Numerical Solution MA217/2012 Final

1. $(x, y, z) = \left(\frac{4}{3}, \frac{2}{3}, \frac{2}{3}\right)$

2.

(a) $\underline{\mathbf{A}}^{-1} = \frac{1}{4}\begin{bmatrix} -2 & -2 & 2 \\ 9 & 3 & -5 \\ -1 & 1 & 1 \end{bmatrix}$; $\underline{\mathbf{A}} \cdot \underline{\mathbf{A}}^{-1} = \underline{\mathbf{I}}$

(b) $\underline{\mathbf{x}} = \begin{bmatrix} -4 \\ 13 \\ -1 \end{bmatrix}$.

3. $\underline{\mathbf{A}}^{-1} = \frac{1}{(ad-bc)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$; $\underline{\mathbf{A}} \cdot \underline{\mathbf{A}}^{-1} = \underline{\mathbf{I}}$

4. (a) $a = 7$; $b = 17$ and $c = -24$

(b) $\left(\underline{\mathbf{A}}^{\mathbf{T}}\right)^{-1} = \left(\underline{\mathbf{A}}^{-1}\right)^{\mathbf{T}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -8 & 1 & 0 & 0 & 0 \\ 8 & 0 & 1 & 0 & 0 \\ 7 & -4 & -2 & 1 & 0 \\ 17 & -24 & -5 & 5 & 1 \end{bmatrix}$

5. (a) $|\underline{\mathbf{A}}| = C(C-2)(7-C) = 0$. Hence, C = 0 or C = 2 or C = 7.

(b) Free variable is x_3. $\underline{\mathbf{x}} = \begin{bmatrix} -1 \\ 9/7 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} C$; $C \in \Re$.

6. (a) $|\underline{\mathbf{A}}| = \begin{vmatrix} a \\ b \\ a+b \end{vmatrix}$ … row operations … $|\underline{\mathbf{A}}| = \begin{vmatrix} a \\ b \\ 0 \end{vmatrix}$. Hence, $|\underline{\mathbf{A}}| = 0$, $\underline{\mathbf{A}}$ is singular.

(b) Doing the same row operations as (a) gives, $\left[\begin{array}{c|c} a & 1 \\ b & 0 \\ 0 & -1 \end{array}\right]$. Third row is 0=-1 so the system is inconsistent. Hence, it has no solution.

(c) $\underline{\mathbf{b}} = \begin{bmatrix} a \\ b \\ a+b \end{bmatrix}$ for example $\underline{\mathbf{b}} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. This will make sure that the system is consistent. Hence, it could be unique solution or infinite number of solutions depending on number of pivots and number of unknowns.

(i) If the number of pivots is 1 or 2 and number of unknowns = number of pivots, it is unique solution.

(ii) If the number of unknowns is 3 or more, there will be infinite solution.

(iii) If the number of unknowns is 2 or more and the number of pivots is 1, there will be infinite solutions.

7. (a) $|\underline{\mathbf{A}}| = 2$.

   (b) Matrix B is size 6x6 and matrix A is size 5x5 so the multiplication is incompatable. Det is undefined.

8. (a) $|\underline{\mathbf{A}}| = 1 + at - a$

   (b) $x_2 = \dfrac{b - aT - a(i+g)(t-1)}{1 + at - a}$

9. I = 13,329,083.88
10. I = 2/5
11. *Profit* = 7,400,000,000 and $Profit_{avg}$=370,000.