FN211: Lecture Note 1

Review of Basic Statistics and Calculus

Dr. Winai Homsombat

Bachelor of Economics, International Program

Thammasat University

Basic Statistics

Definition:

- Statistics is the science of presenting and interpreting data.

Descriptive Stats	Inferential Stats
- collecting,	- interpreting,
- describing,	- modeling,
- etc.	- Making decisions.

FN211

- FN211 lies entirely in the subset of inferential statistics (sometimes called Mathematical Statistics)
- Two stages in the mind of a statistician.
  - Design of an experiment and subsequent investigation.
  - Forming a statistical model and inference.

FN211

- Definition: Statistics provides the model that are needed to study situations involving uncertainties.

- A paramount issue is formulating appropriate and valid models depending, of course, on the proposed experiment.

Some questions and ideas we will pose and answer:

Q1: What is the probability of an event?

Q2: What is a probability model?

Q3: Discrete random variables VS Continuous random variables?

Q4: Counting and the principle of choices?

etc.

Casually speaking, the term probability is a measure of one's belief in the occurrence of a future event.

Statistics

- In your first course in Statistics, you probably began the course by discussing data:
    1. Histograms, pie charts,
    2. Measurements of center (mean, median),
    3. Measurements of dispersion (variance, quartiles).

Then stepped back and discussed Probability, etc.

- For example, you most likely encountered the famous discrete distribution : Binomial distribution.

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} \qquad ; x = 0, 1, 2, \ldots, n$$

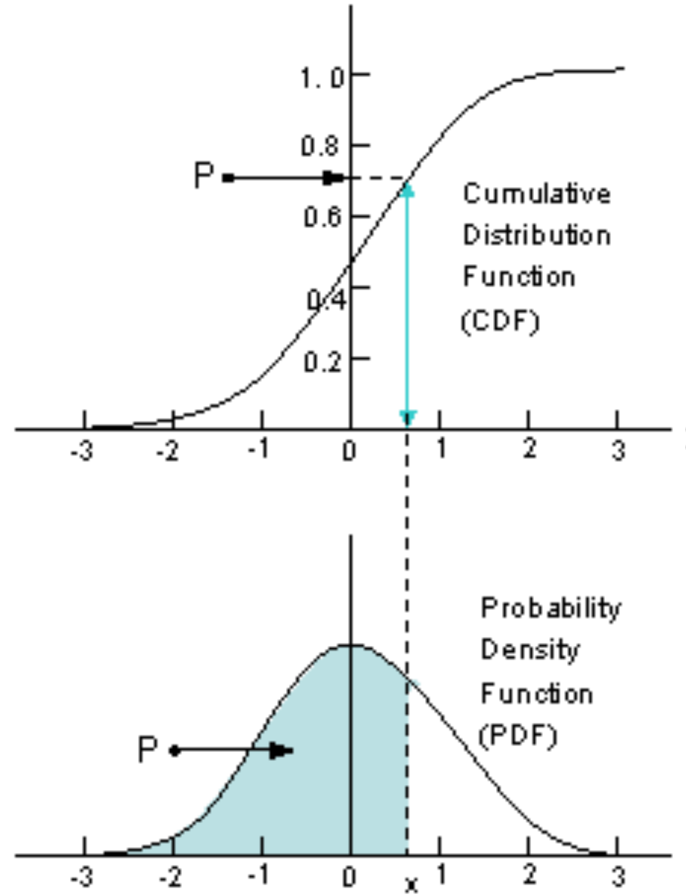
- In addition, you also discussed the normal distribution

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad , -\infty < x < \infty$$

Normal Distribution

$$F(x) = \Phi\left(\frac{x-\mu}{\sigma}\right) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

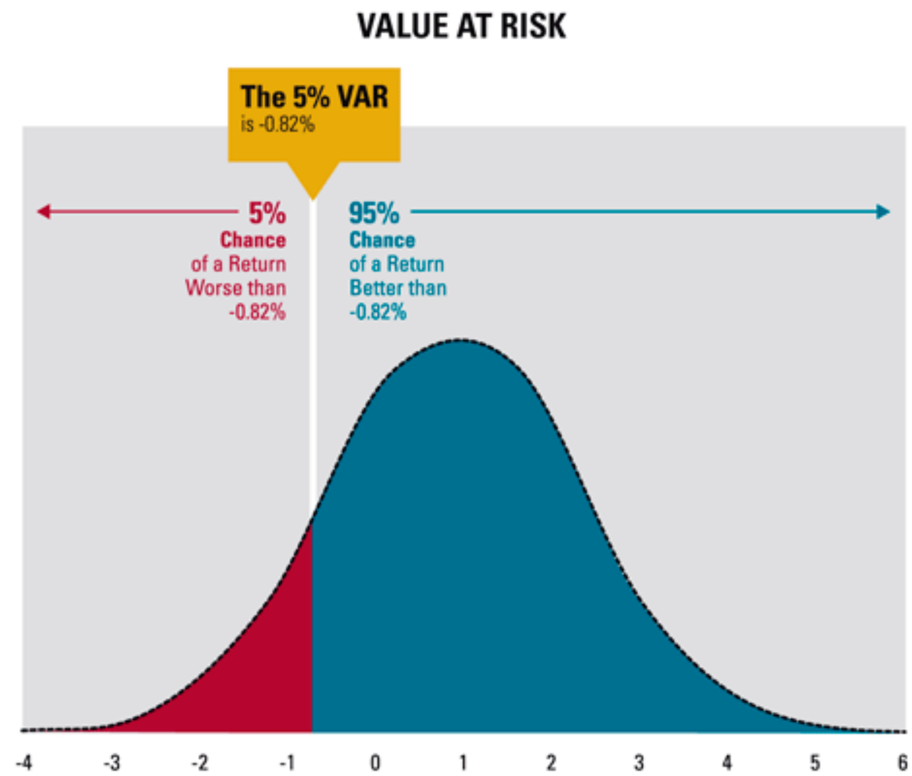


Relations Between Two Different Typical Representations of a Population

Basic Statistics and Application in Finance

Value at Risk (VaR)

“What loss level is such that we are X% confident it will not be exceeded in N business days?”



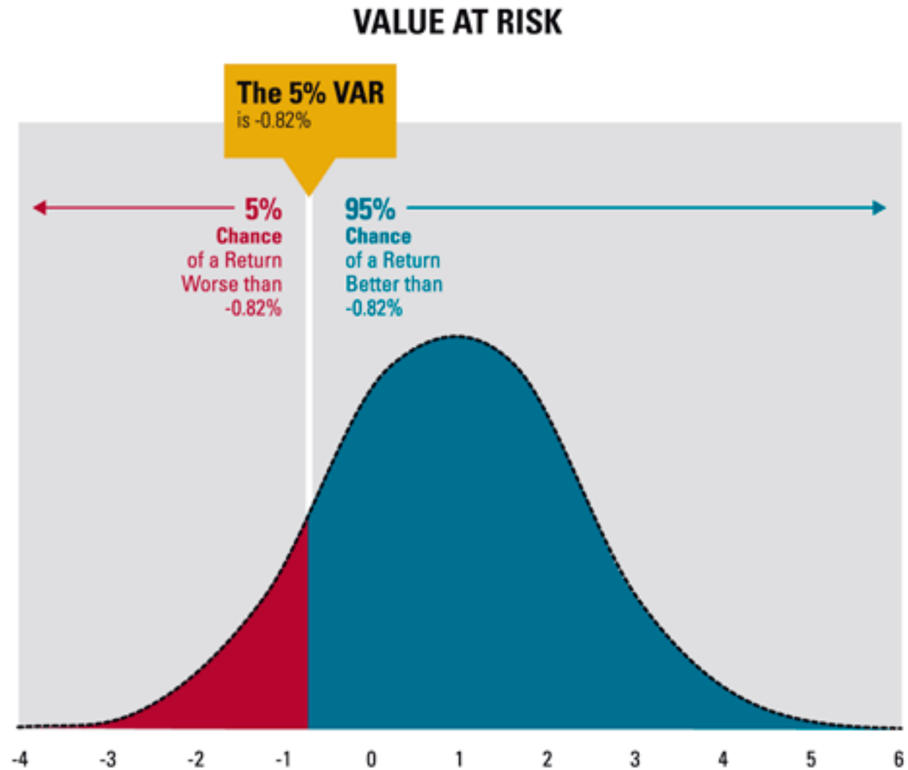
Application: VaR and Regulatory Capital

- Regulators base the capital they require banks to keep on VaR

Basic Statistics and Application in Finance (2)

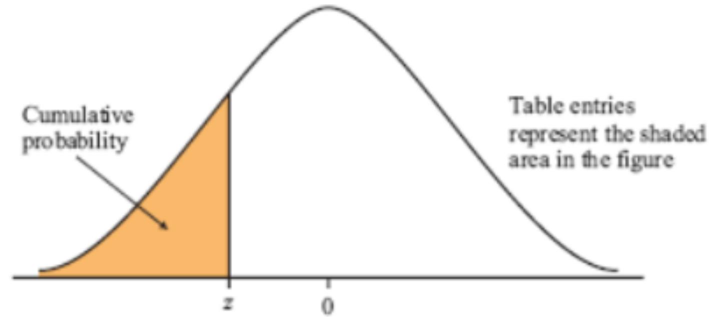
Example:

- The gain from a portfolio during six month is normally distributed with mean \$2 million and standard deviation \$10 million.
- What is the VaR for the portfolio with a six-months time horizon and a 99% confidence level?



$$\text{VaR for the portfolio} = \mu - Z_{\alpha} \cdot \sigma$$

Cumulative Probabilities for the Standard Normal Distribution



FIRST DIGIT OF *z*	SECOND DIGIT OF *z*										FIRST DIGIT OF *z*
z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**	*z*
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010	−3.0
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014	−2.9
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019	−2.8
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026	−2.7
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036	−2.6
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048	−2.5
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064	−2.4
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084	−2.3
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110	−2.2
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143	−2.1
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183	−2.0
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233	−1.9
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294	−1.8
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367	−1.7

Calculus

Limit

Definition of Limit

If $f(x)$ becomes arbitrarily close to a unique number L as x approaches c from either side, then the limit of $f(x)$ as x approaches c is L. This is written as

$$\lim_{x \to c} f(x) = L.$$

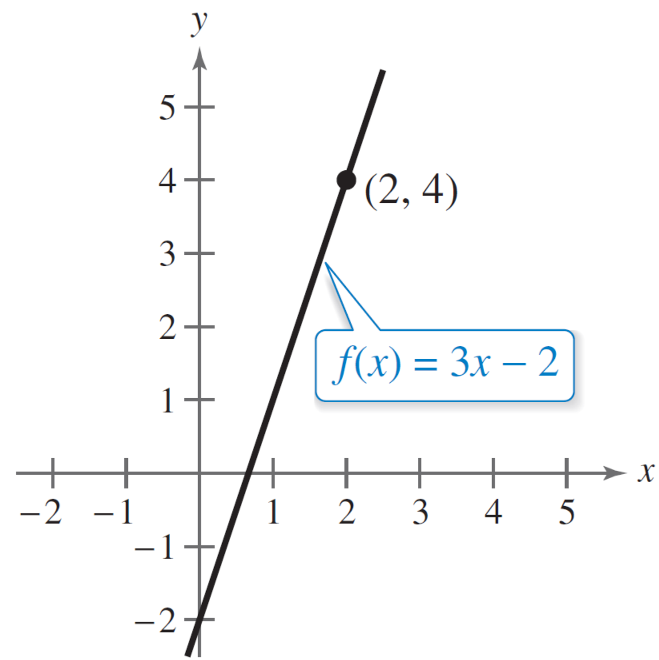
Example: Use a table to estimate numerically the limit:

$$\lim_{x \to 2} (3x - 2)$$

Limit: Example

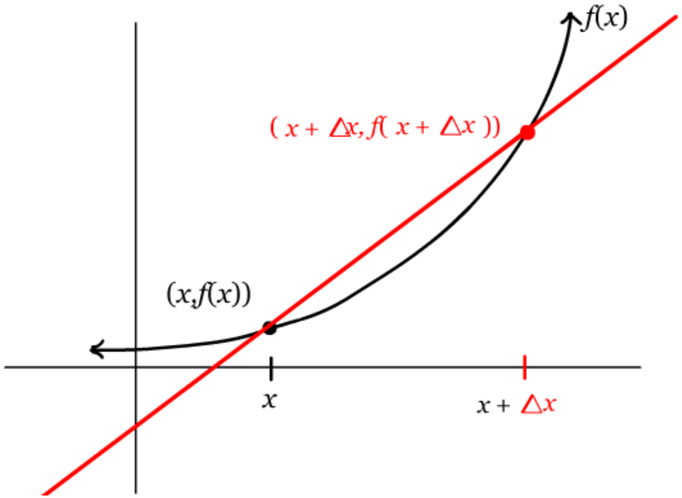
Let $f(x) = 3x - 2$

Then construct a table that shows values of $f(x)$ for two sets of x-values—one set that approaches 2 from the left and one that approaches 2 from the right.



x	1.9	1.99	1.999	2.0	2.001	2.01	2.1
$f(x)$	3.700	3.970	3.997	?	4.003	4.030	4.300

Limit Definition of the Derivative



The derivative is the formula which gives the slope of the tangent line at any point x for $f(x)$, and is denoted

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

provided this limit exists.

Example: $f(x) = x^2 - 3x + 5$

The Chain Rule

- The Chain Rule is a technique for differentiating <u>composite functions</u>.
- <u>Composite functions</u> are made up of layers of functions inside of functions.

If y = f(u) and u = g(x) then y = f(g(x)) and

$$\frac{dy}{dx} = \frac{dy}{du} \bullet \frac{du}{dx} \qquad \text{or} \qquad \frac{dy}{dx} = \frac{d}{dx}\Big(f(g(x))\Big) = f'(g(x)) \bullet g'(x)$$

The Chain Rule: Example

$$1)\ f(x) = (3x - 5x^2)^7$$

$$2)\ f(x) = \sqrt[3]{(x^2 - 1)^2}$$

Basic Calculus and Application in Finance

Options (derivatives product)

- Options are financial instruments that are derivatives based on the value of underlying securities such as stocks.
- Options contract offers the buyer the right to buy or sell - depending on the type of contract they hold - the underlying asset.

Call Options: Position of Buyer vs. Writter

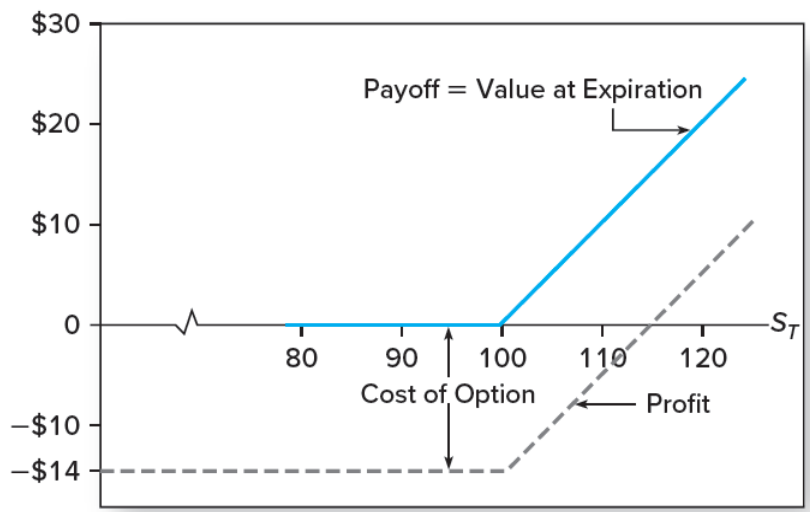


Figure 20.2 Payoff and profit to call option at expiration

Put Options: Position of Buyer vs. Writter

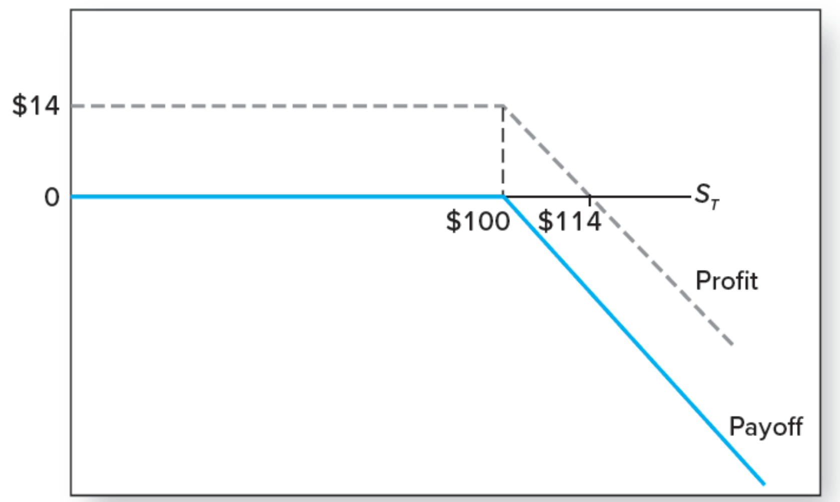


Figure 20.3 Payoff and profit to call writer at expiration

Basic Calculus and Application in Finance

- The Black-Scholes-Merton Formulas
  - An investment asset provides no income

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2) \quad \text{and} \quad p = Ke^{-rT} N(-d_2) - S_0 N(-d_1)$$

where:

$$d_1 = \frac{\ln(S_0 / K) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = \frac{\ln(S_0 / K) + (r - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

and

N(x) is the cumulative probability distribution function for a standardized normal distribution

S_0	is today's asset price	T	is time to maturity of the option
K	is strike price	σ	is stock price volatility
r	is the continuously compounded risk-free rate		

Basic Calculus and Application in Finance: Greek Letters

Delta (Δ)

= the rate of change of the value of the portfolio with respect to

a market variable:

$$Delta = \frac{\partial P}{\partial S}$$

where P is the value of the portfolio

S is the small increase in the value of the variable

Basic Calculus and Application in Finance: Greek Letters

Delta (Δ) Hedging

Note: Hedging is a risk management strategy employed to offset losses in investments by taking an opposite position in a related asset.

Example: The financial institution writes call options with the current delta is 0.5216 per one stock. If it writes 10,000 stocks per contract, how does it formulate hedging strategy?

BACHELOR OF ECONOMICS
THAMMASAT UNIVERSITY

Question?