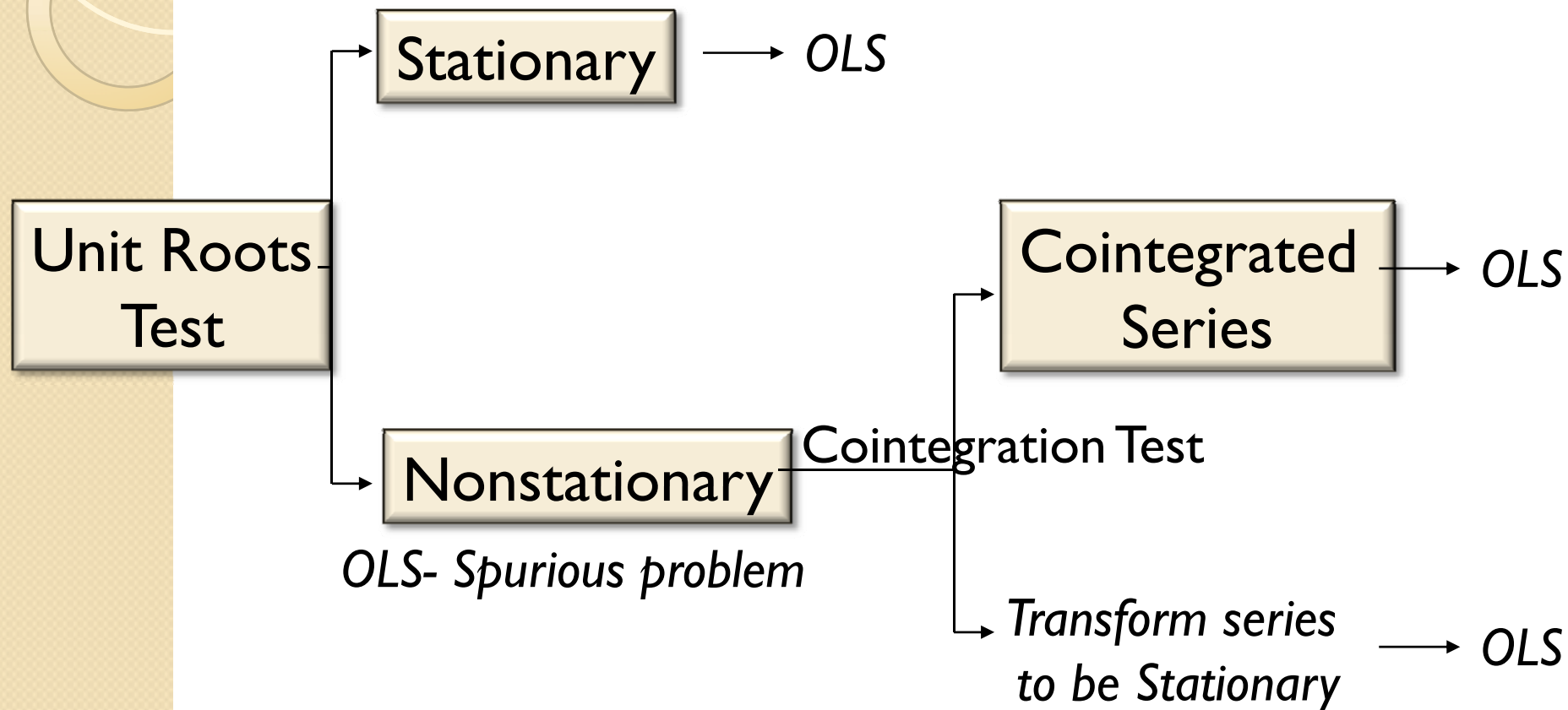




# Time Series Econometrics

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# Basic Concept



# Stochastic Processes

OLS assume that  $X_s$  must be nonstochastic process variables

Time-series is a sequence of random variables ordered in time.

## Stochastic Process

The probability structure of a sequence of random variables is determined by the joint distribution of a stochastic process.

i.e.  $Y_t$  is stochastic series and white-noise process if  $Y_t = u_t, \quad u_t \sim n.i.d.(0, \sigma^2)$

# Stochastic Processes

## Stochastic Process

- Stationary Process
- Nonstationary Process

## Properties of Stationary Process

1. Mean of series must be stationary

$$E(Y_t) = \mu$$

2. Variance of series must be stationary

$$\text{var}(Y_t) = E(Y_t - \mu)^2 = \sigma^2$$

3. Covariance of Series must be stationary

$$\gamma_k = E[(Y_t - \mu)(Y_{t+k} - \mu)]$$

# Nonstationary Processes

## Random Walk without Drift

$$Y_t = Y_{t-1} + u_t$$

We can derive:  $Y_1 = Y_0 + u_1$

$$Y_2 = Y_1 + u_2 = Y_0 + u_1 + u_2$$

Then,  $Y_t = Y_0 + \sum u_t$

Therefore,  $E(Y_t) = E\left(Y_0 + \sum u_t\right) = Y_0$

Then,  $\text{var}(Y_t) = t\sigma^2$

and  $(Y_t - Y_{t-1}) = \Delta Y_t = u_t$

# Nonstationary Processes

## Random Walk with Drift

$$Y_t = \delta + Y_{t-1} + u_t$$

where  $\delta$  is drift parameter

Then,  $(Y_t - Y_{t-1}) = \Delta Y_t = \delta + u_t$

Therefore,  $E(Y_t) = Y_0 + t \cdot \delta$

and  $\text{var}(Y_t) = t\sigma^2$

# Unit Root Stochastic Processes

From random walk model:

$$Y_t = \rho Y_{t-1} + u_t \quad -1 \leq \rho \leq 1$$

If  $\rho = 1$ , the model is random walk without drift.

The model then is known as unit root problem, or nonstationary.

However, if  $|\rho| < 1$ , the model is stationary.

These terms – nonstationary, random walk, and unit root – can be treated as synonymous.

# Trend vs Difference Stationary

## Deterministic vs Stochastic

Deterministic – if trend of time series is completely predictable and not variable.

Stochastic – if trend of time series is not predictable.

$$Y_t = \beta_1 + \beta_2 t + \beta_3 Y_{t-1} + u_t$$

where  $u_t$  is a white noise error term.

$t$  is time trend



# Trend vs Difference Stationary

From:  $Y_t = \beta_1 + \beta_2 t + \beta_3 Y_{t-1} + u_t$

Pure random walk  $\beta_1 = 0, \beta_2 = 0, \beta_3 = 1$

$$Y_t = Y_{t-1} + u_t \quad \text{and} \quad \Delta Y_t = (Y_t - Y_{t-1}) = u_t$$

Thus, random walk without drift is nonstationary while its first difference is stationary – called as **difference stationary process (DSP)**.

# Trend vs Difference Stationary

From:  $Y_t = \beta_1 + \beta_2 t + \beta_3 Y_{t-1} + u_t$

Random walk with drift  $\beta_1 \neq 0$ ,  $\beta_2 = 0$ ,  $\beta_3 = 1$

$$Y_t = \beta_1 + Y_{t-1} + u_t \quad \text{and} \quad \Delta Y_t = (Y_t - Y_{t-1}) = \beta_1 + u_t$$

Thus, random walk with drift is nonstationary.

However, the series shows positive ( $\beta_1 > 0$ ) or negative ( $\beta_1 < 0$ ), then it is called **stochastic trend**.

Its first difference is stationary – also **DSP process**.

# Trend vs Difference Stationary

From: 
$$Y_t = \beta_1 + \beta_2 t + \beta_3 Y_{t-1} + u_t$$

Deterministic Trend  $\beta_1 \neq 0, \beta_2 \neq 0, \beta_3 = 0$

$$Y_t = \beta_1 + \beta_2 t + u_t$$

This series is **trend stationary process (TSP)**  
and mean equal  $\bar{Y}_t = \beta_1 + \beta_2 t$ , which is not  
constant, its variance is constant.

We can **detrend** the series by:  $Y_t - \bar{Y}_t$

# Trend vs Difference Stationary

From: 
$$Y_t = \beta_1 + \beta_2 t + \beta_3 Y_{t-1} + u_t$$

Random walk with drift and deterministic trend

$$\beta_1 \neq 0, \beta_2 \neq 0, \beta_3 = 1$$

$$Y_t = \beta_1 + \beta_2 t + Y_{t-1} + u_t \quad \text{and} \quad \Delta Y_t = \beta_1 + \beta_2 t + u_t$$

$Y_t$  is nonstationary process.

# Trend vs Difference Stationary

From:  $Y_t = \beta_1 + \beta_2 t + \beta_3 Y_{t-1} + u_t$

Deterministic trend with stationary AR(I) component  $\beta_1 \neq 0, \beta_2 \neq 0, \beta_3 < 1$

$$Y_t = \beta_1 + \beta_2 t + \beta_3 Y_{t-1} + u_t$$

$Y_t$  is stationary around deterministic trend.

# Integrated Stochastic Processes

## Integrated Series

Nonstationary series can be integrated series if the series is differentiated one or more times, the resulting series will be stationary.

RWMM without drift is nonstationary, but its first difference is stationary, the series is integrated of order 1 or  $I(1)$ .

If  $Y_t$  and its first difference ( $\Delta Y_t$ ) are nonstationary, but its second difference is stationary, the series is integrated of order 2 or  $I(2)$ .

If  $Y_t$  is nonstationary, but its  $d^{th}$  difference is stationary, the series is integrated of order  $d$  or  $I(d)$ .

# Integrated Stochastic Processes

## Properties of Integrated Series

1. If  $X_t \sim I(0)$  and  $Y_t \sim I(1)$ , then  $Z_t = (X_t + Y_t) \sim I(1)$

Linear combination of stationary and nonstationary time series is nonstationary.

2. If  $X_t \sim I(d)$ , then  $Z_t = (a + bX_t) \sim I(d)$

Linear combination of an  $I(d)$  series is  $I(d)$ .

3. If  $X_t \sim I(d_1)$  and  $Y_t \sim I(d_2)$ , then  $Z_t = (aX_t + bY_t) \sim I(d_2)$   
where  $d_1 < d_2$ .

4. If  $X_t \sim I(d)$  and  $Y_t \sim I(d)$ , then  $Z_t = (aX_t + bY_t) \sim I(d^*)$ ;  
 $d^*$  is generally equal to  $d$ , but in some case  $d^* < d$ .

# Spurious Problem

If  $X_t$  and  $Y_t$  are uncorrelated nonstationary series, OLS estimated result of model

$$Y_t = \beta_1 + \beta_2 X_t + u_t$$

can lead to spurious problem.

Spurious regression has a high  $R^2$  and  $t$ -statistics appear to be significant, but the results are without any theoretical meaning.

An  $R^2 > DW$  is good rule of thumb to suspect that the estimated regression is spurious.



# Unit Roots Test

Statistical test that test whether the series is stationary or nonstationary

- Dickey-Fuller (DF) Test
- Augmented Dickey-Fuller (ADF) Test
- Dickey-Fuller GLS (ERS)
- Phillips-Perron Test
- Kwiatkowski-Phillips-Schmidt-Shin
- Elliott-Rothenberg Stock Point-Optimal
- Ng-Perron

# Unit Roots Test – DF Test

Suppose  $Y_t = \alpha + \beta Y_{t-1} + u_t$

When,  $t = 1$  then,  $Y_1 = \alpha + \beta Y_0 + u_1$

$$\begin{aligned} t = 2, \quad Y_2 &= \alpha + \beta(\alpha + \beta Y_0 + u_1) + u_2 \\ &= \alpha(1 + \beta) + \beta^2 Y_0 + (u_2 + \beta u_1) \end{aligned}$$

Then,  $Y_t = \alpha(1 + \beta + \beta^2 + \dots + \beta^{t-1}) + \beta^t Y_0$   
 $+ (u_t + \beta u_{t-1} + \beta^2 u_{t-2} + \dots + \beta^{t-1} u_1)$

For stationary condition,  $|\beta| < 1$ , then,

$$E(Y_t) = \mu = \frac{\alpha}{1 - \beta}$$

# Unit Roots Test – DF Test

If the series is stochastic trend process:

$$Y_t = \delta_0 + \delta_1 t + u_t \quad \text{and} \quad u_t = \beta u_{t-1} + \varepsilon_t$$

Then, 
$$Y_t = [\delta_0(1-\beta) + \beta\delta_1] + \delta_1(1-\beta)t + \beta Y_{t-1} + \varepsilon_t$$

DF test suggests to estimate the model:

$$Y_t - Y_{t-1} = \Delta Y_t = \underbrace{[\delta_0(1-\beta) + \beta\delta_1]}_{\gamma = \beta - 1} + \underbrace{\delta_1(1-\beta)t}_{\alpha_1 t} + \varepsilon_t$$

Where:  $\gamma = \beta - 1$

Then, 
$$\Delta Y_t = \alpha_0 + \alpha_1 t + \gamma Y_{t-1} + \varepsilon_t$$

DF test --  $\tau$  (tau) statistic is t-test of  $\gamma$  using critical value from MacKinnon.

# Unit Roots Test – ADF Test

DF test assumes  $1^{st}$  order autocorrelation.

ADF test assumes higher order.

For example,  $2^{nd}$  order autocorrelation:

$$Y_t = \delta_0 + \delta_1 t + u_t \quad \text{and} \quad u_t = \beta_1 u_{t-1} + \beta_2 u_{t-2} + \varepsilon_t$$

Then, the test equation:

$$\Delta Y_t = \underbrace{\left[ \delta_0 (1 - \beta_1 - \beta_2) + (\beta_1 + \beta_2) \delta_1 \right]}_{\substack{+ \gamma Y_{t-1} + \beta_2 \Delta Y_{t-1} \\ \text{Intercept} \\ \text{Lags}}} + \underbrace{\delta_1 (1 - \beta_1 - \beta_2) t}_{\text{Trend}} + \varepsilon_t$$

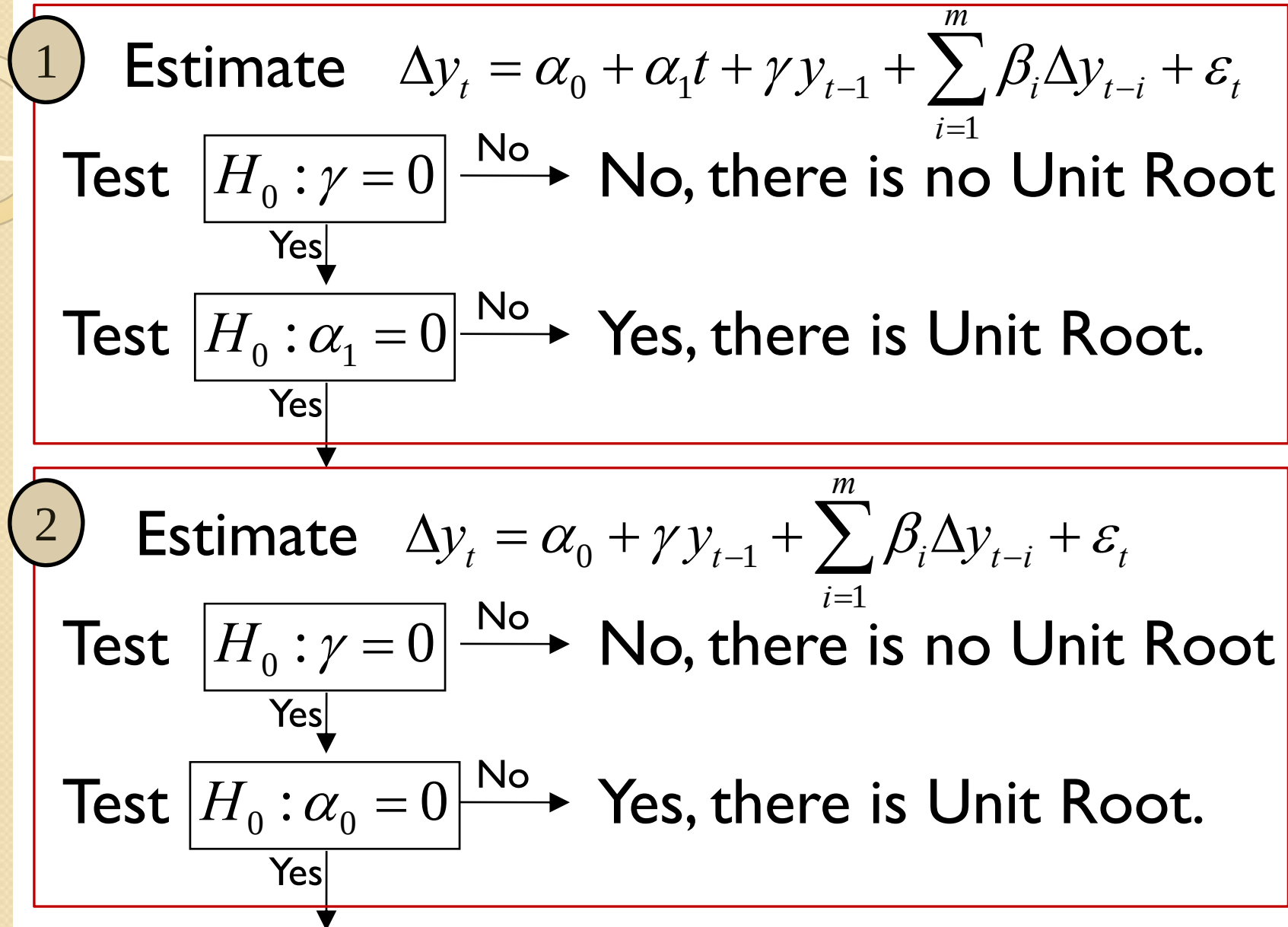


# Unit Roots Test in Practice

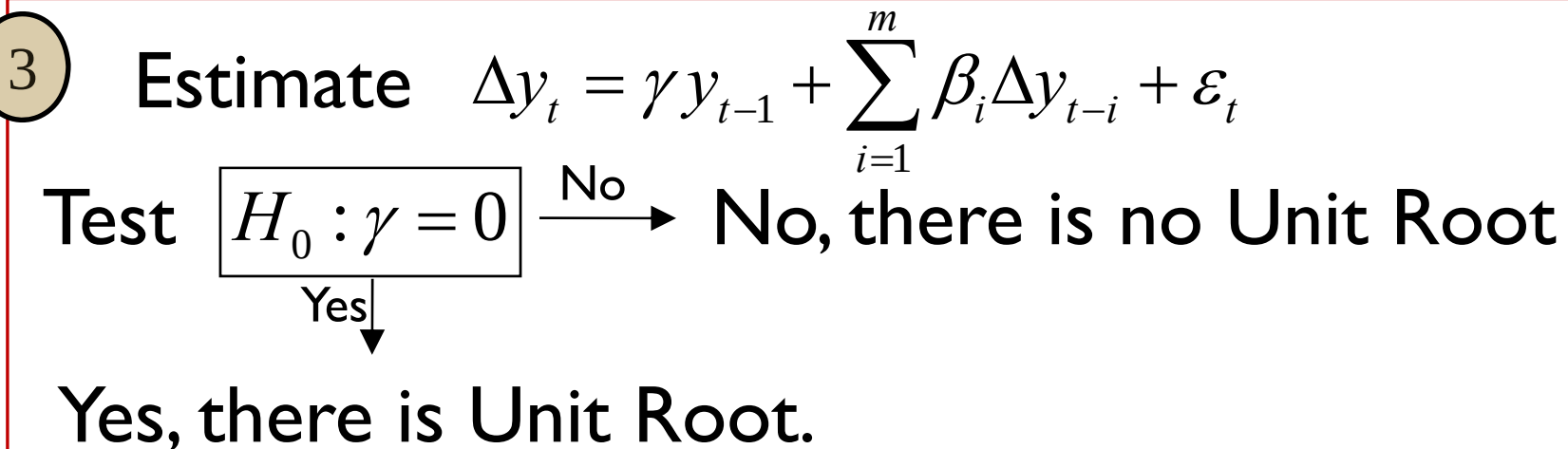
Setting up the test:

- Testing Method
- Level of test
- Testing equation
- Optimal lags criteria

# Unit-Test Process



# Unit-Test Process



# Cointegration

## Cointegrated Time-series

If  $X_t$  and  $Y_t$  are nonstationary, but their linear combination  $u_t = Y_t - \beta_1 - \beta_2 X_t$  is stationary.

Then,  $X_t$  and  $Y_t$  are cointegrated time-series.

## Cointegration Test

Statistical test that tests whether the series are cointegrated or not.

- Augmented Engle-Granger (AEG) Test
- Multivariate (Johansen) Test



# Cointegration and Error Correction Mechanism (ECM)

## Cointegration Regression

Long-run relationship of cointegrated time series can be estimated using OLS.

$$Y_t = \beta_1 + \beta_2 X_t + u_t$$

## Error Correction Mechanism (ECM)

ECM can be stated as:

$$\Delta Y_t = \alpha_0 + \alpha_1 \Delta X_t + \delta \hat{u}_{t-1} + \varepsilon_t$$

$\delta$  determines speed of adjustment to equilibrium and is expected to be negative and  $-1 < \delta < 0$  .