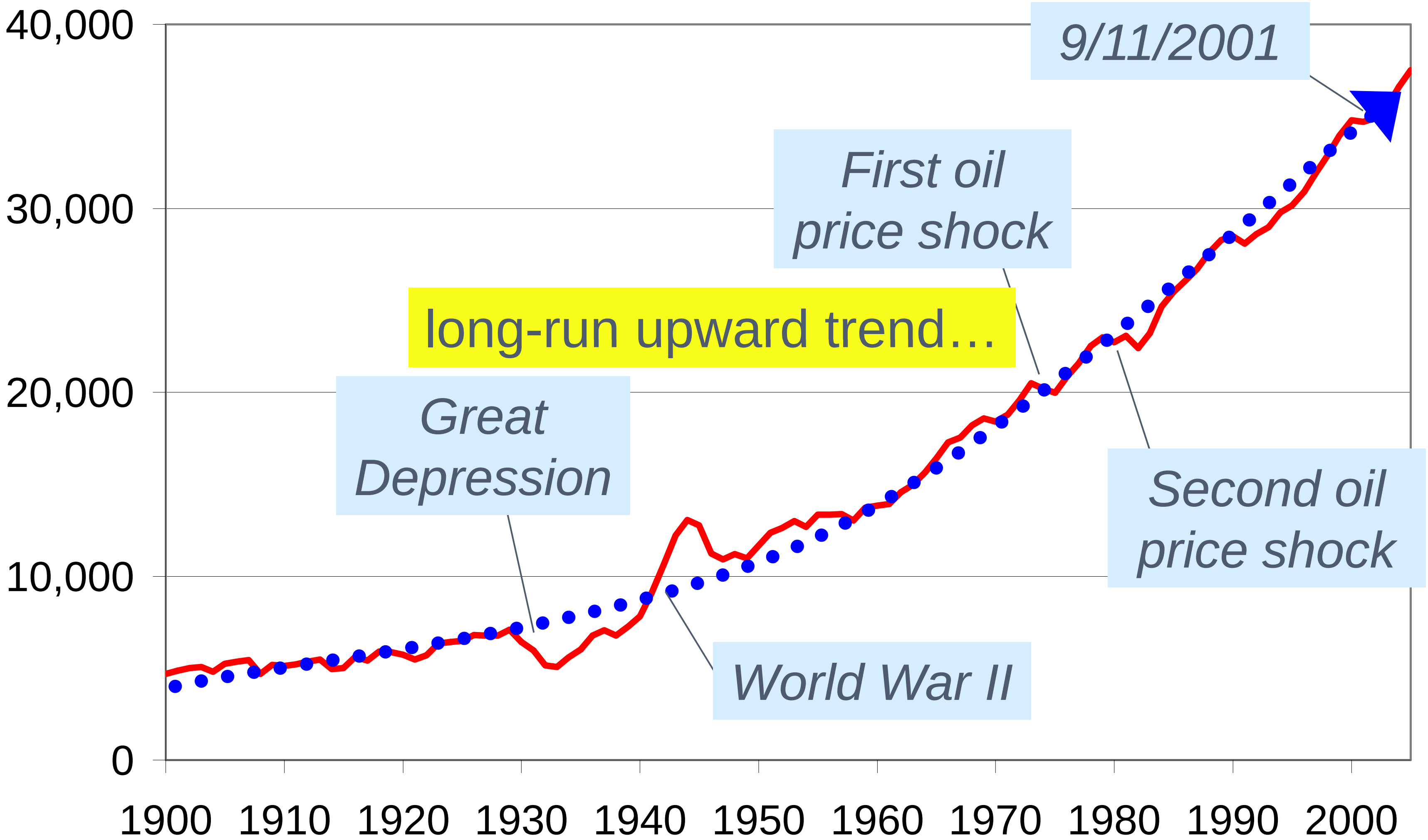
Introduction to the framework of dynamic macroeconomic analysis

MACROECONOMIC ANALYSIS (EE412)

Dynamic Macroeconomic Model

- Dynamic macroeconomic model is an analytical framework that economists use to explain certain long-term and short-term macroeconomic facts,
- (1) Persistent long term growth in modern economic history,
- (2) There are both expansion and recession periods in the short run (known as "Business cycle").
- (3) sustained diversity in income levels among different countries.

U.S. Real GDP per capita
(2000 dollars)



Persistent Long Term Growth of Modern Economic Growth v.s. Malthusian Dilemma

- Sir Thomas Malthus wrote in 1798 the famous "Essay on the Principle of Population"
- He predicted that the world would experience exponential growth but only arithmetical food growth.
- As soon as the population levels exceed food levels by some critical amount, a Malthusian dilemma (doom day) occurs.

Sustained diversity in income levels

- World Bank's World Development Report 1983
  - The diversity across countries measured in per capita income levels is quite large
    - (US. $10,000, India $240) (1980)
  - This is a difference of a factor of 40 in living standard.

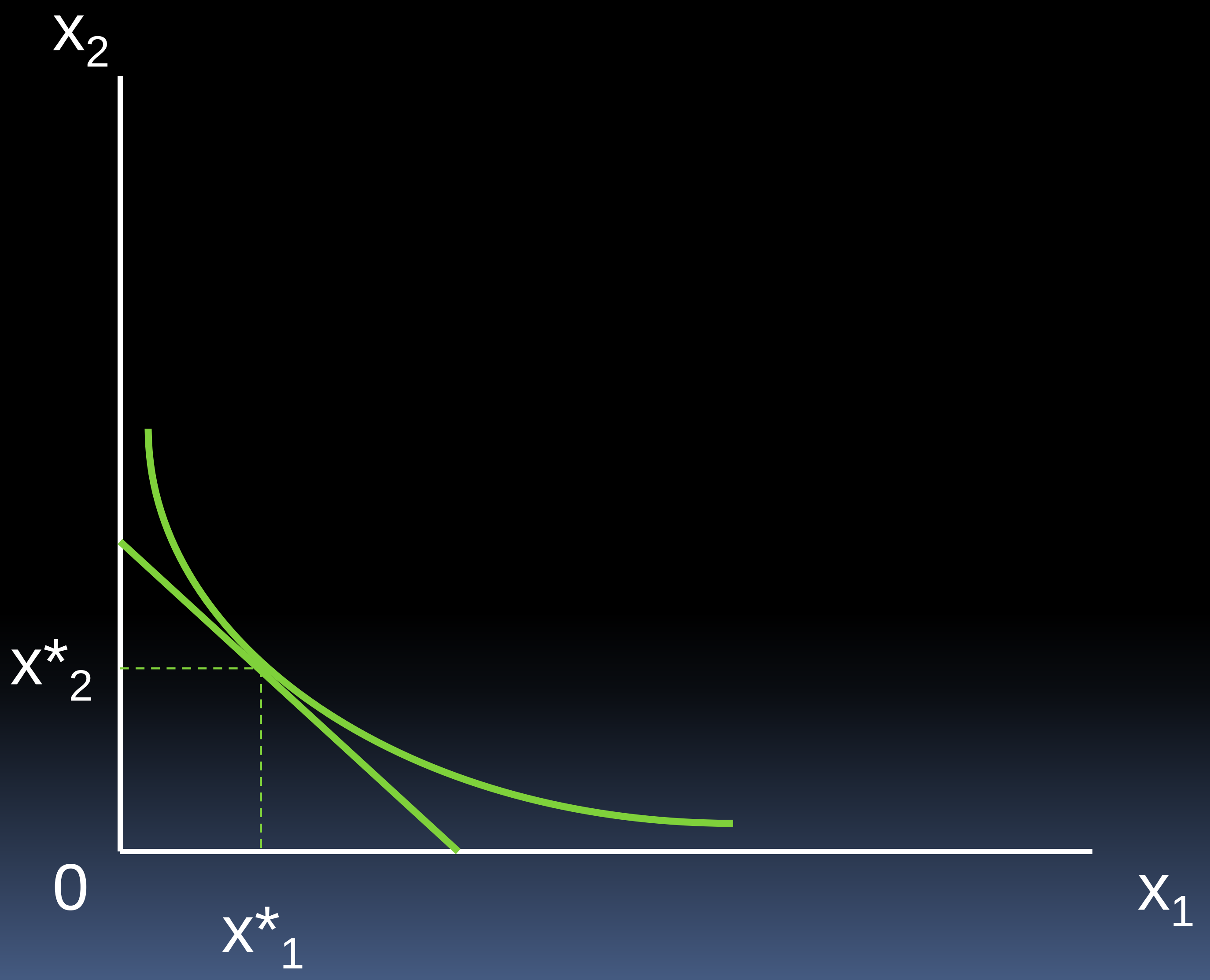
Static vs. Dynamic Problems

Static problem:
single period Problem

$$\underset{x_1, x_2}{Max} \; U(x_1, x_2)$$

$s.t.$

$$p_{x_1} . x_1 + p_{x_2} . x_2 = Y$$


x2
x*2
0
x*1
x1

$$L = U\left(x_1, x_2\right) + \lambda.\left[Y - p_{x_1} x_1 - p_{x_2} x_2\right],$$

$$first - order\ conditions,$$

$$\frac{\partial L}{\partial x_1} = 0,$$

$$\frac{\partial L}{\partial x_2} = 0,$$

$$\frac{\partial L}{\partial \lambda} = 0.$$

How do we solve a 2-period problem (without uncertainty)?

$$\max_{\{x_1(t),x_2(t)\}_{t=0}^{1}} \sum_{t=0}^{1}\left\{(1+r)^{-t}.U_t\left(x_1(t),x_2(t)\right)\right\},\quad 0<r<1$$

$s.t.$

$$p_{x_1(0)}.x_1(0)+p_{x_2(0)}.x_2(0)\leq Y(0),$$

$$(1+r)^{-1}.\left[p_{x_1(1)}.x_1(1)+p_{x_2(1)}.x_2(1)\right]=\left[Y(0)-p_{x_1(0)}.x_1(0)-p_{x_2(0)}.x_2(0)\right]$$

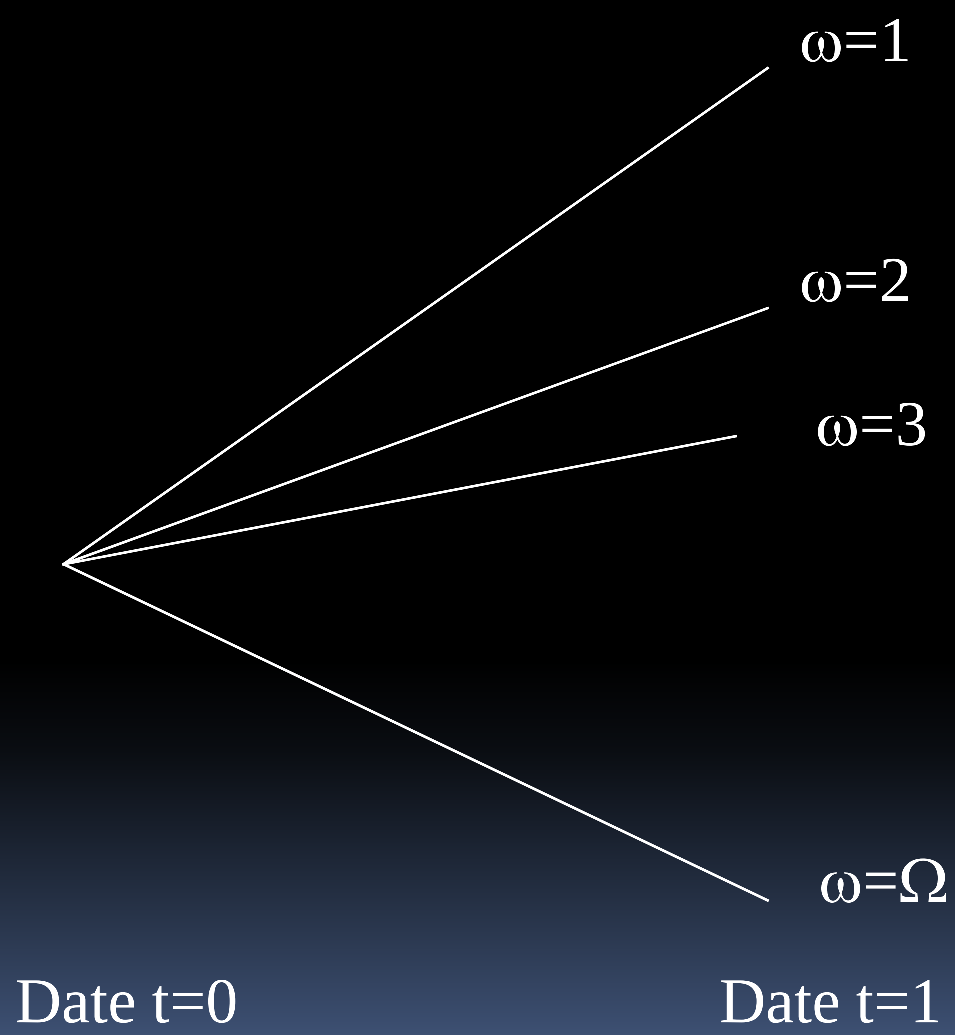
Should we solve each period problem separately?

Why?

Dynamic Model,
Two-Date Model: Complete Markets

- Consider two dates: t=0 and t=1.
- There is uncertainty at t=1,

  with ω = 1, 2, 3,…., Ω states of the world.
- Let assume that there is only 1 physical commodity, at each date/state.
- In this case, complete markets means the number of markets for trading this commodity must be Ω+1.


ω=1
ω=2
ω=3
ω=Ω
Date t=0
Date t=1

Complete Market

- In a complete markets, there exists an "Arrow-Debreu security" for each possible state of the world.
- An Arrow-Debreu security for state ω pays one unit of consumption only in state ω and nothing otherwise.
- The price of these Arrow-Debreu securities, $p(\omega)$, are called state prices.

Two-Date Model: Complete Markets

- Under these conditions, we can use the standard tools of price theory (in Microeconomics) under certainty to analyze an economy with *contingent consumption.*
- Each consumer i=1,2,3,…,I., has the problem:

Two-Date Model: Complete Markets

$$\underset{\{x_i \in X_i\}}{Max}\, U_i\left(x_{0i}, x_{1i}, \ldots, x_{\Omega i}\right)$$

$$s.t. \quad p_0 x_{0i} + \sum_{\omega=1}^{\Omega} p(\omega).x_{\omega i} \equiv Y_{0i}$$

where

p_0 *is the price at* $t = 0$ *of the commodity;*

$p(\omega)$ *is the price at* $t = 0$ *of the contingent commodity* ω;

x_{0i} *is the consumer* i*'s demand for commodity at* $t = 0$;

$x_{\omega i}$ *is the consumer* i*'s demand for contingent commodity* ω;

Y_{0i} *is* $t = 0$ *wealth of the consumer* i.

A Competitive Equilibrium

- A competitive equilibrium for the contingent claims economy is a price vector $\left(p_0^*, p^*(1),..,p^*(\omega),..,p^*(\Omega)\right)$; and consumer demands $x_i^* \equiv \left(x_{0i}^*, x_{1i}^*,....,x_{\Omega i}^*\right), i \in I$ such that :

$$x_i^* \;\; solves: \;\; Max \;\; U_i\left(x_i\right)$$

(Note: this definition can also be extended to cover the case of an economy with production sector)

Multiple dates Model (More than two periods; i.e., t=0,1,2,….,T): Complete Market

- Problem: The number of all markets that must be cleared simultaneously (at t=o) will increase with the number of time periods.
- So, we may have to solve simultaneously a very large consumer problem.
- To avoid such problem, we may limit our analysis of consumer problems to those cases that have some special utility functions (to assume a much more restrictive type of preference).

Special Utility Functions

- For example, assuming for additive and separable utility over time (as well as over states)

$$U_i\left(x_{0i}, x_{1i}, \ldots, x_{\Omega i}\right) = u_i\left(x_{0i}\right) + \sum_{\omega=1}^{\Omega} u_i\left(x_{\omega i}\right)\pi_{\omega},$$

$$\textit{where} \quad u_i^{'}\left(\bullet\right) > 0, \;\; u_i^{''}\left(\bullet\right) \le 0.$$

$$0 < \pi_{\omega} < 1, \qquad \sum_{\omega=1}^{\Omega} \pi_{\omega} = 1$$