In your example, discuss what implications this may have towards the market portfolio.

The portfolio optimization can be seperated into two steps
1. ↳ Investors can identify a unique ~~market~~ risky portfolio independent of their risk preferences.
   ↳ (This risky portfolio is the same for all investors) = market portfolio
2. Then, depending on risk preferences, investors will hold a combination of the risk-free and risky portfolio

A scenario where this may not hold is when the investor can not borrow and lend at the same rate. In this situation, the ~~ma~~ risky portfolio is not the same for all investors and will depend on their risk preferences.