

# Consumption under Uncertainty

# Tools for Describing Risky Outcomes

Definition: A **lottery** is any event with an uncertain outcome.

Examples: Investment, Roulette, Football Game.

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Definition: A **probability** of an outcome (*of a lottery*) is the likelihood that this outcome occurs.

Example: The probability often is estimated by the historical frequency of the outcome.

# Tools for Describing Risky Outcomes

Definition: The **probability distribution** of the lottery depicts all possible payoffs in the lottery and their associated probabilities.

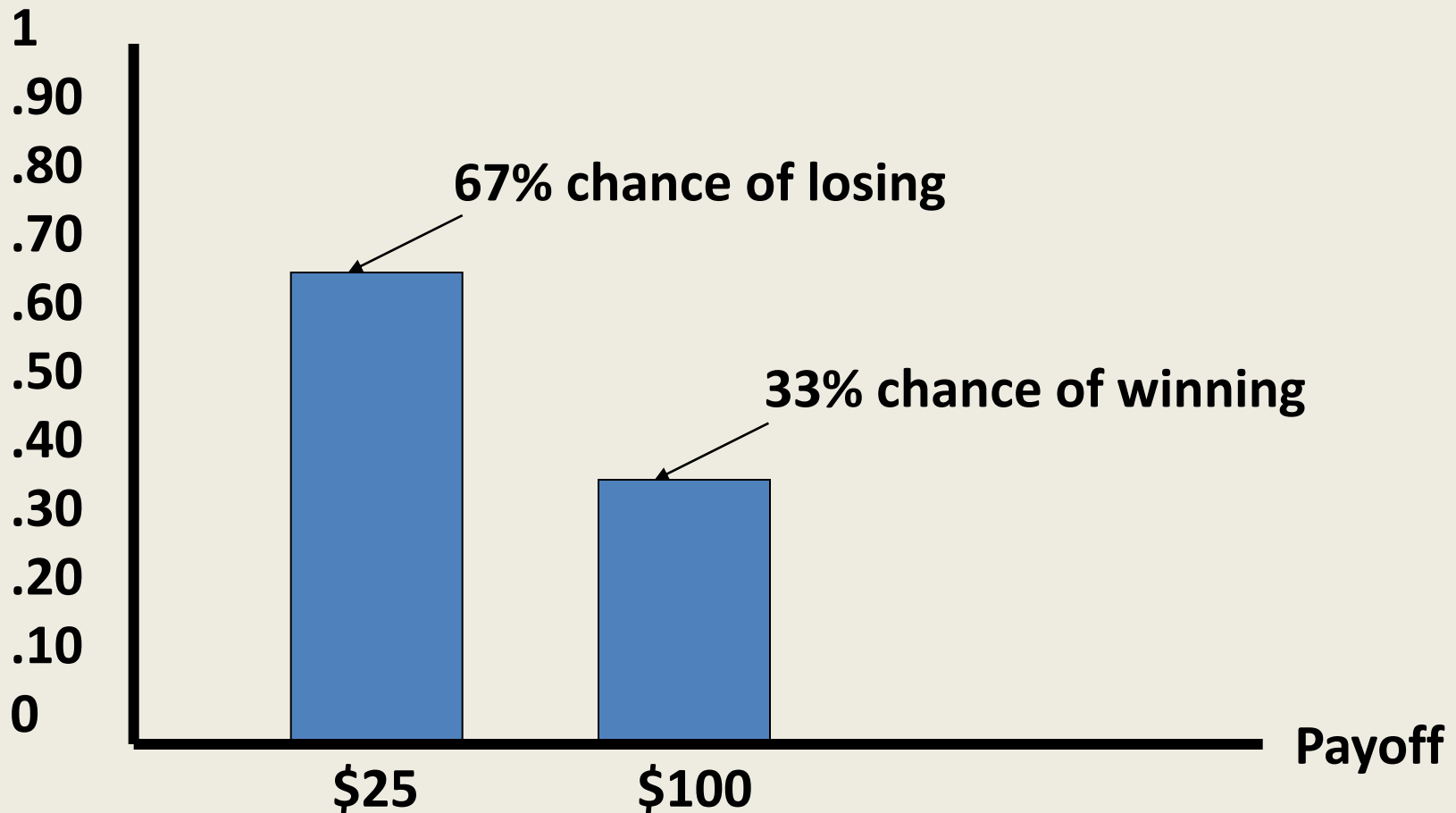
Property:

- The probability of any particular outcome is between 0 and 1.
  - The sum of the probabilities of all possible outcomes equals 1.
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Definition: Probabilities that reflect subjective beliefs about risky events are called **subjective probabilities**.

# Probability Distribution

Probability



# Expected Value and Expected Utility

Definition: The **expected value** of a lottery is a measure of the average payoff that the lottery will generate.

Let  $X$  be a lottery with three possible outcomes  $A$ ,  $B$ , and  $C$ .

$$E[X] = \text{Pr}(A) \cdot A + \text{Pr}(B) \cdot B + \text{Pr}(C) \cdot C$$

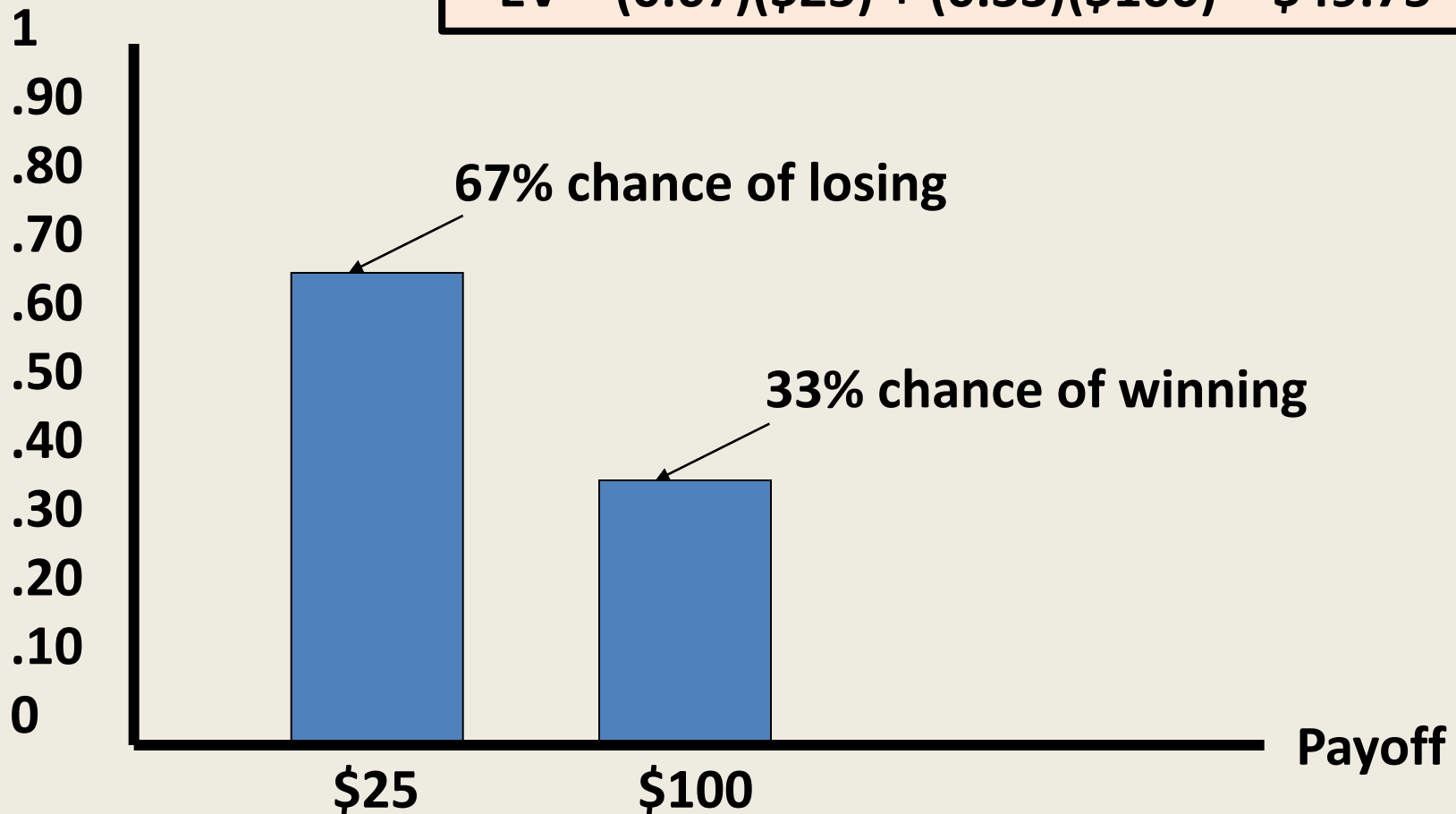
Definition: The **expected utility** of a lottery is the average value of the utility levels that the lottery will generate.

$$E[U(X)] = \text{Pr}(A) \cdot U(A) + \text{Pr}(B) \cdot U(B) + \text{Pr}(C) \cdot U(C)$$

# Expected Value

Probability

$$EV = (0.67)(\$25) + (0.33)(\$100) = \$49.75$$



# Variance & Standard Deviation

Definition: The **variance** of a lottery is a measure of the lottery's riskiness.

$$\text{Var}[X] = E[X^2] - (E[X])^2$$

$$\text{Var}[X] = (x - E[X])^2 \cdot \text{Pr}(X = x)$$

Definition: The **standard deviation** of a lottery is the square root of the variance. It is an alternative measure of risk.

# Example

$X = x$	$X = 0$	$X = 1$	$X = 2$	$X = 3$
$\Pr(X = x)$	0.1	0.2	0.4	0.3

$$\mathbf{E[X]} = 0(0.1) + 1(0.2) + 2(0.4) + 3(0.3) = 1.9$$

$$\begin{aligned}\mathbf{Var[X]} &= (\mathbf{x} - \mathbf{E[X]})^2 \cdot \mathbf{Pr(X = x)} \\ &= (0.1)(0-1.9)^2 + (0.2)(1-1.9)^2 + (0.4)(2-1.9)^2 + (0.3)(3-1.9)^2 \\ &= 0.89\end{aligned}$$

$$\mathbf{Var[X]} = \mathbf{E[X^2]} - (\mathbf{E[X]})^2$$

$$\mathbf{E[X^2]} = (0^2)(0.1) + (1^2)(0.2) + (2^2)(0.4) + (3^2)(0.3) = 4.5$$

$$\mathbf{Var[X]} = 4.5 - 1.9^2 = 0.89$$

# Risk Preferences

## Notes:

- Utility as a function of income only

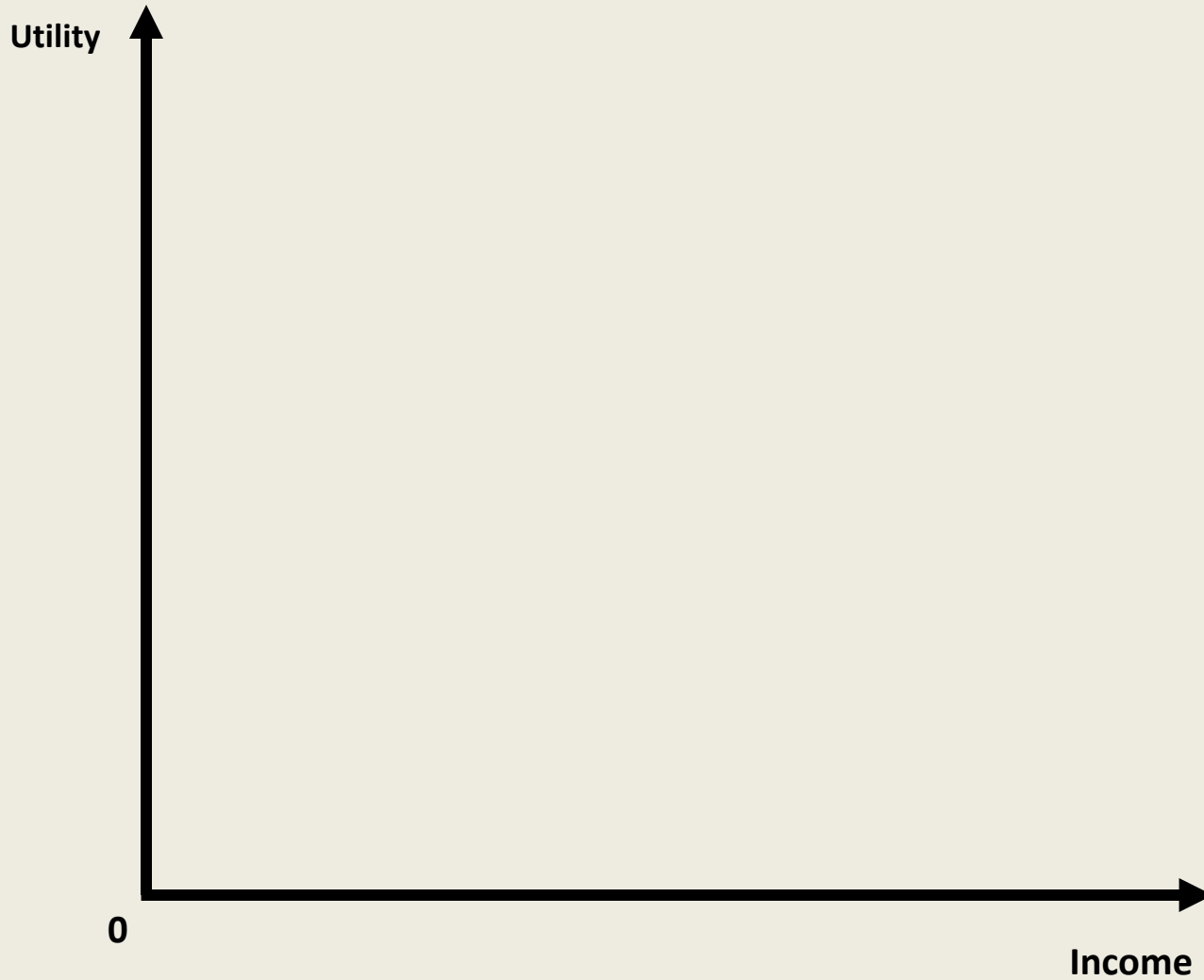
Definition: The risk preferences can be classified as follows:

An individual who prefers a sure thing to a lottery with the same expected value is **risk-averse**.

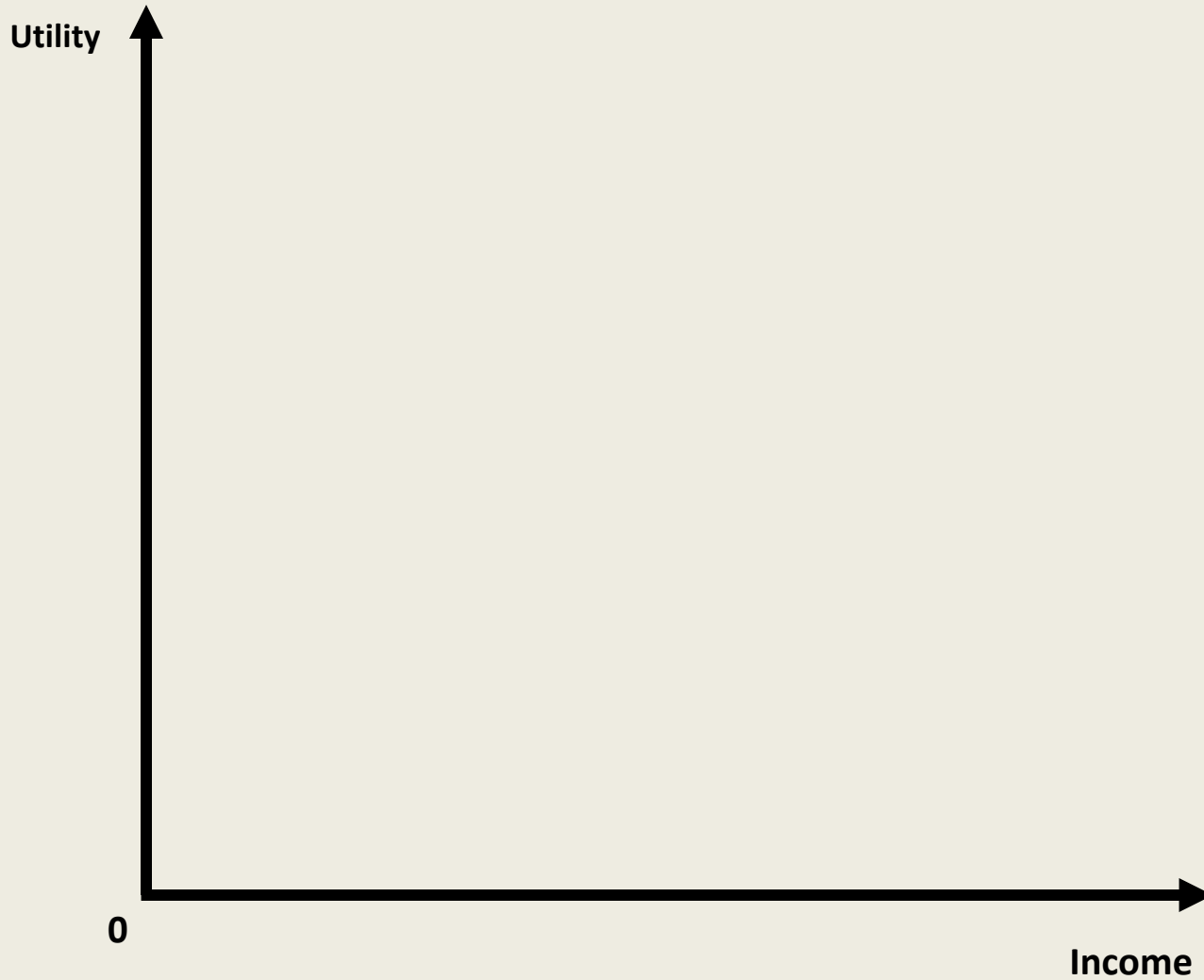
An individual who is indifferent about a sure thing or a lottery with the same expected value is **risk-neutral**.

An individual who prefers a lottery to a sure thing that equals the expected value of the lottery is **risk-loving**.

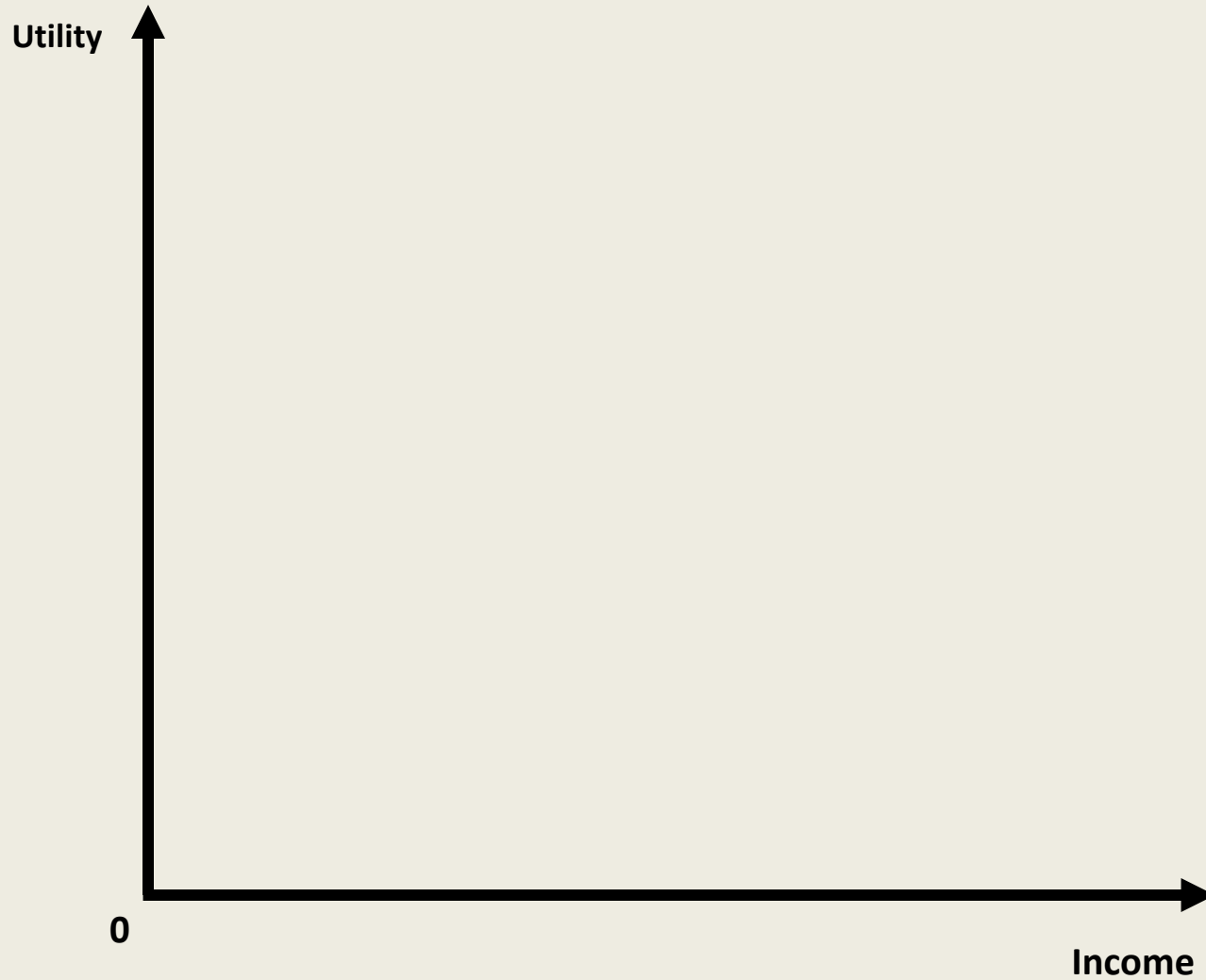
# Risk-Averse Individuals



# Risk-Neutral Individuals



# Risk-Loving Individuals



# Risk Preferences

Let  $X$  be a lottery.

$U(E[X])$  denotes **Utility of the EV (sure thing) of the lottery  $X$ .**

$E[U(X)]$  denotes **Expected Utility of the lottery  $X$ .**

**risk-averse:**  $U(E[X]) > E[U(X)]$

happiness from sure thing  $>$  expected happiness from lottery

**risk-neutral:**  $U(E[X]) = E[U(X)]$

happiness from sure thing  $=$  expected happiness from lottery

**risk-loving:**  $U(E[X]) < E[U(X)]$

happiness from sure thing  $<$  expected happiness from lottery

## Computing the Expected Utility for Two Lotteries

Consider the two lotteries depicted in Figure 15.3. They have the same expected value, but the first (investing in the Internet company's stock) has a larger variance than the second (investing in the public utility company's stock). This tells us that the first lottery is riskier than the second lottery. Suppose that a risk-averse decision maker has the utility function  $U(I) = \sqrt{100I}$ , where  $I$  denotes the payoff of the lottery.

**Problem** Which lottery does the decision maker prefer—that is, which one has the bigger expected utility?

One decision maker is risk neutral, with the utility function  $U(I) = 100I$ , while the other is risk loving, with the utility function  $U(I) = 100I^2$ , where  $I$  denotes the payoff of the lottery.

### Problem

- (a) Which lottery does the risk-neutral decision maker prefer?
- (b) Which lottery does the risk-loving decision maker prefer?

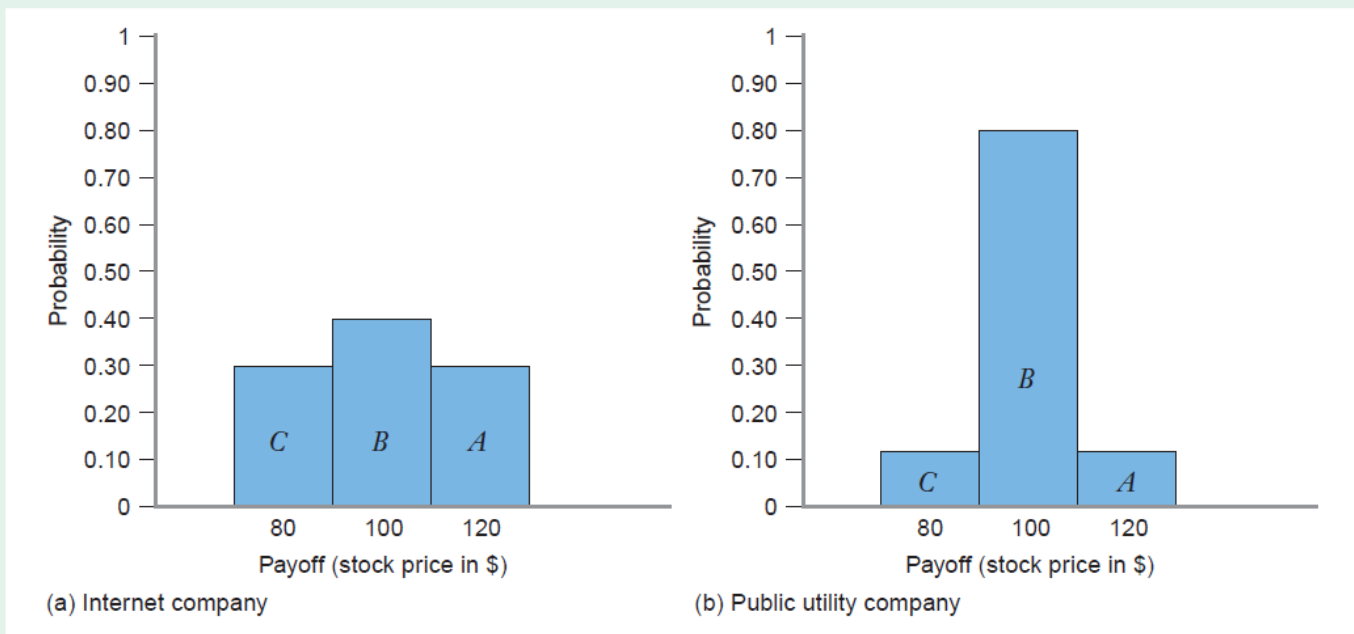


FIGURE 15.3 Probability Distributions, Riskiness, and Variances

# Risk Premium

Most people are risk-averse, but some are willing to take risks if they are fairly compensated. The money needed as the compensation is called the **risk premium**.

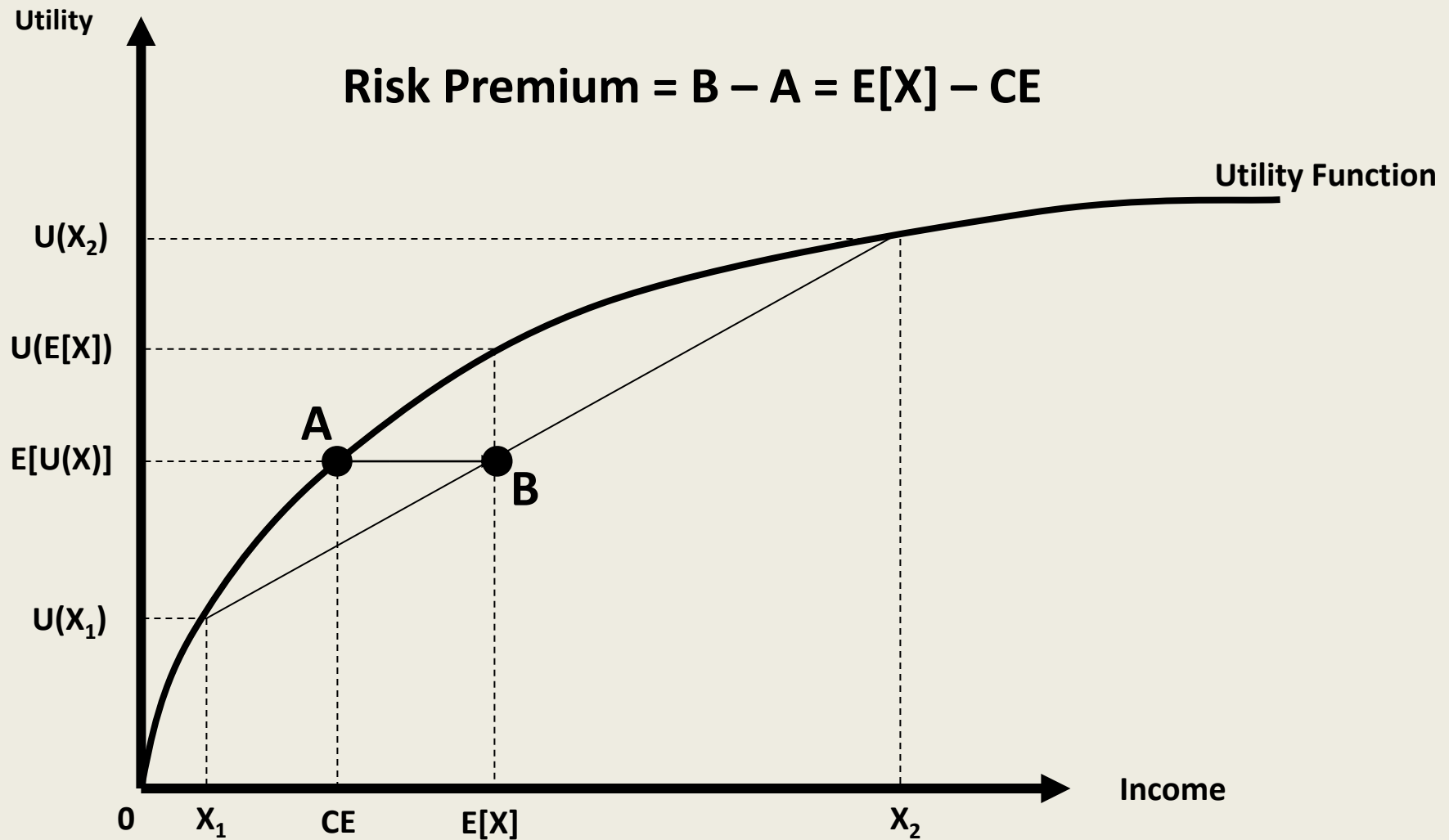
Definition: The **certainty equivalent (CE)** is the guaranteed amount of money (sure thing) such that an individual is indifferent between the lottery and the said money.

$$U(\text{CE}) = E[U(X)]$$

Definition: The **risk premium (RP)** of a lottery is the difference between the EV of the lottery and its CE.

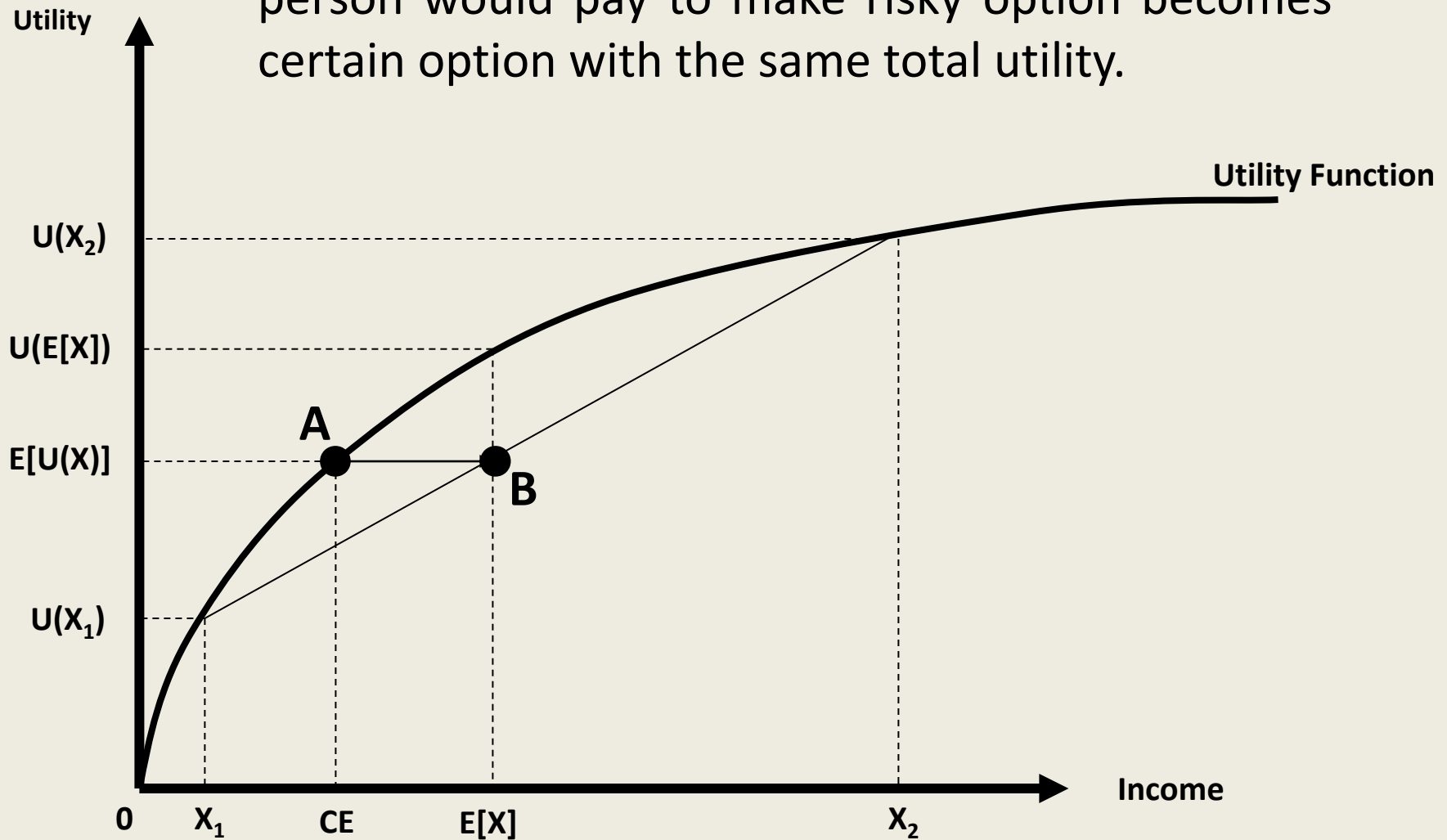
$$\text{RP} = E[X] - \text{CE}$$

# Risk Premium



## Risk Premium (B-A)

The maximum amount of money that a risk-averse person would pay to make risky option becomes certain option with the same total utility.



# Risk Premium

**Risk-Averse:  $CE < E[X]$       Risk Premium is positive**

They are willing to accept a smaller sure thing rather than the lottery.

**Risk-Loving:  $CE > E[X]$       Risk Premium is negative**

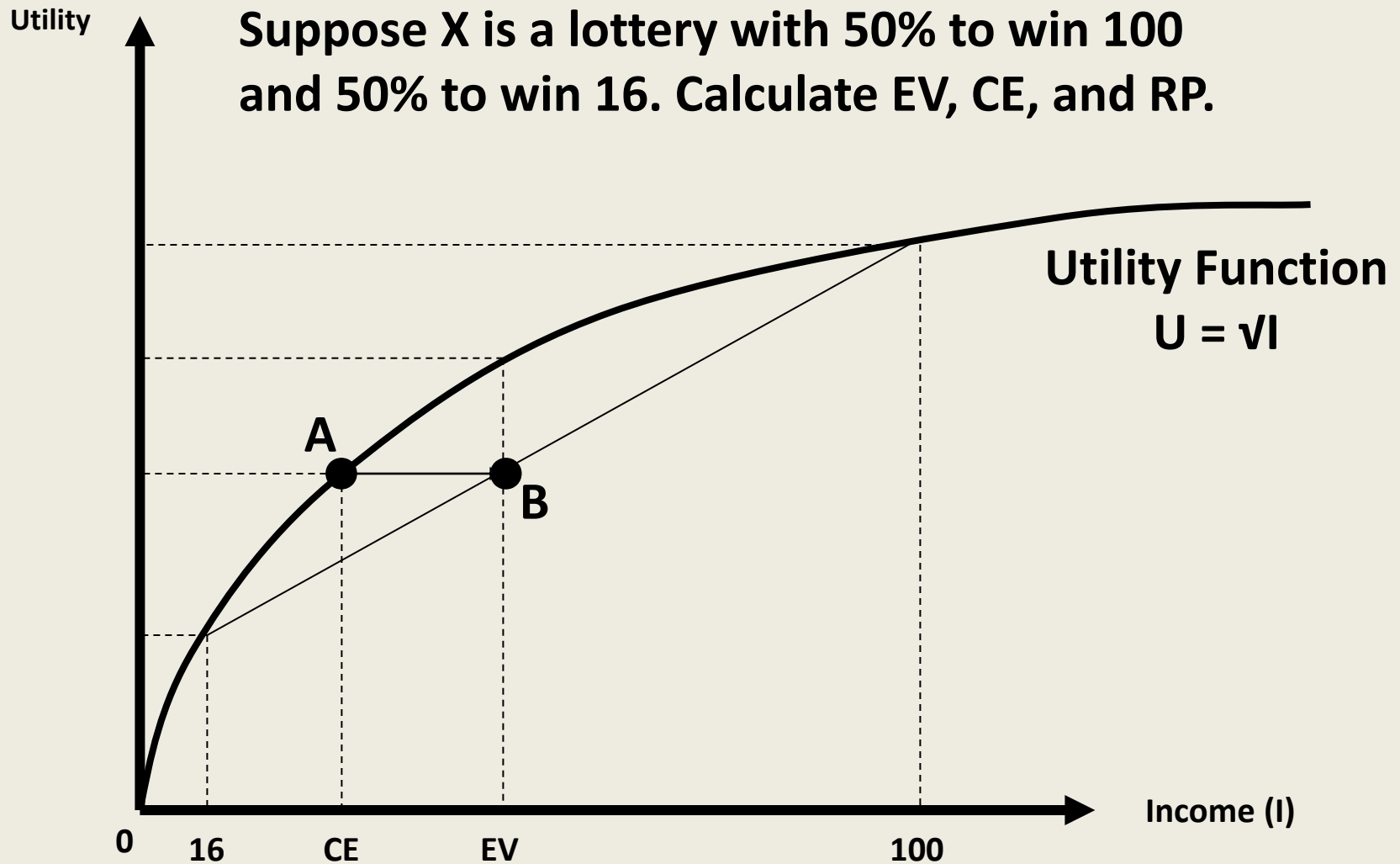
They require a larger sure thing because they love the lottery.

**Risk-Neutral:  $CE = E[X]$       Risk Premium is zero**

They are indifferent between the sure thing and the lottery.

The larger the variance of the lottery, the riskier the lottery, and the larger the risk premium

# Risk Premium – Example



# Insurance

Risk Premium is the money that risk-averse people accept to take risks.

**Insurance Premium** is the “price” of the insurance that risk-averse people buy in order to eliminate the risks.

**Fairly-Priced Insurance Policy** is a policy where the insurance premium is equal to the expected value of the insurance payment.

# Fairly-Priced Insurance – Example

## Example

Insurance Premium (price) = \$500

It promises to pay \$10,000 if Bad Thing happens.  
It promises to pay \$0 if Nothing happens.

$\text{Pr}(\text{BadThings}) = 5\%$

$\text{Pr}(\text{Nothing}) = 95\%$

$$500 = .05(10,000) + .95(0)$$

That is, insurance premium = expected payment.  
Thus, this insurance is fairly priced.



## LEARNING-BY-DOING EXERCISE 15.4

### The Willingness to Pay for Insurance

Your current disposable income is \$90,000.

Suppose that there is a 1 percent chance that your house may burn down, and if it does, the cost of repairing it will be \$80,000, reducing your disposable income to \$10,000. Suppose, too, that your utility function is  $U = \sqrt{I}$ .

#### Problem

- (a) Would you be willing to spend \$500 to purchase an insurance policy that fully insures you against your loss?
- (b) What is the highest price that you would be willing to pay for an insurance policy that fully insures you in the event that your house burns down?

The following section is based on  
the lecture slides by  
Prof. Chayun Tantivasadakarn

# The Demand for Risky Assets

- The higher the return, the greater the risk.
- Risk-averse investors must balance risk relative to return.
- In other words, there is a trade-off between return and risk.

# Trade-off: Risk and Return

- An investor is choosing between Treasury-bills (riskless), Stocks (risky), and some combination of both.

Let

- $R_f$  = risk-free return on T-bills
- $R_m$  = expected return on stocks
- Assume  $R_m > R_f$  or no risk averse investor would buy the stocks

# Trade-off: Risk and Return

Let

- $b$  = fraction of funds placed in stocks
- $(1-b)$  = fraction of funds placed in T-bills
- The expected return on the portfolio is the weighted average of expected returns on the two assets:

$$R_P = bR_m + (1-b)R_f$$

# Trade-off: Risk and Return

For example,

Suppose  $R_m = 12\%$ ,  $R_f = 4\%$ , and  $b = 1/2$ .

Then,

$$R_P = bR_m + (1 - b)R_f$$

$$R_P = (1/2)(12\%) + (1 - 1/2)(4\%)$$

$$R_P = 8\%$$

# Trade-off: Risk and Return

- How risky is the portfolio?
  - As stated before, one measure of risk is standard deviation.
  - Standard deviation of the risk-free asset = 0
  - Standard deviation of the risky asset,  $\sigma_m$
  - Standard deviation of the risky portfolio,  $\sigma_p$
  - Thus,

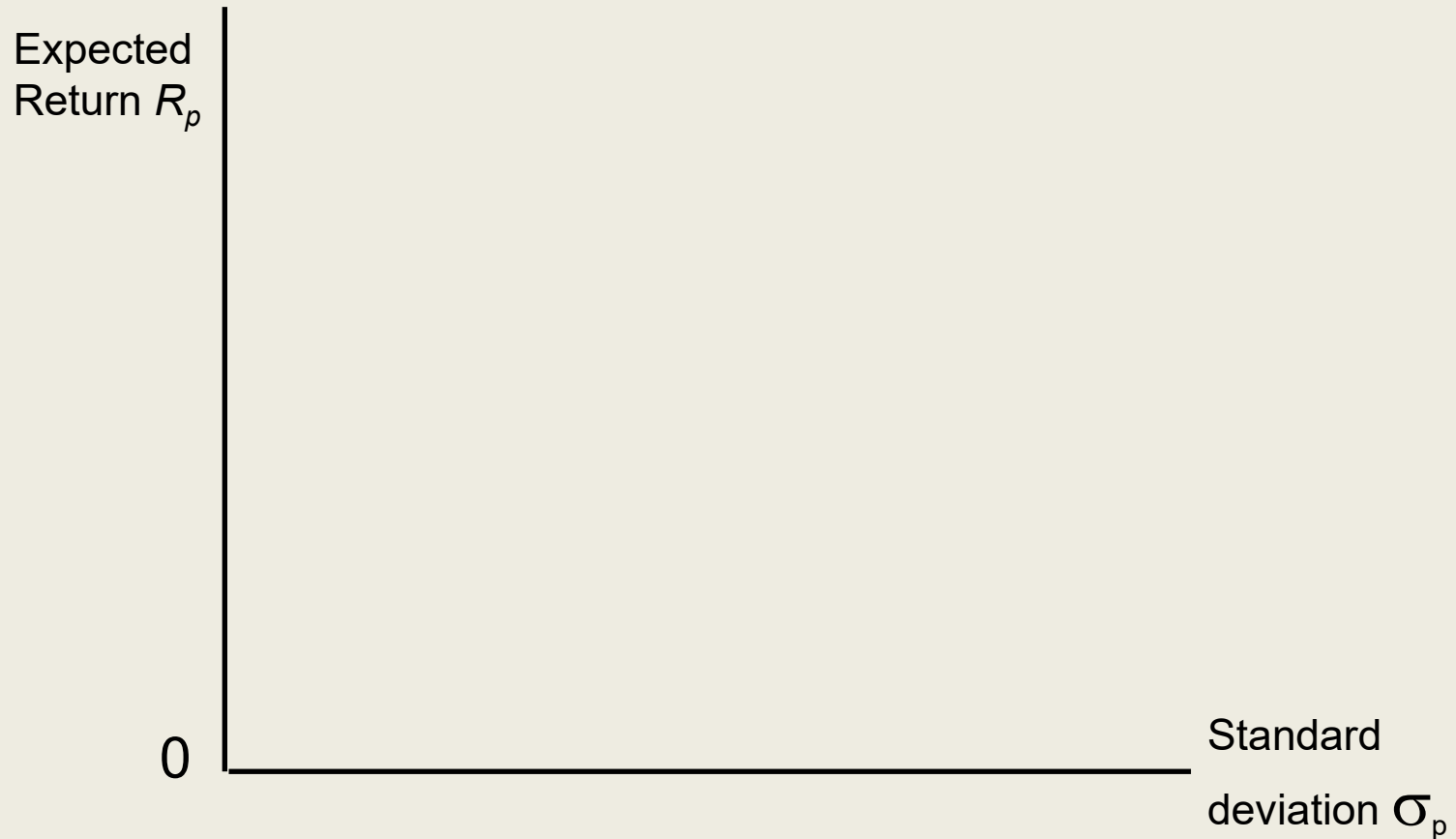
$$\sigma_p = b \sigma_m$$

# Trade-off: Risk and Return

- To see how the investor allocates the portfolio, we can apply the theory of consumer choice, using a budget line and ICs.
- A type of budget line can be constructed describing the trade-off between risk and expected return.

$$R_p = bR_m + (1 - b)R_f$$
$$R_p = R_f + \frac{(R_m - R_f)}{\sigma_m} \sigma_p$$

# Trade-off: Risk and Return



- Expected return on the portfolio,  $R_p$  increases as the standard deviation,  $\sigma_p$  of that return increases

# Trade-off: Risk and Return

- The slope of the line is called the price of risk.
- It tells how much extra risk an investor must incur to enjoy a higher expected return.

$$\textit{Slope} = (R_m - R_f) / \sigma_m$$

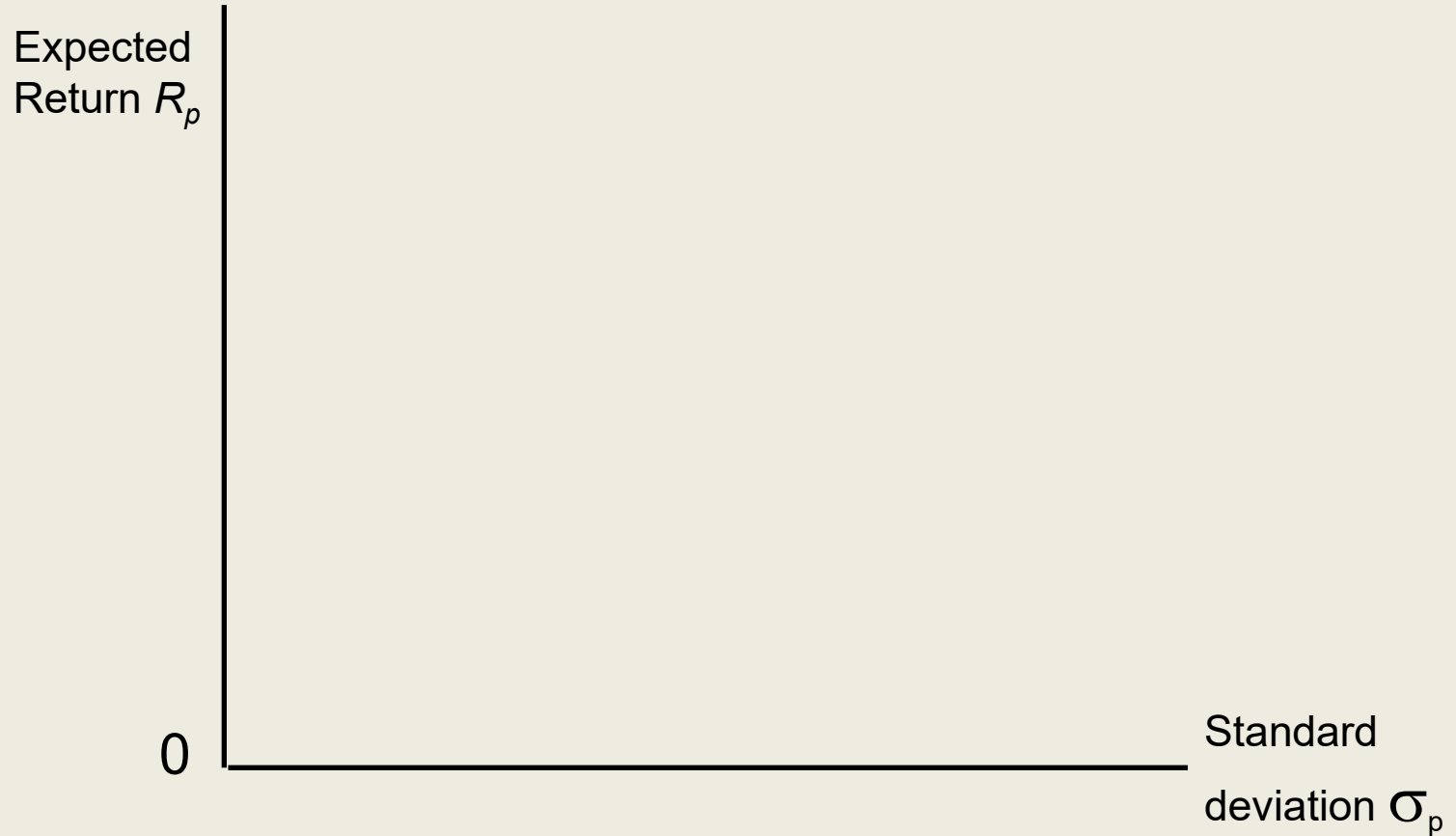
# Trade-off: Risk and Return

- If all funds are invested in T-bills ( $b=0$ ), expected return is  $R_f$
- If all funds are invested in stocks ( $b=1$ ), expected return is  $R_m$  but with standard deviation of  $\sigma_m$
- Funds may be invested between the assets with expected return between  $R_f$  and  $R_m$ , with standard deviation between  $\sigma_m$  and 0.

# Trade-off: Risk and Return

- We can draw indifference curves showing combinations of risk and return that leave an investor equally satisfied.
- Comparing the pay-offs and risk between the two investment choices and the preferences of the investor, the optimal portfolio choice can be determined.

# Trade-off: Risk and Return



# Trade-off: Risk and Return

- Different investors have different attitudes toward risk.
- If we consider a very risk averse investor,
  - Portfolio will contain mostly T-bills and less stocks with a return slightly larger than  $R_f$
- If we consider a riskier investor,
  - Portfolio will contain mostly stocks and less T-bills with a higher return  $R_b$  but with higher standard deviation

# Trade-off: Risk and Return

