② $\sum_{i=1}^{n} k_i = 0$

Proof: $\sum_{i=1}^{n} k_i = \dfrac{\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} x_i^2}$ where $x_i = X_i - \bar{X}$

$$= \frac{\sum_{i=1}^{n} (X_i - \bar{X})}{\sum x_i^2}$$

$$= \frac{\left(\sum_{i=1}^{n} X_i - \sum_{i=1}^{n} \bar{X}\right)}{\sum_{i=1}^{n} x_i^2}$$

$$= \frac{\left(\sum_{i=1}^{n} X_i - n \cdot \bar{X}\right)}{\sum_{i=1}^{n} x_i^2}$$

$$= \frac{\left(\sum_{i=1}^{n} X_i - \cancel{n} \cdot \frac{\sum_{i=1}^{n} X_i}{\cancel{n}}\right)}{\sum_{i=1}^{n} x_i^2}$$

$$= 0. \quad \#$$

③ $\sum_{i=1}^{n} k_i^2 = \dfrac{1}{\sum_{i=1}^{n} x_i^2}$.

Proof: $\sum_{i=1}^{n} k_i^2 = \sum_{i=1}^{n} \left(\dfrac{x_i}{\sum_{i=1}^{n} x_i^2} \right)^2 = \dfrac{\sum_{i=1}^{n} x_i^2}{\left(\sum_{i=1}^{n} x_i^2\right)^2} = \dfrac{1}{\sum_{i=1}^{n} x_i^2}$. #

④ $\sum_{i=1}^{n} k x_i = \sum_{i=1}^{n} k_i X_i = 1$.

Proof: $\sum_{i=1}^{n} k_i x_i = \sum_{i=1}^{n} k_i (X_i - \bar{X}) = \sum_{i=1}^{n} k_i X_i - \underbrace{\sum_{i=1}^{n} k_i}_{=0 \text{ (Property \#2)}} \bar{X} = \sum_{i=1}^{n} k_i X_i$

$$\sum_{i=1}^{n} k_i x_i = \sum_{i=1}^{n} \frac{x_i}{\sum_{i=1}^{n} x_i^2} \cdot x_i = \frac{\sum_{i=1}^{n} x_i^2}{\sum_{i=1}^{n} x_i^2} = 1. \quad \#$$