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BACHELOR OF ECONOMICS
THAMMASAT UNIVERSITY

**EE320(sec 046402)**

Semester 1/2020

**Quiz#3 Solution**

**This quiz has two questions. Each question will be treated as if they are separated quizzes which will be added to the pool of your quizzes throughout the semester that you can drop the lowest score.**

**Question 1** Let the production function be $Q = 4K^{1/5}L^{3/5}$, $Q$ is total output, $K$ is capital, and $L$ is labor.

(a.) Show that production function is decreasing return to scale

***Solution:***

$$Q(tK, tL) = 4(tK)^{\frac{1}{5}}(tL)^{\frac{3}{5}} = t^{\frac{4}{5}}4K^{\frac{1}{5}}L^{\frac{3}{5}} = t^{\frac{4}{5}}Q(K, L)$$

Since $t^{\frac{4}{5}}Q(K, L) < tQ(K, L)$ because $\frac{4}{5} < 1$, the given Cobb-Douglas production function is decreasing return to scale.

(b.) Find $MP_K$ and $MP_L$

***Solution:***

$$MP_K = \frac{\partial Q}{\partial K} = \frac{4}{5}K^{-\frac{4}{5}}L^{\frac{3}{5}}$$

$$MP_L = \frac{\partial Q}{\partial L} = \frac{12}{5}K^{\frac{1}{5}}L^{\frac{-2}{5}}$$

(c.) Show that the slope of isoquant curve, which is the number of units of factor capital (K) that can be substituted by one unit of factor labor (L) keeping the same level of output), is equal to $-MP_L/MP_K$.

***Solution:***

On isoquant curve, changing labor and capital along the isoquant curve yield zero change in quantity.

$$dQ = \frac{\partial Q}{\partial K}dK + \frac{\partial Q}{\partial L}dL = 0$$

$$dQ = MP_K\, dK + MP_L dL = 0$$

Hence,

$$\frac{dK}{dL} = -\frac{MP_L}{MP_K}$$

(d.) What is the slope of isoquant curve, when the production function is $Q = 4K^{1/5}L^{3/5}$ ?

***Solution***

From answer in (b.) and (c.),

$$\frac{dK}{dL} = -\frac{MP_L}{MP_K} = -\frac{\frac{12}{5}K^{\frac{1}{5}}L^{\frac{-2}{5}}}{\frac{4}{5}K^{-\frac{4}{5}}L^{\frac{3}{5}}} = -\frac{3K}{L}$$

**Question 2**

Consider a firm who produces two snacks: $A$ (Apple pie) and $B$ (Boba). Suppose this firm is the monopolist in both markets of $A$ (Apple pie) and $B$ (Boba). Market demand for $A$ (Apple pie) and $B$ (Boba) are:

Market demand for $A$ (Apple pie): $Q_A = 4(P_B - P_A)$

Market demand for $B$ (Boba): $Q_B = 4(9 + P_A - 2P_B)$

Total cost of this firm is $TC = 2Q_A + 3Q_B + 100$. Find $P_A, P_B, Q_A, Q_B$ that maximize firm's profit.

***Solution***

From the question:

$$Q_A = 4(P_B - P_A) \qquad - (1)$$

$$Q_B = 4(9 + P_A - 2P_B) \qquad - (2)$$

$$TC = 2Q_A + 3Q_B + 100$$

Profit function is:

$$\pi = TR_A + TR_B - TC$$

$$= P_A Q_A + P_B Q_B - TC$$

Since the firm is a monopolist, we need to find the inverse market demand function in each market to get the total revenue from each market:

From (1) and (2), we have:

$$-P_A + P_B = \frac{1}{4}Q_A \qquad - (3)$$

$$P_A - 2P_B = \frac{1}{4}Q_B - 9 \qquad - (4)$$

(3) + (4):
$P_B = -\frac{1}{4} Q_A - \frac{1}{4} Q_B + 9$ -(5)
From (3) & (5)
$P_A = -\frac{1}{2} Q_A - \frac{1}{4} Q_B + 9$ -(6)

Use inverse market demand function in (5) and (6) to get at profit function:

$$\pi = P_A Q_A + P_B Q_B - TC$$
$$= (-\frac{1}{2}Q_A - \frac{1}{4}Q_B + 9)Q_A + (-\frac{1}{4}Q_A - \frac{1}{4}Q_B + 9)Q_B - 2Q_A - 3Q_B - 100$$

Rearrange and get the objective function:

$$Max\,\pi = -\frac{1}{2}Q_A^2 - \frac{1}{4}Q_B^2 - \frac{1}{2}Q_A Q_B + 7Q_A + 6Q_B - 100$$

Necessary Condition:

$$\frac{\partial \pi}{\partial Q_A} = -Q_A - \frac{1}{2}Q_B + 7 = 0 \quad \text{---->} \quad Q_A + \frac{1}{2}Q_B = 7 \qquad \text{- (7)}$$
$$\frac{\partial \pi}{\partial Q_B} = -\frac{1}{2}Q_B - \frac{1}{2}Q_A + 6 = 0 \quad \text{---->} \quad \frac{1}{2}Q_A + \frac{1}{2}Q_B = 6 \qquad \text{- (8)}$$

(7) – (8):

$$\frac{1}{2}Q_A = 1 \quad \text{---->} \quad Q_A^* = 2,$$
$$Q_B^* = 2(7 - Q_A^*) = 2(7-2) = 10$$

Substitute $Q_A^*$ and $Q_B^*$ into inverse market demand function to get price for each market:

$$\overline{P_A} = -\frac{1}{2}Q_A - \frac{1}{4}Q_B + 9 = -\frac{1}{2}(2) - \frac{1}{4}(10) + 9 = 5.5$$
$$\overline{P_B} = -\frac{1}{4}Q_A - \frac{1}{4}Q_B + 9 = -\frac{1}{4}(2) - \frac{1}{4}(10) + 9 = 6$$

Sufficient Condition:
The Hessian matrix is:

$$H = \begin{bmatrix} \pi_{AA} & \pi_{AB} \\ \pi_{BA} & \pi_{BB} \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 \pi}{\partial Q_A^2} & \frac{\partial^2 \pi}{\partial Q_B \partial Q_A} \\ \frac{\partial^2 \pi}{\partial Q_A \partial Q_B} & \frac{\partial^2 \pi}{\partial Q_B^2} \end{bmatrix}$$
$$= \begin{bmatrix} -1 & -1/2 \\ -1/2 & -1/2 \end{bmatrix}$$

Check the definiteness of Hessian matrix:

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$$|H_1| = \pi_{AA} = -1 < 0$$

$$|H_2| = \pi_{AA}\pi_{BB} - (\pi_{BA}\pi_{AB})^2$$

$$= (-1)(-1/2) - (-1/2)^2 = 1/4 > 0$$

Hence, at $Q_A^* = 2$ and $Q_B^* = 10$ and price of A and B equal to 5.5 and 6 accordingly, firm maximizes profit.