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EE431 Economics of Financial Markets and Institutions, 2/2016

Problem Sets 8 : Convexity, Excessive Risk and Bank Regulation

Please submit at the BE office, 5th floor department of Economics building.
Deadline of submission : May 9, 2016, before 15.00 hrs.
Late submission will not be accepted.
In the context of convexity, excessive risk and bank regulations, answer all the following questions.

1. Suppose that there are two investment projects, A and B. Each requires initial investment of 1,000.

   Project A returns 1,500 with probability 0.5 and 700 with probability 0.5.

   Project B returns 1,300 with probability 0.5 and 900 with probability 0.5.

   Answer all parts of this question.

   (a) Suppose that the project is financed by 100% equity. If the owner is risk neutral, which project the owner of the project will prefer? Why? Explain.

   ANSWER. If the project is financed by 100% equity and the owner is risk neutral, the owner will be indifferent between the two projects.

   Expected payoff on a gamble = $\sum_i p_i Y_i$, where p_i is the probability of the outcome Y_i to occur

   Expected payoff on project A = $(0.5 \times 1,500) + (0.5 \times 700) = 1,100$.

   Expected payoff on project B = $(0.5 \times 1,300) + (0.5 \times 900) = 1,100$.

   The project is financed by 100% equity, therefore Equity's payoff is equal to the expected payoff on the project.

   Risk neutral is indifferent with the risk. They prefer higher return. For a risk neutral, return is good, risk is neither good or bad.

   The two projects has the same expected payoffs but different level of the risk. **Project A is riskier** (higher spread, higher variance).

   Therefore, the owner who is assumed to be risk neutral would be **indifferent between the two projects.**

   (b) Now suppose that the owner (shareholder) of the investment project does not have any initial capital. Therefore, it has to borrow from a lender (bondholder) and repay 1100. Limited liability is assumed. Suppose that the lenders and the owner are risk-neutral. Which project the owner would prefer? Which project the lender would prefer?

   ANSWER.

   ANSWER 1. Which project the owner would prefer? If the owner does not have initial capital, the owner's payoff will not equal to the expected payoff on the project. The owner would **prefer Project A.**

   The owner's liability is limited. The owner's payoff (γ) = $Max(0, Y - L)$, where Y is the payoff of the project and L is the value of debt.

   L = $1,000 \times 1.1 = 1,100$.

   The owner's payoff on project A =γ_A

   $$\begin{aligned} \gamma_A &= 0.5Max[0, 1500 - 1100] + 0.5Max[0, 700 - 1100] \\ &= (0.5 \times 400) + (0.5 \times 0) \\ &= 200 \end{aligned}$$

The owner's payoff on project B $=\gamma_B$

$$\begin{aligned}\gamma_B &= 0.5Max[0, 1300-1100] + 0.5Max[0, 900-1100] \\ &= (0.5 \times 200) + (0.5 \times 0) \\ &= 100\end{aligned}$$

$\gamma_A > \gamma_B$; Therefore, the owner **would prefer project A**, the riskier project.

ANSWER 2. Which project the lender would prefer? The lender's payoff $(\alpha) = Min(L, Y)$. Lender's payoff is fixed. However, since the owner's liability is limited, the lender will not be paid in full when the project's outcome is lower than the debt value.

The lender's payoff on project A $=\alpha_A$

$$\begin{aligned}\alpha_A &= 0.5Min[1100, 1500] + 0.5Max[1100, 700] \\ &= (0.5 \times 1,100) + (0.5 \times 700) \\ &= 900\end{aligned}$$

The lender's payoff on project B $=\alpha_B$

$$\begin{aligned}\alpha_B &= 0.5Min[1, 100, 1, 300] + 0.5Max[1100, 900] \\ &= (0.5 \times 1100) + (0.5 \times 900) \\ &= 1000\end{aligned}$$

$\alpha_A < \alpha_B$; Therefore, the lender **would prefer project** B, the less riskier project.

(c) Discuss moral hazard and asset substitution problem in the model.

ANSWER. Moral hazard problem arises in the model because lender have no control over which project the borrower will choose. The borrower has an incentive to substitute low risk project (Project B) with high risk one (Project A). This is called **"Asset Substitution Problem"**. In doing so, the owner would gain while the lender would loss. The owner's payoff increases ad the level of risk increases. The lender's payoff decreases as the level of risk increases. The owner would gain $\gamma_A - \gamma_B = 200 - 100 = 200$. Equity gain is equal to 300. Lender would loss $\alpha_B - \alpha_A = 1000 - 900 = 100$. The lender's loss is equal to 300. [Or you may say the lender's payoff is equal to $\alpha_A - \alpha_B = -100$. The minus sign indicates "loss".] The borrower has **limitted liability**. The borrower does not have to take full reponsibility when the project fails. Thus, the borrower's payoff is convex and the lender's payoff is concave. The borrower, hence, gains when the level of risk increases and the lender loses when the level of risk increases. There is a conflict of interest between the borrower and the lender on the level of risk. Moral hazard problem emerges.

(d) Suggest one way to design a loan contract that reduces asset substitution problem.

ANSWER. There are various ways in which the lenders of the firm can protect themselves from moral hazard. We may write covenants in the loan contract that limit the scope for asset substitution.

- Lenders may limit the borrower's new debt issues. Moral hazard problem worsens as debt level rises. To litmit the new debt issues would limit the moral hazard problem.
- Ask for collateral. Collateral would have two-fold benefit. The owner provide some property to be pledged as collaeral. Then, now the owener will lose collateral when the project's payoff is lower than debt value. The lender can seize the collateral in case of default.
  - The owner will have to take responsibility when project's yield is low. This reduces moral hazard problem.
  - The lender will receive payment when the project return is low by liquidating the collateral.