Answer: Exercise 4

Application of the Derivative: Related Rates, Linearization & Differentials

1 Linearization & Differentials

1. $L(\theta) = f(0) + f'(0)(\theta - 0) = 0 + 1(\theta - 0) \quad \Longrightarrow \quad L(\theta) = \theta.$
2. (a) $L(x) = x - 1$
   (b) $L(x) = 2x - 4.$
3. (a) $(8.1)^{2/3} \approx L(8.1) = 4 + \frac{1}{3}(8.1 - 8) = 4 + 1/30 = \frac{121}{30}$
   (b) $\frac{(-0.9)^3}{1+(-0.9)^2} \approx L(-0.9) = -\frac{1}{2} + \frac{5}{4}(-0.9 + 1) = \frac{15}{40}$
4. $dy = [2x^2 \cos(2x) + 2x \sin(2x)]dx$
5. $dy = f'(4)dx = -\frac{1}{16} \cdot 0.02 = -\frac{1}{800}$, $\triangle y = f(4.02) - f(4) = \frac{1}{\sqrt{4.02}} - \frac{1}{2}.$
6. (i) $L(x) = 1 - 6x.$
   (ii) 0.7
7. (i) 9π
   (ii) 8π

2 Related Rates

1. - The velocity function is $v(t) = \frac{d}{dt}s(t) = t^{-1/2} - \pi \sin(\pi t).$
   - The acceleration function is $a(t) = \frac{d}{dt}v(t) = -\frac{1}{2}t^{-3/2} - \pi^2 \cos(\pi t).$
2. (a) – The velocity function is $v(t) = 6t^2 - 24t.$
       – The acceleration function is $a(t) = \frac{d}{dt}v(t) = 12t - 24.$
   (b) 16
3. $\frac{5}{32\pi}$ m/min
4. $8\sqrt{3}$ cm^2/hr.
5. 17 mi/hr .