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EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.),2020

Use 4 decimal places for numerical answers

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

Table 1.a

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

$\bar{X} = \frac{621}{8} = 77.625$

$\bar{Y} = \frac{25.7}{8} = 3.2125$

$X_i Y_i = 2012.4$

$\Sigma X_i^2 = 48719$

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1. (Note: *NIID* = Normally, Identically, and Independently Distributed).

$$\hat{\beta}_1 = \frac{\Sigma X_i Y_i - n\bar{X}\bar{Y}}{\Sigma X_i^2 - n\bar{X}^2}$$

$$= \frac{2012.4 - 8(77.625)(3.2125)}{48719 - 8(77.625)^2} = \frac{2012.4 - 1994.9625}{48719 - 48203.125} = \frac{31}{910} = 0.0341 \checkmark$$

$$\therefore \quad \hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

$$= 3.2125 - 0.0341(77.625)$$

$$= 0.5655 \checkmark$$

1.2 For each observation i, find $\widehat{Y_i}$ and $\widehat{u_i}$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i$

$i = 1 \; ; \; \hat{Y}_1 = 2.714$

"–" $= 2 \; ; \; \hat{Y}_2 = 3.021$

"–" $= 3 \; ; \; \hat{Y}_3 = 3.225$

"–" $= 4 \; ; \; \hat{Y}_4 = 3.327$

"–" $= 5 \; ; \; \hat{Y}_5 = 3.532$

"–" $= 6 \; ; \; \hat{Y}_6 = 3.123$

"–" $= 7 \; ; \; \hat{Y}_7 = 3.123$

"–" $= 8 \; ; \; \hat{Y}_8 = 3.634$

$\Rightarrow$

$\hat{u}_i = Y_i - \hat{Y}_i$

$\hat{u}_1 = 2.8 - 2.714 = 0.086$

$\hat{u}_2 = 3.4 - 3.021 = 0.379$

$\hat{u}_3 = 3.0 - 3.225 = -0.225$

$\hat{u}_4 = 3.5 - 3.327 = 0.173$

$\hat{u}_5 = 3.6 - 3.532 = 0.068$

$\hat{u}_6 = 3.0 - 3.123 = -0.123$

$\hat{u}_7 = 2.7 - 3.123 = -0.423$

$\hat{u}_8 = 3.7 - 3.634 = 0.066$

$\sum_{i=1}^{8} \hat{u}_i = 0 \checkmark$

σ^2

1.3 Find $var(\widehat{u_i}), var(\widehat{\beta_0}), var(\widehat{\beta_1})$

$$Var(\hat{u}_i) = \frac{\sum \hat{u}_i^2}{n-2} = \frac{0.4347}{8-2} = 0.07245 ✓$$

$$Var(\hat{\beta}_0) = \frac{\sigma^2}{\sum (x_i-\bar{x})^2} = \frac{0.07245}{511.875} = 0.000141 ✓$$

$$Var(\hat{\beta}_1) = \frac{\sum x_i^2}{n\sum (x_i-\bar{x})^2}(\sigma^2) = \frac{48717(0.07245)}{8(511.875)} = 0.8625 ✓$$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

$\bar{X} = 20$

$\bar{Y} = 9.1$

$\sum X_i Y_i = 2214$

$\sum X_i^2 = 4440$

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

$$\hat{\beta}_1 = \frac{\sum X_i Y_i - n\bar{X}\bar{Y}}{\sum X_i^2 - n\bar{X}^2}$$

$$= \frac{2214 - 10(20)(9.1)}{4440 - 10(20^2)} = \frac{2214 - 1820}{4440 - 4000} = 0.895 ✓$$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

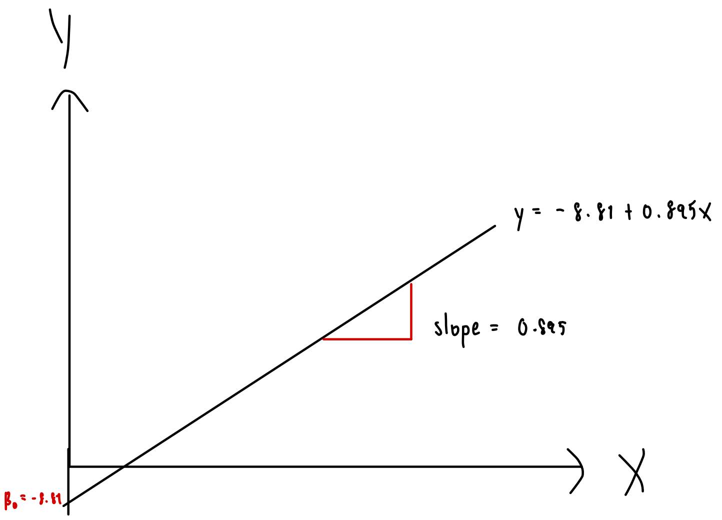
$$= 9.1 - 0.895(20) = -8.81 ✓$$

2.2 Find the value of $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$i=1$; $\hat{Y}_1 = 0.1455$		$\hat{u}_1 = -0.145$
$i=2$; $\hat{Y}_2 = 1.9364$		$\hat{u}_2 = 0.064$
$i=3$; $\hat{Y}_3 = 3.7273$		$\hat{u}_3 = 1.273$
$i=4$; $\hat{Y}_4 = 5.5182$		$\hat{u}_4 = 0.482$
$i=5$; $\hat{Y}_5 = 7.3091$	$\Rightarrow$	$\hat{u}_5 = -0.309$
$i=6$; $\hat{Y}_6 = 10.8909$		$\hat{u}_6 = -0.891$
$i=7$; $\hat{Y}_7 = 12.6818$		$\hat{u}_7 = -2.682$
$i=8$; $\hat{Y}_8 = 14.4727$		$\hat{u}_8 = 0.527$
$i=9$; $\hat{Y}_9 = 16.2636$		$\hat{u}_9 = -0.264$
$i=10$; $\hat{Y}_{10} = 18.0545$		$\hat{u}_{10} = 1.945$

$$\sum_{i=1}^{10} u_i = 0$$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?



If $\bar{x} = 20$ and $\bar{y} = 9.1$ can be perfectly fit in regression function, the line pass $(\bar{x}, \bar{y})$

$\because$ $Y = -8.81 + 0.895(20)$

$= Y = 9.1$ ✓

$\therefore$ The line pass $(\bar{x}, \bar{y})$

2.4 If X_i =16, what is the predicted Y?

If $x_i = 16$; $\hat{Y}_i = \beta_0 + \beta_1(16)$

$= -8.81 + 0.895(16) = 5.51$ ✓

2.5 Find $\overset{\sigma^2}{var(\widehat{u}_i)}$, $var(\widehat{\beta_0})$, $var(\widehat{\beta_1})$

$$Var(\hat{u}_i) = \frac{\sum \hat{u}_i^2}{n-2} = \frac{14.091}{10-2} = 1.761375 ✓$$

$$Var(\hat{\beta}_0) = \frac{\sigma^2}{\sum(x_i - \bar{x})^2} = \frac{1.761375}{440} = 0.004003 ✓$$

$$Var(\hat{\beta}_1) = \frac{\sum x_i^2(\sigma^2)}{n\sum(x_i - \bar{x})^2} = \frac{4440(1.761375)}{10(440)} = 1.7774 ✓$$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i \,,$$

Where $u_i \sim NIID(0, \sigma^2)$. Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

In order to prove that $\hat{\beta}_1$ is unbiased estimator, we have to use assumption SLR 1-4

$$\hat{\beta}_1 = \frac{\Sigma (y_i - \bar{y})(x_i - \bar{x})}{\Sigma (x_i - \bar{x})^2} \,; \quad \text{Let } x_i = x_i - \bar{x}$$

$$k_i = \frac{x_i}{\Sigma_i x_i^2} = \frac{x_i - \bar{x}}{\Sigma_i (x_i - \bar{x})^2}$$

Now, write as; $\hat{\beta}_1 = \sum_{i=1}^{n} (y_i - \bar{y}) k_i$

$$= \sum_{i=1}^{n} (\beta_0 + \beta_1 x_i + u_i - \beta_0 - \beta_1 \bar{x}) k_i$$

$$= \Sigma_i \beta_1 (x_i - \bar{x}) k_i + \Sigma_i u_i k_i$$

$$= \beta_1 \Sigma_i x_i k_i + \Sigma_i u_i k_i$$

$$= \beta_1 \Sigma_i x_i \left(\frac{x_i}{\Sigma_i x_i^2}\right) + \Sigma_i u_i k_i$$

$$= \beta_1 \frac{\Sigma_i x_i^2}{\Sigma_i x_i^2} + \Sigma_i u_i k_i$$

$$E(\hat{\beta}_1) = E[\beta_1 + \Sigma_i u_i k_i] = \beta_1 + E[\Sigma_i u_i k_i]$$

SLR 4: $E(u_i | x_i) = 0 \rightarrow$ This assumption takes the value of x as given (or fixed) so that we can treat x_i as a constant

$$E(\hat{\beta}_1) = \beta_1 + \Sigma_i k_i E(u_i)$$

$$= \beta_1$$

$\therefore$ β_1 is an unbiased estimator