Macroeconomics

Lecture # 12

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The New Growth Theory

- The first view:
  - The driving force of growth is the accumulation of knowledge.
- The second view:
  - Extending capital to include human capital.

Part A: Research and Development Models

- To introduce an explicit research and development sector, and model the production of new technologies.
- Assumptions:
  - Both the R&D and goods production functions are generalized Cobb-Douglas functions.
  - Takes the fraction of output saved and the fractions of the labor force and the capital stock used in the R&D sector as exogenous.

The Model

- Fraction a_L of labor force is used in the R&D sector and fraction 1 - a_L in the good-producing sector.
- Fraction a_K of capital stock is used in R&D and the rest in goods production.
- Both sectors use the full stock of knowledge, A.

The quantity of output produced at time t is

$$Y(t)=\left[(1-a_K)K(t)\right]^{\alpha}\left[A(t)(1-a_L)L(t)\right]^{1-\alpha}, \quad 0<\alpha<1. \quad (7)$$

The production of new ideas depended on K, L and A,

$$\dot{A}(t)=G\left(a_K K(t), a_L L(t), A(t)\right) \quad (8)$$

Under the assumption of generalized Cobb – Douglas production,

$$\dot{A}(t)=B\left[a_K K(t)\right]^{\beta}\left[a_L L(t)\right]^{\gamma} A(t)^{\theta}, \quad B>0, \beta\geq 0, \gamma\geq 0, \quad (9)$$

The production function of knowledge is not assumed to have constant returns to scale in capital and labor.

Depreciation is set to zero, thus

$$\dot{K}(t) = sY(t) \qquad (10)$$

Population growth is

$$\dot{L}(t) = nL(t), \qquad n \geq 0 \qquad (11)$$

Sustituting the production function (7) *into* (10)

$$\dot{K}(t) = s(1-a_K)^{\alpha}(1-a_L)^{1-\alpha} K(t)^{\alpha} A(t)^{1-\alpha} L(t)^{1-\alpha} \qquad (12)$$

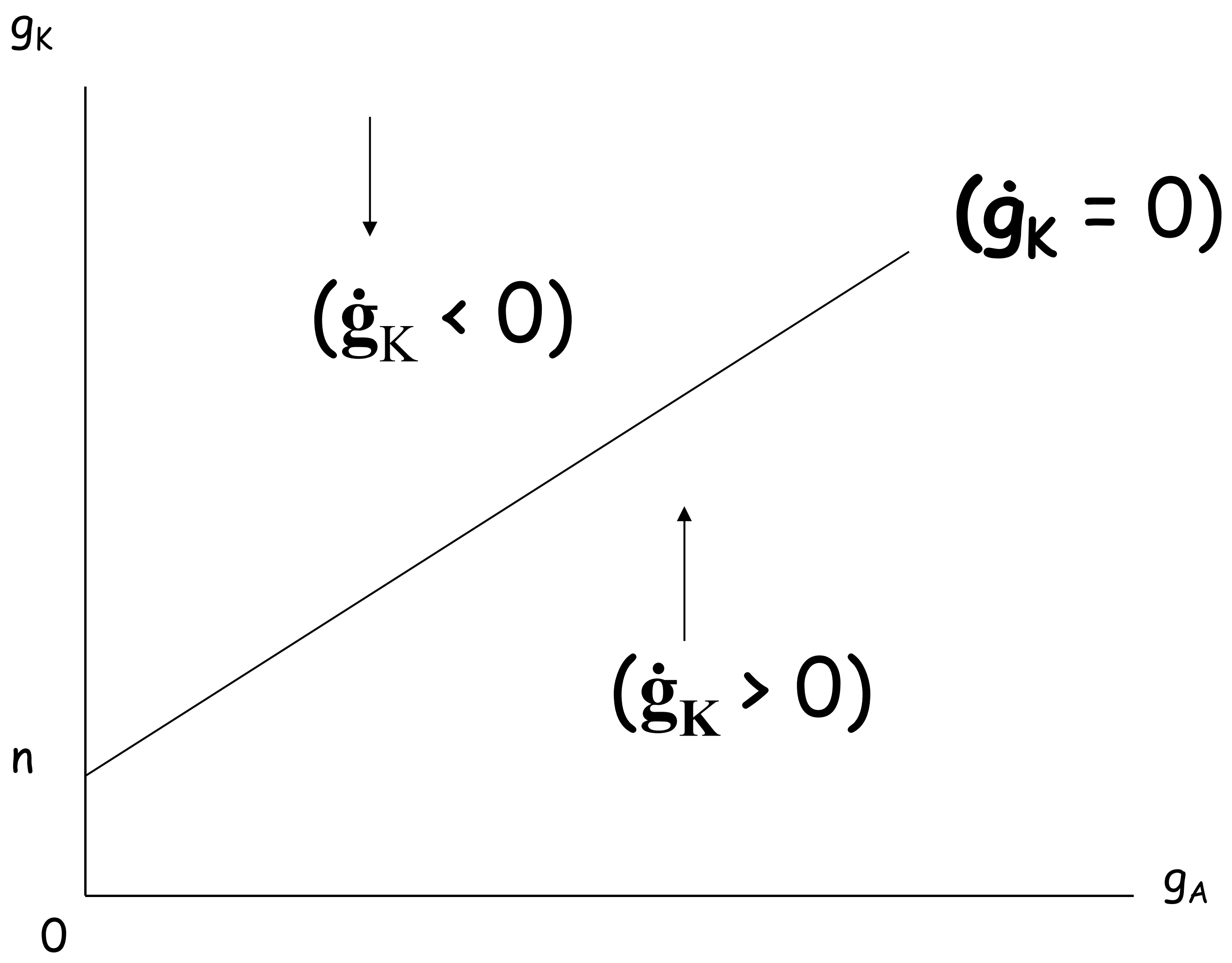
Dividing both sides by $K(t)$ *and defining* $c_K = s(1-a_K)^{\alpha}(1-a_L)^{1-\alpha}$,

$$g_K(t) \equiv \frac{\dot{K}(t)}{K(t)} = c_K \left[\frac{A(t)L(t)}{K(t)}\right]^{1-\alpha} \qquad (13)$$

The growth rate of $\frac{A(t)L(t)}{K(t)}$ *is* $g_A + n - g_K$. *Thus* g_K *is rising*

if $g_A + n - g_K$ *is positive.*($\because \ln g_K(t) = \ln c_K + (1-\alpha)(\ln A(t) + \ln L(t) - \ln K(t))$,

hence, $\dot{g}_K(t) = 0 = g_A(t) + n - g_K(t)$)



g_K
$(\dot{g}_K = 0)$
$(\dot{g}_K < 0)$
$(\dot{g}_K > 0)$
n
g_A
0

$Similarly,\ dividing\ both\ sides\ of\ (9)\ by\ A(t)\ yields,$

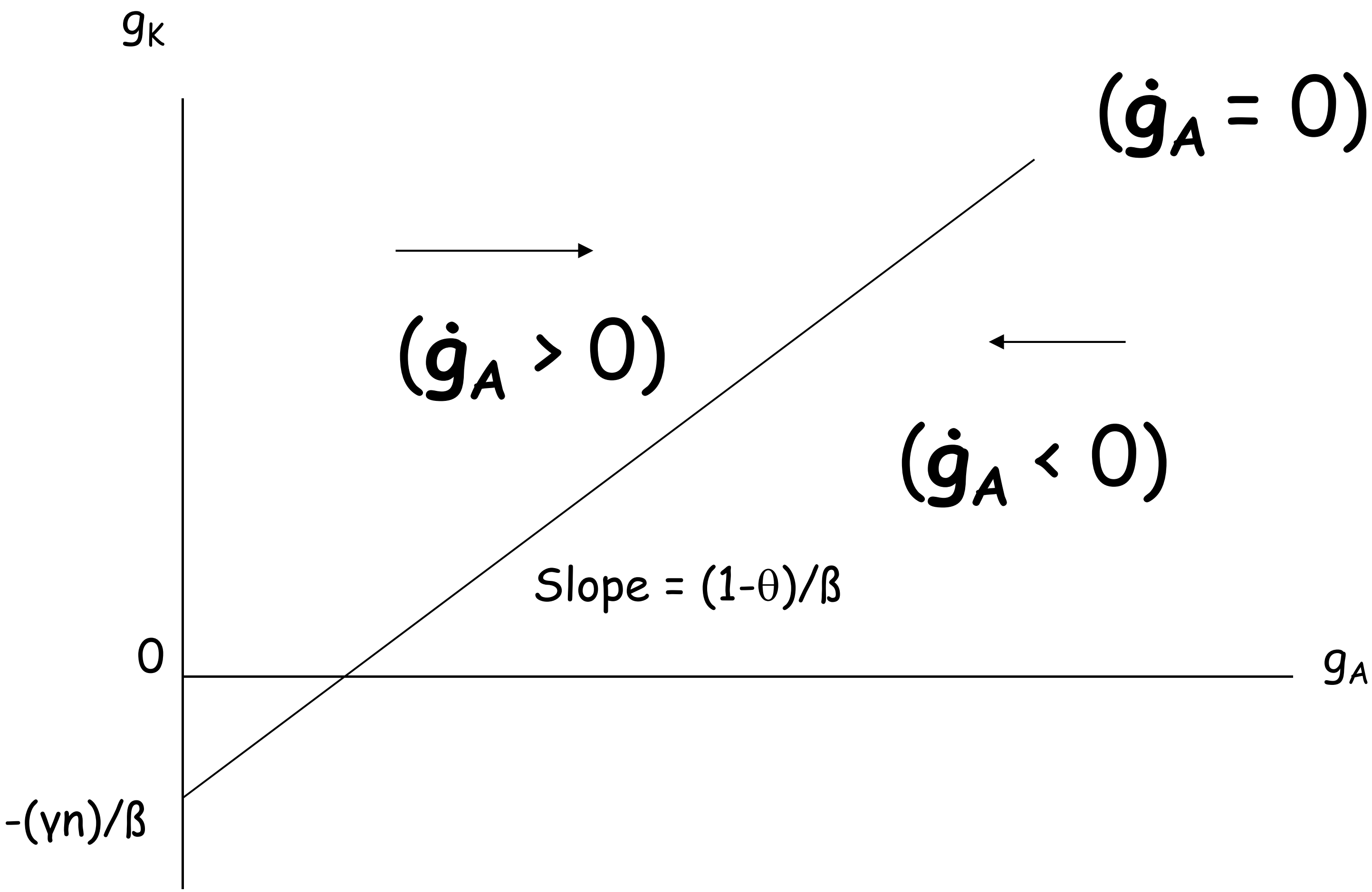
$$g_A(t) = c_A K(t)^{\beta} L(t)^{\gamma} A(t)^{\theta-1} \qquad (14)$$

$where$

$$c_A \equiv B a_K^{\beta} a_L^{\gamma}$$

$The\ behavior\ of\ g_A\ depends\ on\ \beta g_K + \gamma n + (\theta - 1) g_A:$

$g_A\ is\ rising\ if\ this\ expression\ is\ positive.$

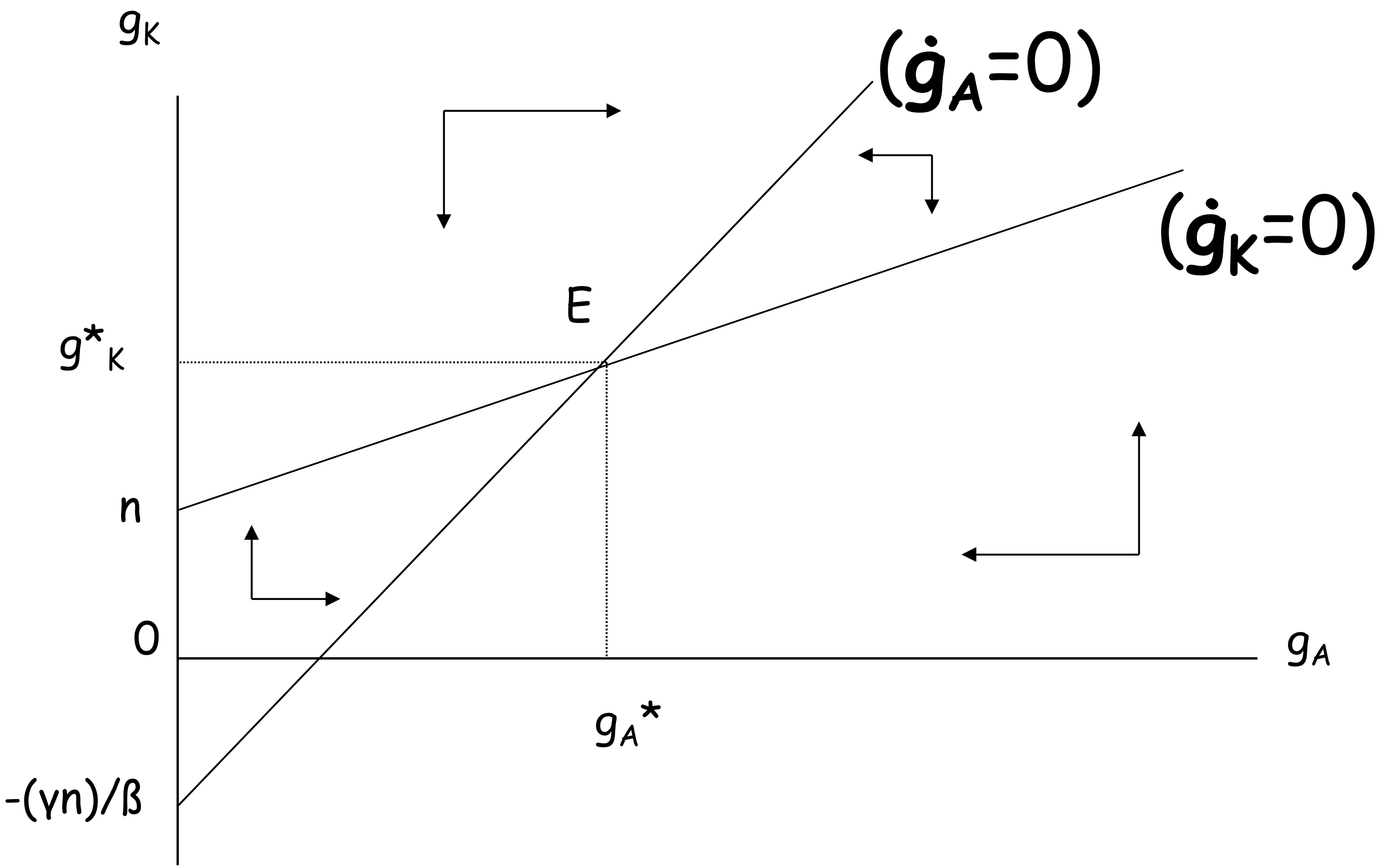

g_K
$(\dot{g}_A = 0)$
$(\dot{g}_A > 0)$
$(\dot{g}_A < 0)$
Slope = (1-θ)/ß
0
g_A
-(γn)/ß

Case 1: ß+θ<1

- If ß+θ<1, (1-θ)/ß is greater than 1. Thus the locus of points where $\dot{g}_A = 0$ is stepper than the locus where $\dot{g}_K = 0$.
- Regardless of where g_A and g_K begin, they converge to Point E. At Point E, we have

$$g_A^* + n - g_K^* = 0 \qquad (15)$$

and


g_K
$(\dot{g}_A=0)$
$(\dot{g}_K=0)$
E
g^*_K
n
0
g_A
g_A^*
-(γn)/ß

Case 1: $\beta+\theta<1$

- $\beta g_K^* + \gamma n - (\theta-1)g_A^* = 0$ (16)
- Rewriting (15) as $g_K^* = n + g_A^*$, and substituting into (16) yields
- $\beta g_A^*+(\beta+\gamma)n+(\theta-1)g_A^*=0.$
- Or

$$g_A^* = \frac{\beta+\gamma}{1-(\theta+\beta)}n \qquad (17)$$

and $$g_K^* = g_A^* + n$$

Case 1: ß+θ<1

- Recall Eq (7)

$$Y(t)=\left[(1-a_K)K(t)\right]^{\alpha}\left[A(t)(1-a_L)L(t)\right]^{1-\alpha},\ 0<\alpha<1. \quad (7)$$

Differentiating (7) *w.r.t time t, (by using chain rule)*

$$\dot{Y}(t)=\alpha\left[(1-a_K)K(t)\right]^{\alpha}\frac{\dot{K}(t)}{K(t)}\left[A(t)(1-a_L)L(t)\right]^{1-\alpha}$$

$$+(1-\alpha)\left[(1-a_K)K(t)\right]^{\alpha}\left[A(t)(1-a_L)L(t)\right]^{1-\alpha}\left\{\frac{\dot{L}(t)}{L(t)}+\frac{\dot{A}(t)}{A(t)}\right\},$$

Then, dividing both sides by $Y(t)$,

Case 1: ß+θ<1

$$\frac{\dot{Y}(t)}{Y(t)} = \alpha \frac{\dot{K}(t)}{K(t)} + (1-\alpha)\left\{\frac{\dot{L}(t)}{L(t)} + \frac{\dot{A}(t)}{A(t)}\right\} \qquad (18)$$

At $\dot{g}_K = \dot{g}_A = 0,\ Eq.(18)$ *becomes*

$$\frac{\dot{Y}(t)}{Y(t)} = \alpha g_K^* + (1-\alpha)\{n + g_A^*\} \qquad (19)$$

where $g_K^* = n + g_A^*$. *Hence, Eq.* (19) *can also be written as*

$$\frac{\dot{Y}(t)}{Y(t)} = \alpha g_K^* + (1-\alpha) g_K^* = g_K^* \qquad (20)$$

Case 1: ß+θ<1

- Hence Eq.(20) states that, if K is growing at g_K*, and A at g_A*, where $g_K^* = n + g_A^*$, then output is growing at rate g_K*. The long-run growth rate of the economy is endogenous, and is an increasing function of population growth.

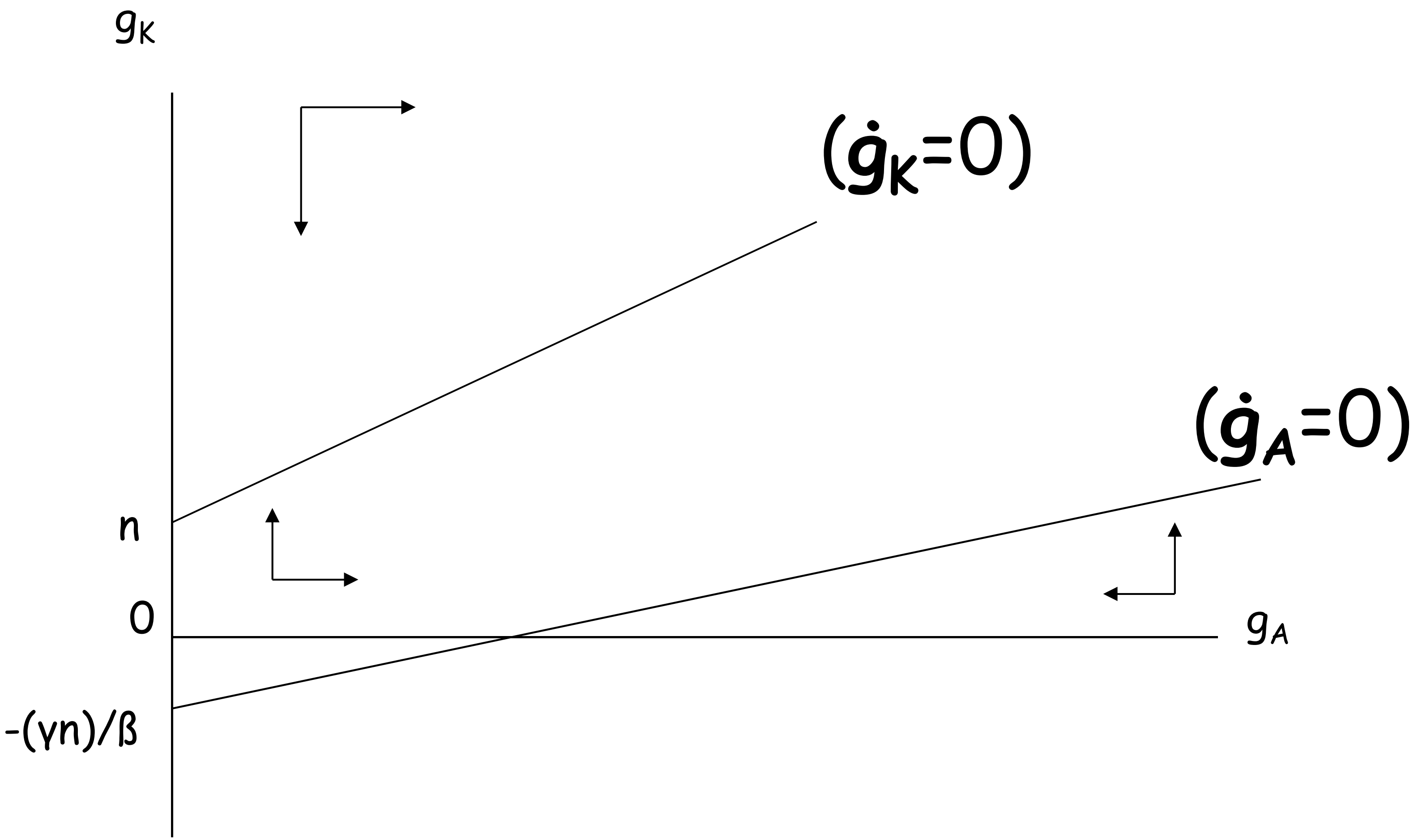
Case 1: ß+θ<1

- Also from Eq.(20) we can see that the fractions of labor force and capital stock engaged in R&D, a_L and a_K, do not affect long-run growth; nor does the saving rate, s.

Case 2: ß+θ>1

- In this case, the loci where both g_K and g_A are constantly diverged.
- Regardless of where the economy starts, the long term growth rates of K, A and Y will continue to increase forever.

Case 2: ß+θ>1



Case 2: ß+θ>1

- By recalling Eq. (14)

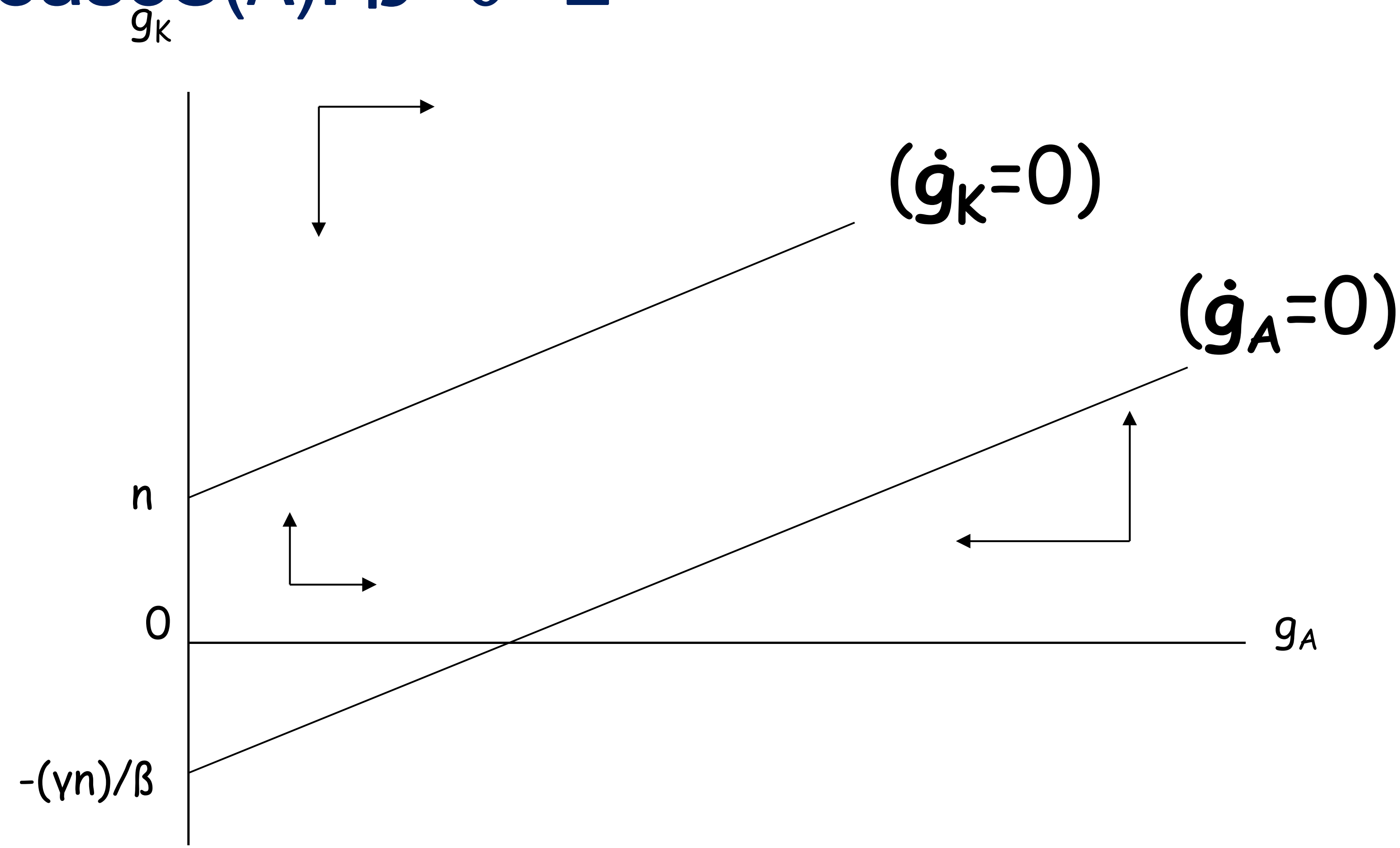
$$g_A(t) = c_A K(t)^{\beta} L(t)^{\gamma} A(t)^{\theta-1} \qquad (14)$$

- In this case, knowledge and capital are so useful in producing new knowledge that each marginal increase in their level result in so much more new knowledge that the growth rate of knowledge rises rather than falls.
- Increase in n and s cause output per worker to rise.

Case3(A): ß+θ=1

- The $\dot{g}_A = 0$ and $\dot{g}_K = 0$ loci have the same slope.
- If n is positive, the dynamics of the economy are similar to those when ß+θ>1.
- If n=0 and θ=1, the phase diagram does not tell us what balance growth path the economy converges to.

Case3(A): $ß+\theta=1$



Case3(B): ß+θ=1 (and n=0)