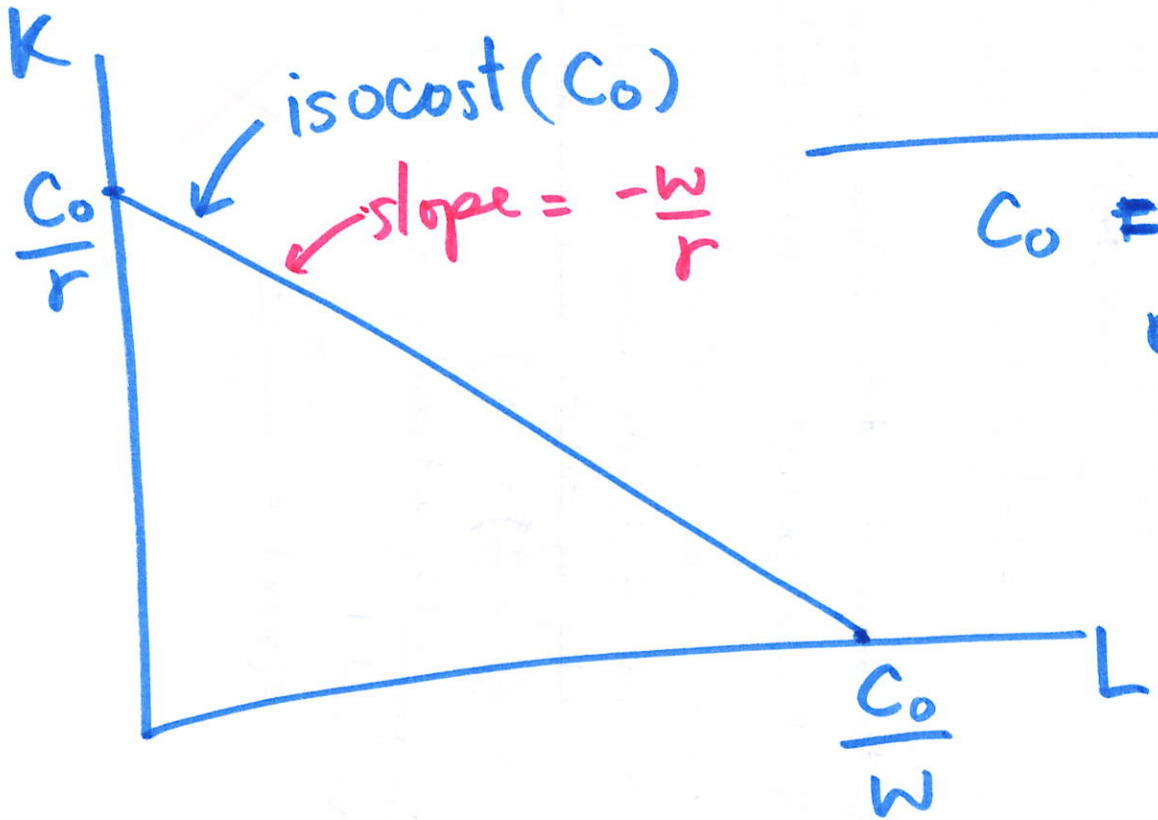
Cost function

$C = wL + rK$ where w = wage = price of labor

r = price of capital (rent)



$C_0 = wL + rK$

$rK = C_0 - wL$

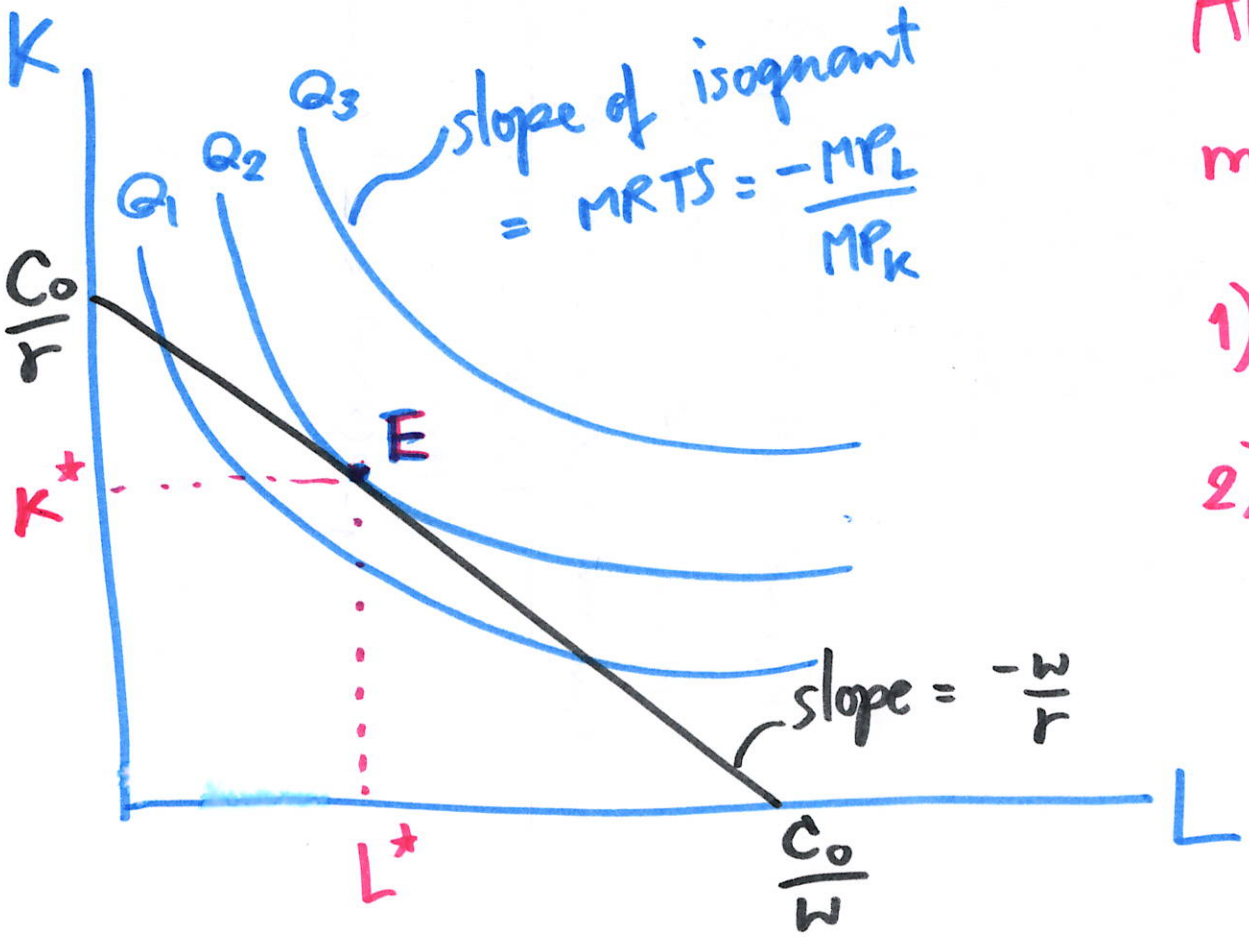
$K = \frac{C_0}{r} - \frac{w}{r} L$

slope of isocost

Output Maximization

Max $Q = f(K, L)$

subject to $C_0 = WL + rK$ (C_0: fixed)



At eqm, (K^*, L^*) must meet 2 criteria:

1) $C_0 = WL^* + rK^*$

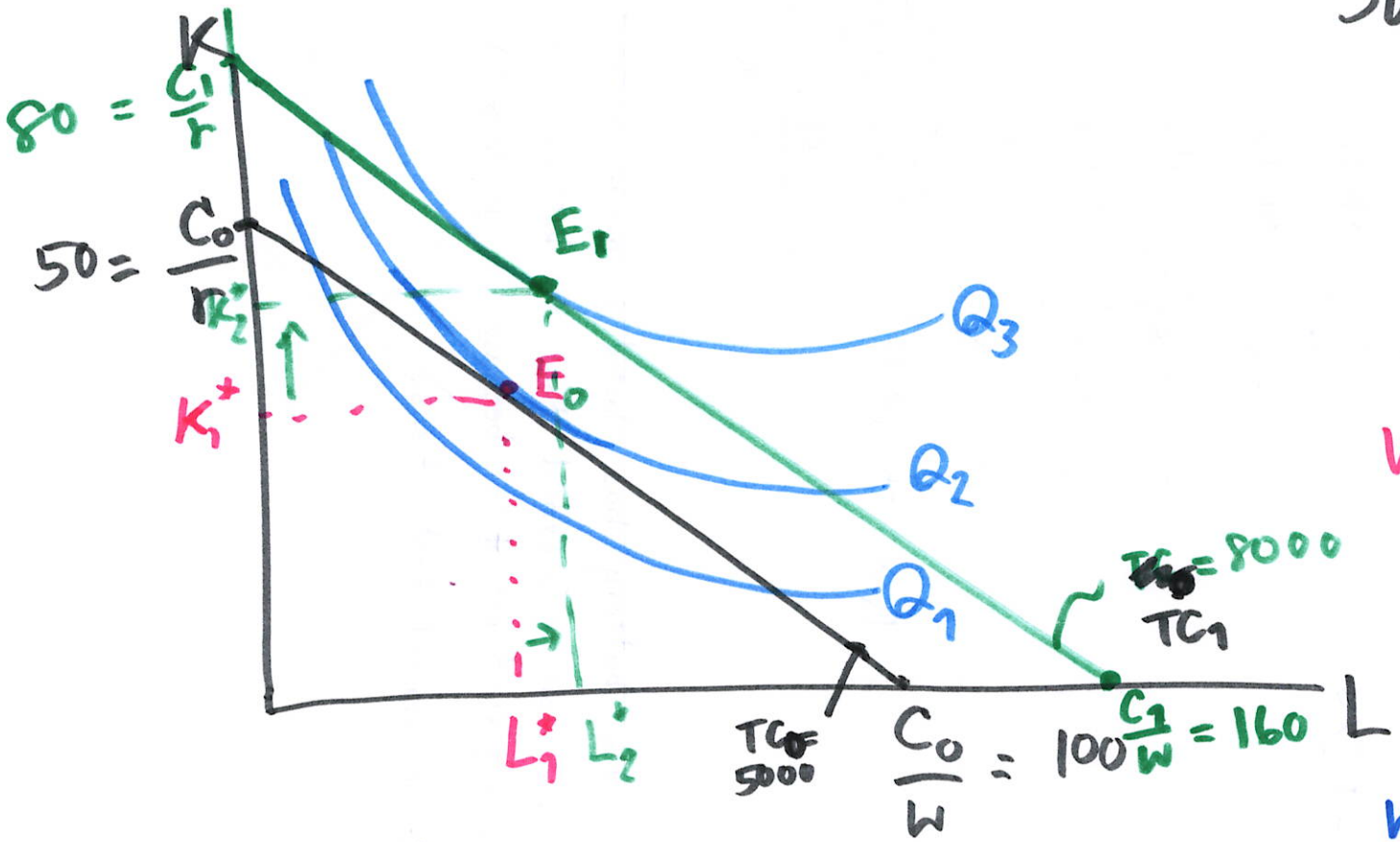
2) $MRTS = -\frac{w}{r}$

$$\frac{-MP_L^{(K^*,L^*)}}{MP_K^{(K^*,L^*)}} = -\frac{w}{r} \quad \leftrightarrow$$

$$\frac{MP_L}{w} = \frac{MP_K}{r}$$

Changes in Eqm

1) Fixed Cost changes from C_0 to C_1 ($C_1 > C_0$).



Suppose
$C_0 = 5000$
$w = 50$
$r = 100$
} Given

When $C = 5000$

Optimal $(L, K) = (L_1^*, K_1^*)$

$\Rightarrow$ Max output $= Q_2$

Suppose $C_1 = 8000$

When $C = 8000$

$\Rightarrow$ Max output $= Q_3$

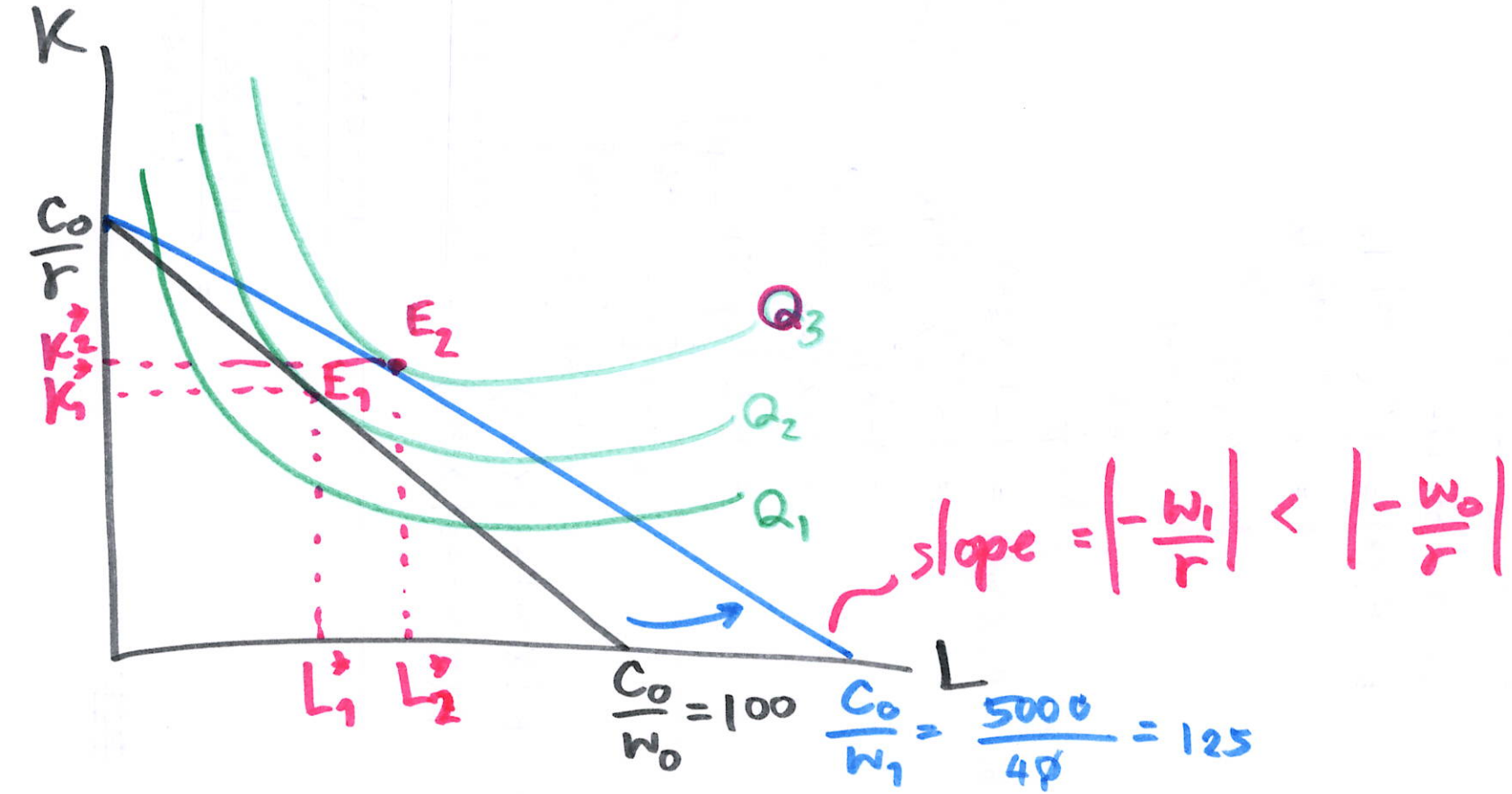
optimal $(L, K) = (L_2^*, K_2^*)$

2) Suppose w changes.

Before $w_0 = 50$, $r = 100$

After $w_1 = 40$, $r = 100$

} $C_0 = 5000$



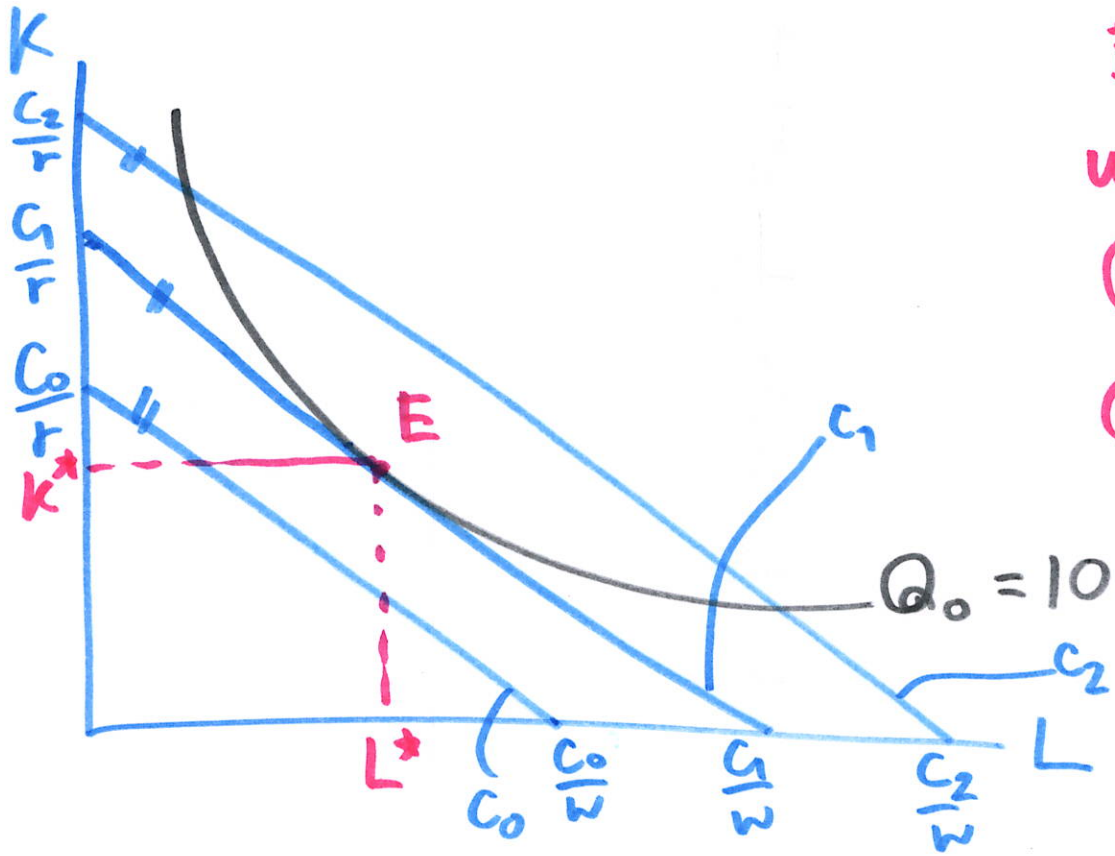
Cost Minimization

$$\text{Min } C = wL + rK$$

$$\text{subject to } Q_0 = f(L, K)$$

(Q_0: fixed)

Let
$w = 50$
$r = 100$



$E(L^*, K^*)$ is eqm under 2 criteria:

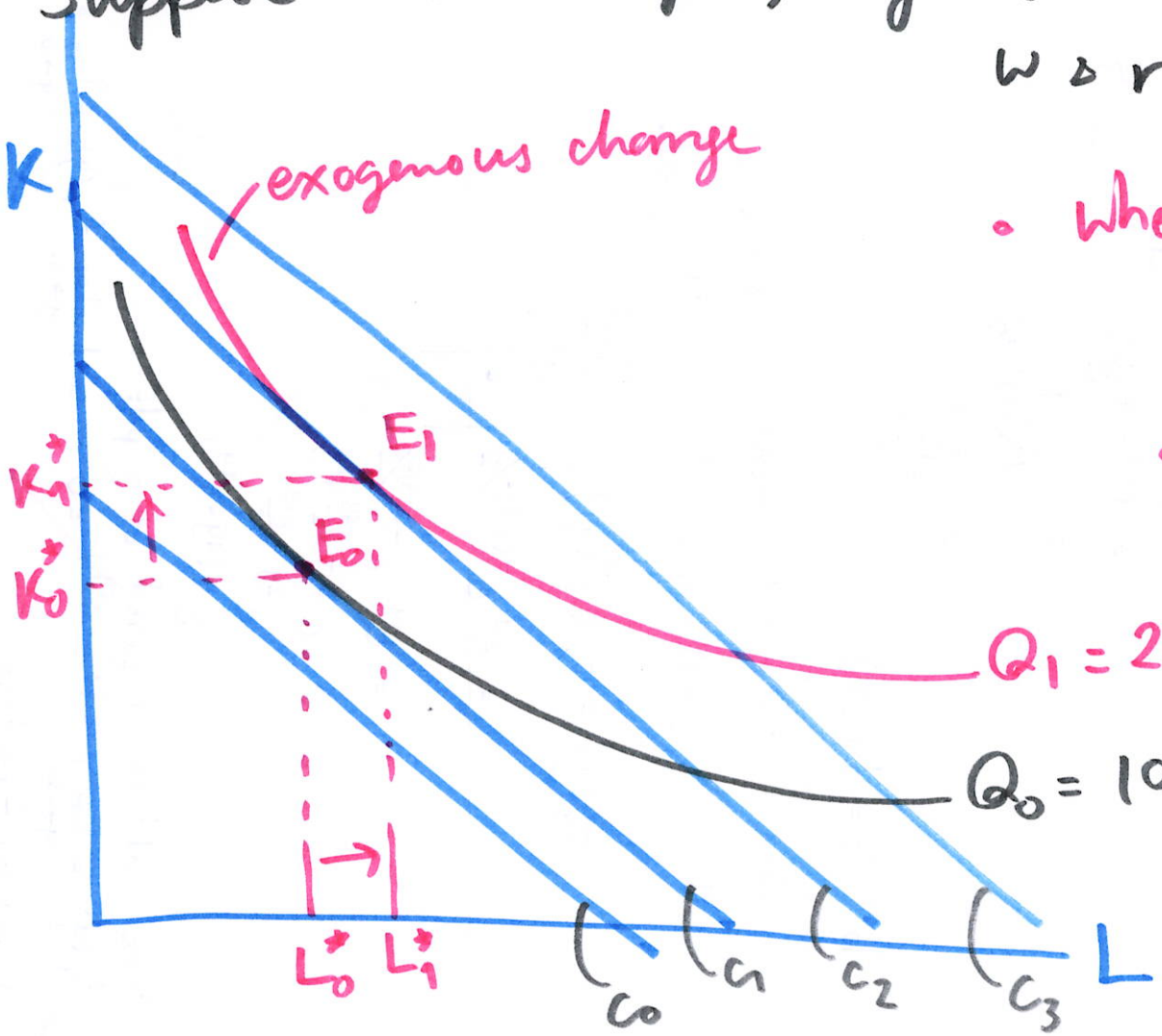
① $f(L^*, K^*) = Q_0 = 100$

② $\dfrac{MP_L(L^*, K^*)}{MP_K(L^*, K^*)} = \dfrac{w}{r}$

Changes in Eqm

1) Suppose Q_0 changes, say $Q_0 = 100$, $Q_1 = 200$

w & r are the same.



- When $Q = Q_0$

  Min cost = C_1

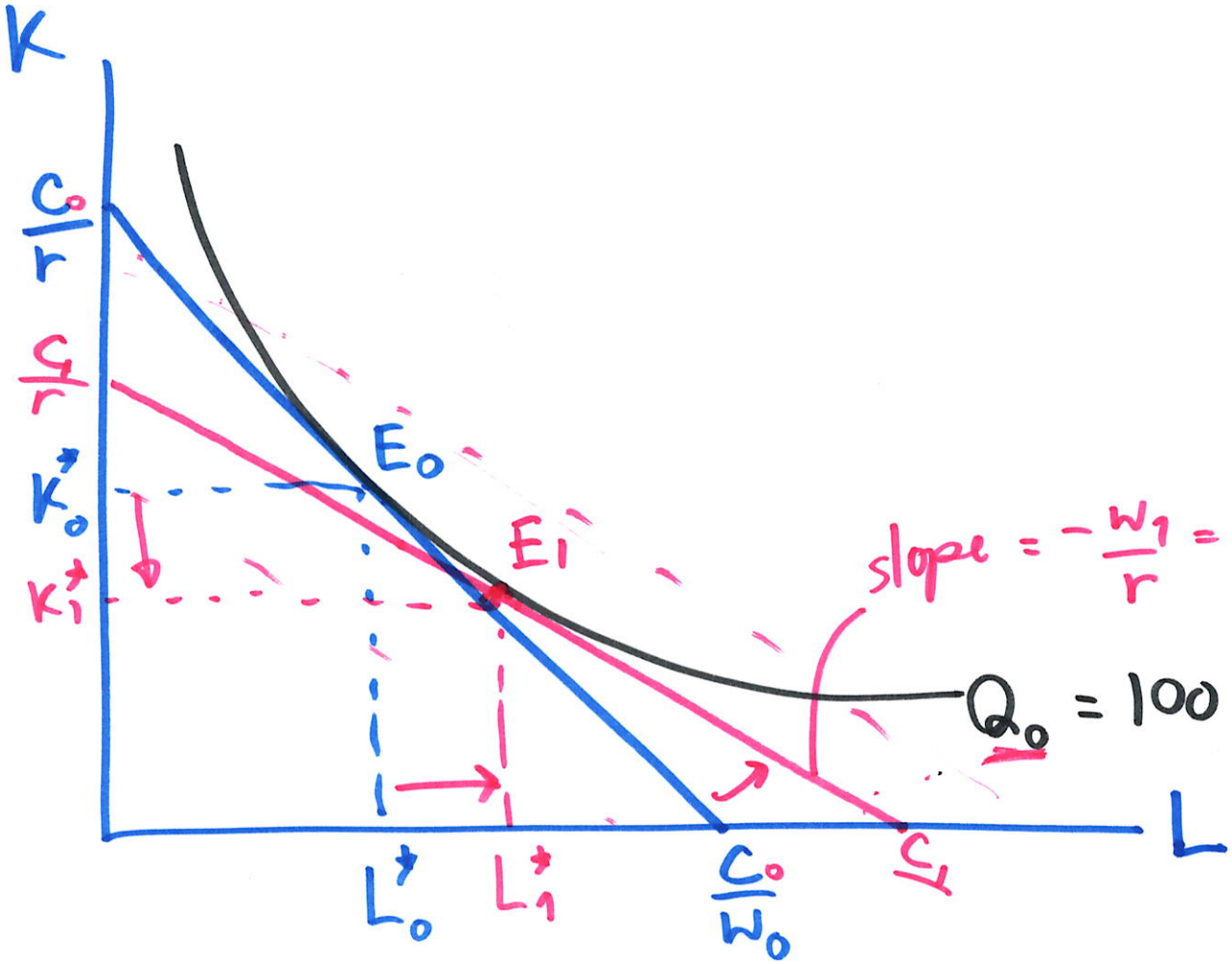
- When $Q = Q_1$

  Min cost = C_2

2) Suppose w changes (everything else constant).

$Q_0 = 100$, $w_0 = 50$, $r = 100$ → $\frac{w_0}{r} = \frac{50}{100} = \frac{1}{2}$

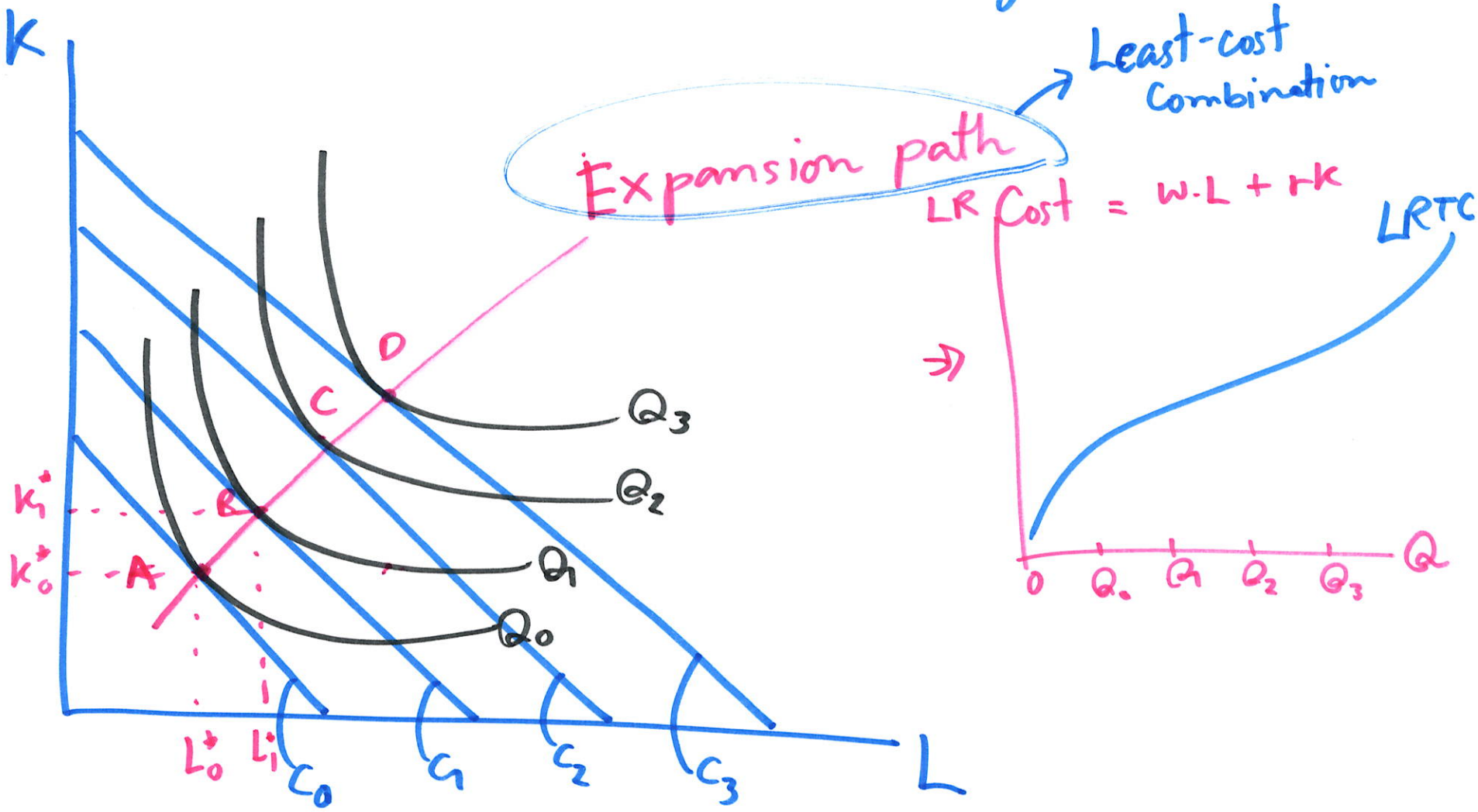
$w_1 = 40$, $r = 100$ → $\frac{w_1}{r} = \frac{40}{100} = \frac{2}{5}$

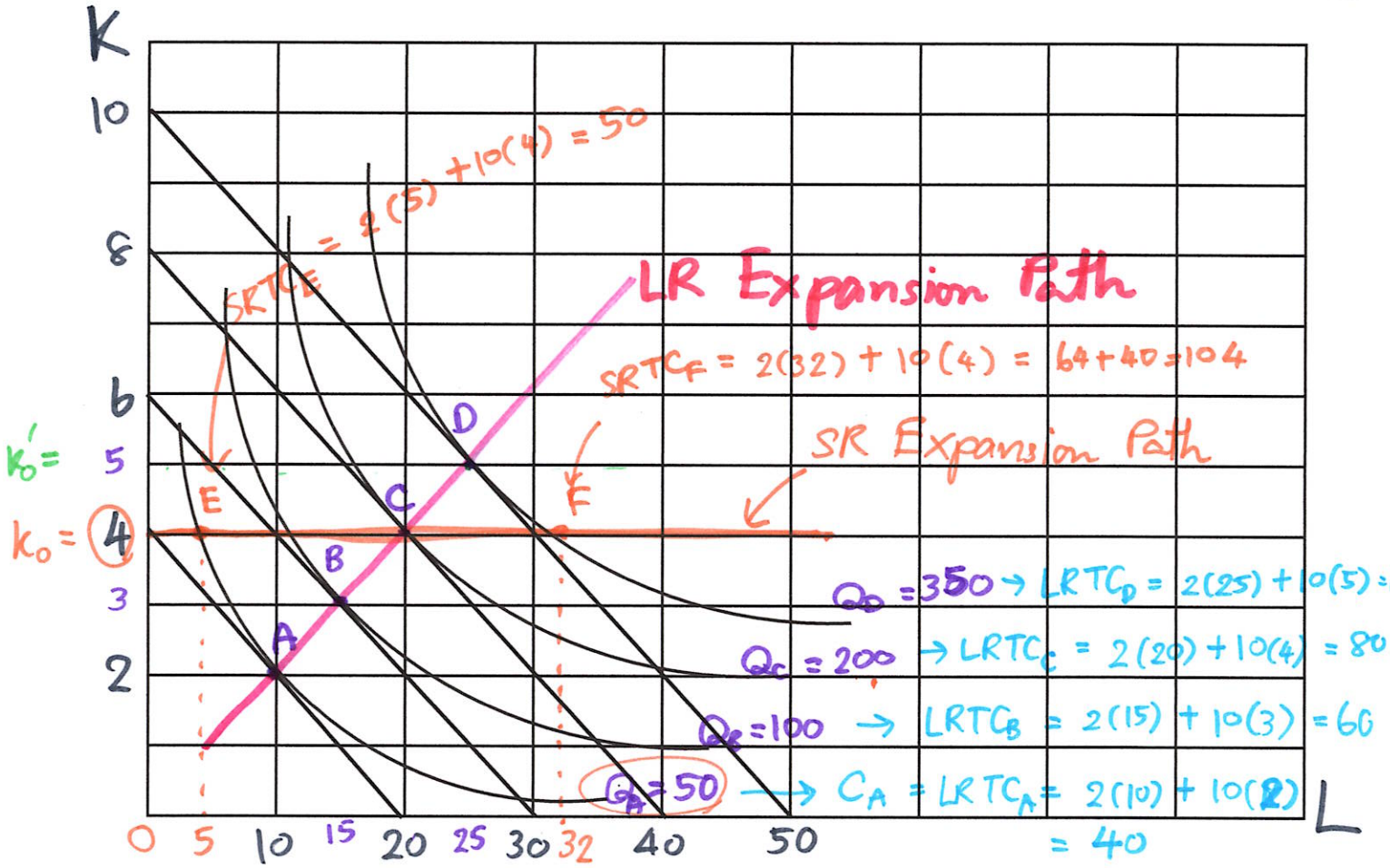


slope $= -\frac{w_1}{r} = \left|\frac{-2}{5}\right| < \left|\frac{-w_0}{r}\right|$

Expansion Path

w and r are given





Cost = $wL + rk$ $(w = 2, r = 10)$