1a) $\hat{\beta}_2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2} = \frac{20.58}{211} = 0.0975$

$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{X} = 2.8278 - 0.0975(21.5556) = 2.8278 - 2.1017 = 0.7261$

SRF model is $\hat{y}_i = 0.7261 + 0.0975 X_i$ meaning that the intercept is 0.7261 and the slope of this function is 0.0975

1b) $r^2 = 1 - \frac{\sum_{i=1}^{n}\hat{u}_i^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} = 1 - \frac{0.5781}{2.5844} = 1 - 0.2237 = 0.7763$

r^2 is the measure of goodness to fit, In this case 77.63% can be explained by the regression model

1C) From $\hat{y}_i = 0.7261 + 0.0975 X_i$

Plug $X_i = 30$ in

$\hat{y}_i = 0.7261 + 0.0975(30)$

$\hat{y}_i = 0.7261 + 2.925$

$\hat{y}_i = 3.6511$

When $X_i = 30$ the average of $\hat{y}_i$ will be 3.6511

1.d) $var(u_i) = \frac{\sum_{i=1}^{n} u_i^2}{n-k} = \frac{0.5781}{18-2} = \frac{0.5781}{16} = 0.0361 = \hat{\sigma}^2$

$var(\hat{\beta}_1) = \frac{\sum_{i=1}^{n}(x_i)^2}{n\sum_{i=1}^{n}(x_i - \bar{x})^2}\hat{\sigma}^2 = \frac{9620}{(18)211}(0.0361) = 2.5329(0.0361) = 0.0914$

$var(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2} = \frac{0.0361}{211} = 0.0002$

1e) Find CI : Using t-test because σ^2 is unknown

$se\,\hat{\beta}_2 = \sqrt{var(\hat{\beta}_2)} = \sqrt{0.0002} = 0.0141$

90% CI ; $\alpha = 0.1$

$t_{\frac{\alpha}{2}} = t_{\frac{0.1}{2}} = t_{0.05,16} = 1.746$

Upper limit $= \hat{\beta}_2 + t_{\frac{\alpha}{2}} \cdot se\,\hat{\beta}_2 = 0.0975 + (1.746 \cdot 0.0141) = 0.0975 + 0.0246 = 0.1221$

lower limit $= \hat{\beta}_2 - t_{\frac{\alpha}{2}} \cdot se\,\hat{\beta}_2 = 0.0975 - (1.746 \cdot 0.0141) = 0.0975 - 0.0246 = 0.0729$

90% CI of $\hat{\beta}_2$ is $[0.0729, 0.1221]$.

It means that there are 90% probability that $\hat{\beta}_2$ will be in the range between 0.0729 and 0.1221 if) Test β_2 ;

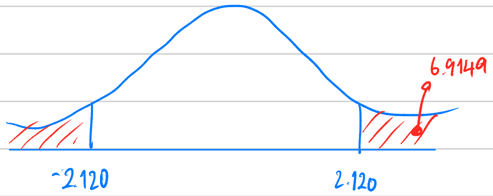
Assume that

H_0 ; $\beta_2 = 0$

H_1 ; $\beta_2 \neq 0$

Compute $t_{cal} = \dfrac{\hat{\beta}_2 - \beta_2}{se(\hat{\beta}_2)} = \dfrac{0.0975 - 0}{0.0141} = 6.9149$

Find CV when $\alpha = 0.05$, $t_{\frac{\alpha}{2}} = t_{0.025, 16} = \pm 2.120$



Tcal is in the rejection region, therefore reject Ho

We can say that β_2 is not zero with 95% CI.

2a) Test β_1

$H_0 ; \beta_1 = 0$
$H_1 ; \beta_1 \neq 0$

$\alpha = 0.05$,

$$t_{cal} = \frac{\hat{\beta}_1 - \beta_1}{se(\hat{\beta}_1)} = \frac{0.4280 - 0}{0.0140} = 30.5714$$

Find CV at $\alpha = 0.05$, $n-k = 27,884$,

$t_{0.025, 27,884} = \pm 1.960$

t_{cal} is more than $t_{0.025, 27,884}$, Reject H_0

β_1 is not equal to zero with 95% CI

Test β_2

$H_0 ; \beta_2 = 0$
$H_1 ; \beta_2 \neq 0$

Compute $t_{cal} = \frac{\hat{\beta}_2 - \beta_2}{se(\hat{\beta}_2)} = \frac{0.0313 - 0}{0.0023} \approx 13.6087$

Find CR ; $t_{\frac{\alpha}{2}, n-k} = t_{0.025, 27,884} = \pm 1.960$

t_{cal} lies beyond the CR, reject H_0

We say that β_2 is not zero with 95% CI

2b) Yes, because higher in age it means that there is higher probability to get sick or going more to hospital

2c) When outp, changes to ln scale, we can reinterprete that when the age increase by 1 unit, the output will increase by

$100 \cdot \hat{\beta}_2$ percent [Approximately], $(100 \cdot e^{\hat{\beta}_2}) - 100$ percent [Exact measure]

2d) When age_i divide by 10, There is no difference when we interpret the coefficents

2e) Mean Prediction $X_0 = 50$

$\hat{Y}_0 = 0.0031 + 0.4280(50) = 0.0031 + 21.4 = 21.4031$

$se_{\hat{y}_0} = \sqrt{var(\hat{y}_0)} = \sqrt{0.00002} = 0.0045$

Find 99% CI ($t_{\frac{\alpha}{2}} = 2.576$) ($\alpha = 0.01, 1-\alpha = 0.99$)

$Pr[21.4031 - (2.576(0.0045)) \leq y_0 \leq 21.4031 + (2.576 \cdot (0.0045))] = 0.99$

$Pr[21.4031 - 0.0116 \leq y_0 \leq 21.4031 + 0.0116] = 0.99$

$Pr[21.3915 \leq y_0 \leq 21.4147] = 0.99$

3) The reason is due to when the further away from the mean is, the larger of the variance will be. As explained by the formula of both mean and individual prediction. As both formula utilised variance as the part in the calculation the further it is, the more error it can be.