MA217 Final 2015

Q1: 5 marks

For a two-variable function

$$f(x,y)=\ln(xy) \text{ subjected to } x+y\le 12,$$

find all possible critical point(s) and the optimum value(s).

Ans: $f(6,6)=\ln(36)$

Q2: 7 marks

For a three-variable function

$$f(x,y,z)=2x^2-2xy-4xz+y^2+2yz+3z^2-2x+2z,$$

find all possible critical point(s) using Lagrange multiplier method and matrix and also show that your answer is correct.

Ans: (1, 1, 0)

Q3: 7 marks

Consider a linear system $\underline{\mathbf{A}}\underline{x}=\underline{b}$ where $\underline{\mathbf{A}}$ is a real 3 × 3 matrix and *a* and *c* are real contants.

$$\underline{\mathbf{A}}=\begin{bmatrix}1 & 2 & 0\\ 1 & a & c\\ 1 & a^2-2 & c^2\end{bmatrix} \qquad \underline{x}=\begin{bmatrix}x\\ y\\ z\end{bmatrix} \qquad \underline{b}=\begin{bmatrix}1\\ -1\\ 1\end{bmatrix}$$

For which values of *a* and *c* do the equations Ax = d have (i) a unique solution, (ii) more than one solution, (iii) no solution? For each pair (*a*, *b*) satisfying (ii), give the solutions in vector form.

Ans:
- unique solution $c\neq 0$ AND $c-a+2\neq 0$
- no solution $[c=0$ OR $c-a+2=0]$ AND $a\neq -2$
- Infinite solutions $[c=0$ OR $c-a+2=0]$ AND $a=-2$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}0\\ \frac{1}{2}\\ 0\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}C_z \text{ where } C_z\in\Re$$

Q4: 6 marks

Find the general solution of

$$\begin{aligned}3x_1 - x_2 + x_3 + 2x_4 &= 16\\ x_1 - x_2 + x_3 + 2x_4 &= 6\\ 2x_1 - 2x_2 + 2x_3 + kx_4 &= 13\end{aligned}$$

where *k* is an arbitrary constant. Express the solution in the parametric vector form. Explain how different values of *k* affect the set of solutions.

Ans: If $k=4$, the linear system has no solution.

If $k\neq 4$, the linear system has infinite number of solutions.

$$\underline{x} = \begin{bmatrix} 5 \\ \frac{2}{k-4}+1 \\ 0 \\ \frac{1}{k-4} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} C_3 \text{ where } C_3 \in \Re$$

Q5: 24 marks

$$\underline{\mathbf{A}} = \begin{bmatrix} k & 1 & -2 \\ 0 & -1 & k \\ 9 & 1 & 0 \end{bmatrix}$$, where k is a real contant.

(a) Find values of k for which $\underline{\mathbf{A}}$ is singular.

(b) Given that $\underline{\mathbf{A}}$ is non-singular, find $\underline{\mathbf{A}}^{-1}$ in terms of k. It is suggested if possible, students should use the determinant and adjoint to calculate $\underline{\mathbf{A}}^{-1}$.

(c) Show that the calculation of $\underline{\mathbf{A}}^{-1}$ in part (b) is correct. It is noted that detailed steps of calculation must be shown.

(d) If $k=1$, use the row operation method to determine $\underline{\mathbf{A}}^{-1}$ and show that the answer is the same as substituting $k=1$ in $\underline{\mathbf{A}}^{-1}$ from (b).

(e) If $k=1$, a linear system $\underline{\mathbf{A}}^T \underline{x} = \underline{b}$ where $\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $\underline{b} = \begin{bmatrix} 20 \\ 10 \\ -10 \end{bmatrix}$, determine the solution $\underline{x}$.

(f) If $k=1$, a linear system $\underline{\mathbf{A}}\underline{x} = \underline{b}$ where $\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $\underline{b} = \begin{bmatrix} -20 \\ 10 \\ 30 \end{bmatrix}$, use Cramer's rule to determine the solution x_2.

(g) If $k=1$, $\underline{\mathbf{C}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 & 0 \\ 4 & 5 & 6 & 0 & 0 \\ 7 & 8 & 9 & 10 & 0 \\ 11 & 12 & 13 & 15 & 15 \end{bmatrix}$ and $-4\underline{\mathbf{A}} = \underline{\mathbf{C}}\underline{\mathbf{D}}^2$, determine $|-4\underline{\mathbf{A}}|, |\underline{\mathbf{C}}|$ and $|\underline{\mathbf{D}}|$.

Ans: (a) $k = 6$ or $k = 3$

(b) $$\underline{\mathbf{A}}^{-1} = \frac{1}{k^2 - 9k + 18} \begin{bmatrix} -k & 2 & k-2 \\ 9k & 18 & -k^2 \\ 9 & 9-k & -k \end{bmatrix}$$

(d) $\underline{\mathbf{A}}^{-1} = \frac{-1}{10}\begin{bmatrix} -1 & 2 & -1 \\ 9 & 18 & -1 \\ 9 & 8 & -1 \end{bmatrix}$

(e) $\underline{x} = \begin{bmatrix} 2 \\ -6 \\ 2 \end{bmatrix}$

(f) -30

(g) $\left|-4\underline{\mathbf{A}}\right| = 640, \left|\underline{\mathbf{C}}\right| = 2700$ and $\left|\underline{\mathbf{D}}\right|$ is undefined.

Q6: 3 marks

Determine this integral $I = \int_0^1 \int_0^x e^{x^2}\ dydx$.

Ans: $I = \frac{1}{2}(e-1)$

Q7: 4 marks

Determine this integral $I = \int_0^1 \int_x^{2x} \frac{y}{x^2+y^2}\ dydx$.

Ans: $I = \frac{1}{2}\ln\left(\frac{5}{2}\right)$

Q8: 4 marks

Determine this integral $I = \int_1^a \int_0^b \int_0^c (x^2+y^2+z^2)\ dxdydz$ where a, b and c are real constant.

Ans: $I = \frac{1}{3}\left(abc^3 + ab^3c + a^3bc - bc^3 - b^3c - bc\right)$

Q9: 6 marks

Determine this integral $I = \int_0^2 \int_0^{3(2-x)} \int_0^{6-3x-y} x\ dzdydx$.

Ans: $I = 6$