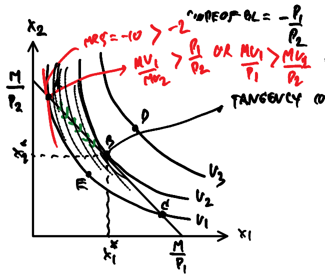
EE311_Wk3

Wednesday, January 29, 2014
11:00 AM

OPTIMAL CHOICE

CONSIDER 2 GOODS: x_1 AND x_2

A CONSUMER'S OPTIMIZATION PROBLEM:
$$\boxed{\begin{array}{l} \max\ U(x_1, x_2) \\ \text{s.t.}\quad P_1x_1 + P_2x_2 = M \end{array}}$$



$\Rightarrow$ BUY MORE OF 1 AND LESS OF 2.

TANGENCY CONDITION: SLOPE OF U_2 = SLOPE OF BL

$$MRS_{x_1x_2} = -\frac{P_1}{P_2}$$

$$\frac{MU_1}{MU_2} = \frac{P_1}{P_2}$$

OR $\boxed{\frac{MU_1}{P_1} = \frac{MU_2}{P_2}}$

MARGINAL UTILITY PER BAHT SPENT ON GOOD 1 — MARGINAL UTILITY PER BAHT SPENT ON GOOD 2

OR $\boxed{MRS_{12} = \left(\frac{P_1}{P_2}\right)}$

MARGINAL BENEFIT OF GOOD 1 — MARGINAL COST OF GOOD 1

EX: $MRS = -10 > \frac{P_1}{P_2} = \frac{100}{50} = 2$

LET'S HAVE AN EXAMPLE:

CONSIDER $\underline{U = x_1^{\alpha} x_2^{1-\alpha}}$ $\rightarrow$ COBB-DOUGLAS UTILITY FUNCTION

$U = x_1 + x_2$

$U = \min(x_1, x_2)$

S.T. $P_1x_1 + P_2x_2 = M$.

Q: FIND (x_1^*, x_2^*).

SOLUTION: $\boxed{\frac{MU_1}{MU_2} = \frac{P_1}{P_2}}$

$$\frac{MU_1}{MU_2} = \frac{\alpha x_1^{\alpha-1} x_2^{1-\alpha}}{(1-\alpha)x_1^{\alpha} x_2^{-\alpha}} = \frac{\alpha}{(1-\alpha)}\frac{x_2}{x_1}$$

$$\frac{\alpha x_2}{(1-\alpha)x_1} = \frac{P_1}{P_2}$$

$$x_2 = \frac{P_1}{P_2}\frac{(1-\alpha)x_1}{\alpha}$$

$$\begin{aligned} M &= P_1x_1 + P_2x_2 \\ &= P_1x_1 + \cancel{P_2}\frac{P_1}{\cancel{P_2}}\frac{(1-\alpha)x_1}{\alpha} \\ &= P_1x_1 + \frac{P_1(1-\alpha)x_1}{\alpha} \\ &= P_1x_1\left[1 + \frac{(1-\alpha)}{\alpha}\right] \\ &= P_1x_1\left[\frac{\cancel{\alpha} + 1 - \cancel{\alpha}}{\alpha}\right] \end{aligned}$$

$$M = \frac{P_1x_1}{\alpha}$$

SO $x_1^* = \frac{\alpha M}{P_1}$. $\Rightarrow$ x_1'S DEMAND FUNCTION

$$x_2^* = \frac{P_1}{P_2}\frac{(1-\alpha)}{\alpha}\cdot x_1 = \frac{\cancel{P_1}}{P_2}\frac{(1-\alpha)}{\cancel{\alpha}}\cdot\left(\frac{\cancel{\alpha}M}{\cancel{P_1}}\right) = (1-\alpha)\frac{M}{P_2}$$ $\Rightarrow$ x_2'S DEMAND FUNCTION

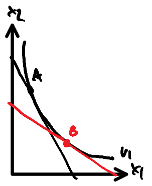
OPTIMAL BUNDLE: $(x_1^*, x_2^*) = \left(\frac{\alpha M}{P_1}, (1-\alpha)\frac{M}{P_2}\right)$.

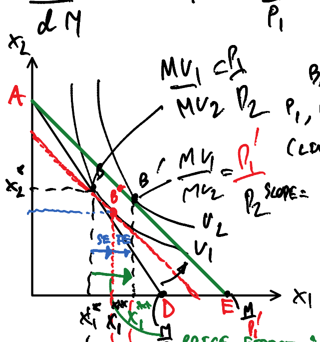
CONSIDER $x_1^* = \alpha\frac{M}{P_1} = \alpha M P_1^{-1}$

$$\frac{dx_1^*}{dP_1} = -1 P_1^{-1-1}\alpha M = -\frac{\alpha M}{P_1^2} \text{ OR } = -\alpha M P_1^{-2} < 0.$$

$$\frac{dx_1^*}{dM} = \alpha P_1^{-1} = \frac{\alpha}{P} > 0.$$ [x_1 IS A NORMAL GOOD]

$$\frac{dx_1^*}{dM} = aP_1^{-1} = \frac{a}{P_1} > 0. \quad [x_1 \text{ is a normal good}]$$





$\frac{MU_1}{MU_2} < \frac{P_1}{P_2}$

B/F: P_1, P_2, M (LINE AD)

A/F: P_1', P_2, M (LINE AE) where $P_1' < P_1$.

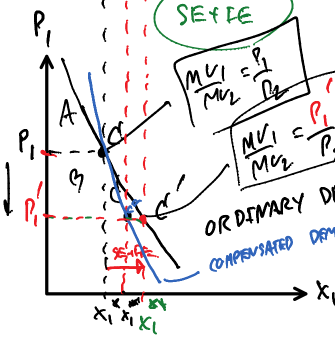
B': $\frac{MU_1}{MU_2} = \frac{P_1'}{P_2}$

SLOPE $= \frac{P_1}{P_2} >$ SLOPE $= \frac{P_1'}{P_2}$

PRICE EFFECT: WHEN P_1 BECOMES CHEAPER, HIS NEW OPTIMAL CHOICE IS AT B': BUY MORE OF GOOD 1.

$$\left[\frac{dx_1^*}{dP_1} < 0\right]$$

SE + IE



$\frac{MU_1}{MU_2} = \frac{P_1}{P_2}$

$\frac{MU_1}{MU_2} = \frac{P_1'}{P_2}$

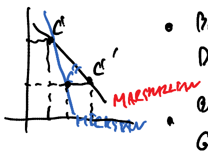
ORDINARY DEMAND CURVE (OR MARSHALLIAN DEMAND CURVE)

TAKE INTO A/C BOTH SE AND IE OF THE PRICE CHANGE.

COMPENSATED DEMAND CURVE (OR HICKSIAN DEMAND CURVE)

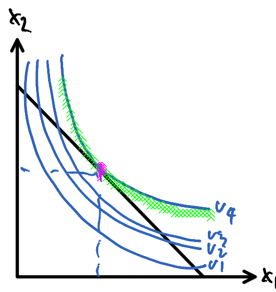
TAKE INTO A/C ONLY SUBSTITUTION EFFECT!

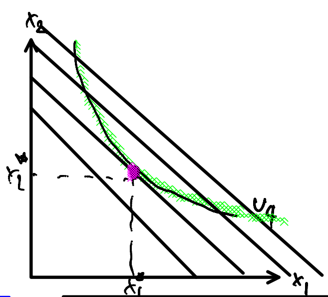
NOTICE THAT
- HICKSIAN D CURVE IS STEEPER THAN THE MARSHALLIAN D CURVE



- BASKET C AND C'' ON HICKSIAN D CURVE DELIVER THE SAME UTILITY LEVEL
- BASKET C' ON MARSHALLIAN D CURVE GIVES HIGHER UTILITY LEVEL COMPARE TO ORIGINAL BASKET.

DUALITY PROBLEM





UTILITY MAXIMIZATION

$$\boxed{\begin{array}{l} \text{MAX } U = x_1^a \cdot x_2^{1-a} \\ \text{S.T. } P_1x_1 + P_2x_2 = M \end{array}}$$

$\Downarrow$

$x_1^* = f(P_1, P_2, M)$

$x_2^* = f(P_1, P_2, M)$

MARSHALLIAN DEMAND CURVE

$$\boxed{\begin{array}{l} \text{MIN } P_1x_1 + P_2x_2 = M \\ \text{S.T. } U = x_1^a x_2^{1-a}. \end{array}}$$

COST MINIMIZATION

WE WILL COME BACK TO THIS AGAIN TOMORROW...

\# $\boxed{\text{PRICE - CONSUMPTION CURVE (PPC)}}$ & $\boxed{\text{INCOME - CONSUMPTION CURVE (ICC)}}$

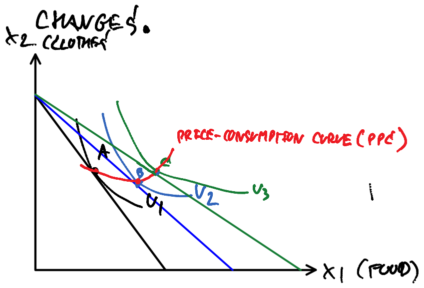
: CURVE TRACING

: CURVE TRACING THE

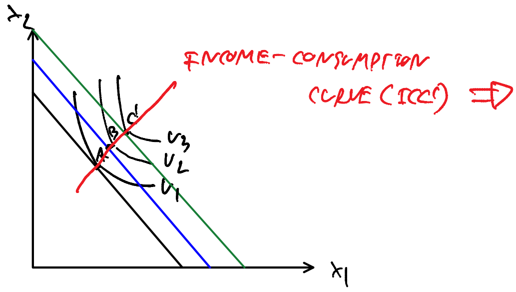
CURVE (PPC)

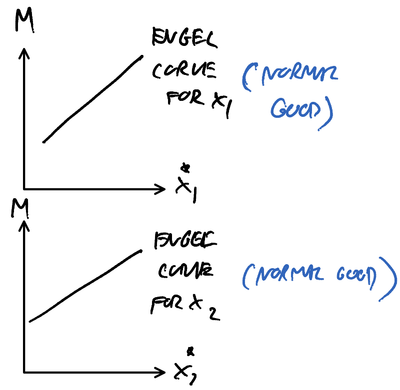
CURVE (ICC)

: CURVE TRACING THE UTILITY-MAXIMIZING COMBINATION OF TWO GOODS AS THE PRICE OF ONE CHANGES.



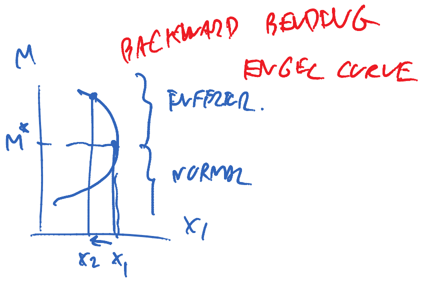
: CURVE TRACING THE UTILITY-MAXIMIZING COMBINATION OF THE TWO GOODS AS INCOME CHANGES.





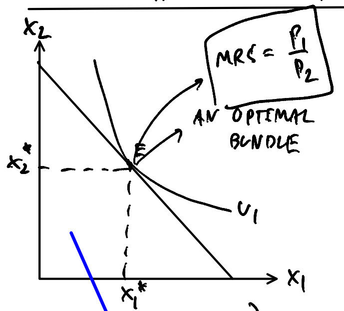
31.01.14 (ON WK 3)

- MANY CASES ON OPTIMAL CHOICE & SOME APPLICATIONS
- UTILITY FUNCTIONS → INDIFFERENCE CURVES → PRICE CONSUMPTION CURVE (PCC)
  - $U = x_1^{2} x_2^{1/2}$
  - $U = \min(x_1, x_2)$ → PERFECT COMPLEMENTS
  - $U = x_1 + x_2$ → → PERFECT COMPLEMENTS → DEMAND CURVE
  - HOW DOES IT LOOK LIKE ?
  - HOW DOES IT LOOK LIKE ?



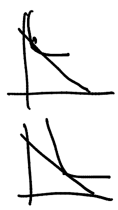
- SLUTSKY'S APPROACH TO DECOMPOSE PRICE EFFECT INTO SUBSTITUTION EFFECT & INCOME EFFECT (OR TOTAL EFFECT OF PRICE CHANGE)

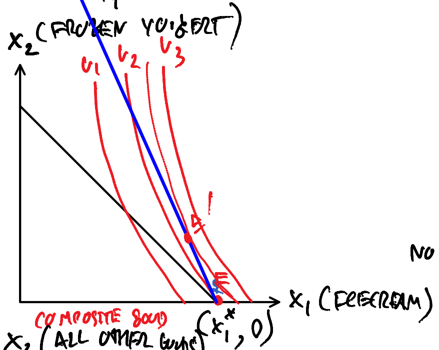
MANY CASES ON OPTIMAL CHOICE



NOTICE THAT AT BASKET E, HE CONSUMES A POSITIVE AMOUNT OF THE TWO GOODS: GOOD 1 & GOOD 2,

" INTERIOR SOLUTION "





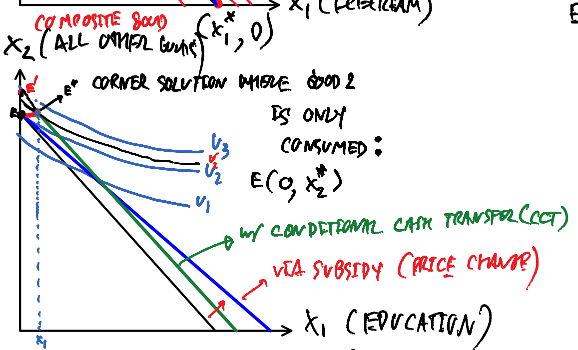
AT THE OPTIMAL BUNDLE E, HE CONSUMES ONLY GOOD 1.

" CORNER SOLUTION "

NOTICE THAT: AT E: $MRS > \frac{P_1}{P_2}$!

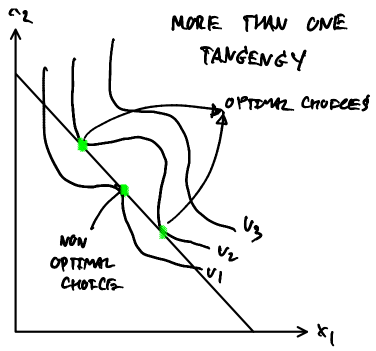
Ex: AT E, $MRS = 10 > \frac{P_1}{P_2} = \frac{2}{1} = 2$.

HE IS WILLING TO WITHDRAW ONE UNIT OF GOOD 1 IF YOU COMPENSATE HIM W/ 10 UNITS OF GOOD 2

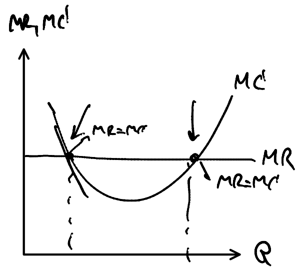


ASSUME THAT EDUCATION IS GOOD (i.e, MORE IS BETTER)

x_1



IMPLICATION FROM THIS IS THAT "TANGENCY CONDITION IS NECESSARY BUT NOT SUFFICIENT FOR UTILITY MAXIMIZATION.

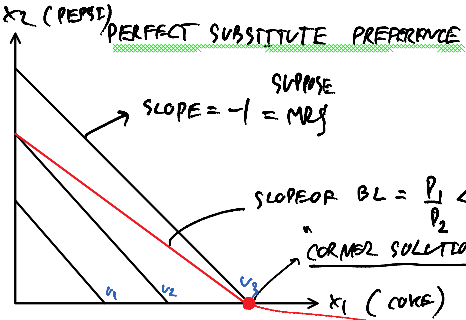


$\boxed{MC = MR}$ + $\boxed{\text{MC MUST BE UPWARD SLOPING}}$

NECESSARY CONDITION — FONC

SUFFICIENT CONDITION — SOC

PERFECT SUBSTITUTE PREFERENCE



SLOPE OF BL = $\frac{P_1}{P_2} < 1 \Rightarrow P_1 < P_2$

"CORNER SOLUTION" : OPTIMAL CHOICE IS ON A BOUNDARY.

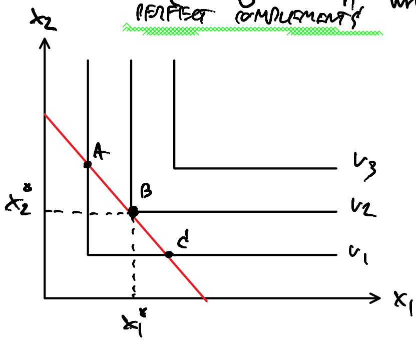
P_1 = PRICE OF GOOD$_1$ (COKE)

P_2 = PRICE OF GOOD 2 (PEPSI)

M = INCOME

$$x_1^* = \begin{cases} \frac{M}{P_1} & \text{WHEN } P_1 < P_2 \\ \text{ANY NUMBER BET. 0 AND } \frac{M}{P_1} & \text{WHEN } P_1 = P_2 \\ 0 & \text{WHEN } P_1 > P_2 \end{cases}$$

PERFECT COMPLEMENTS



THE CONSUMER SELECTS BASKET B (x_1^*, x_2^*) WHERE A FIXED PROPORTION OF 1 AND 2 IS CONSUMED.

UTILITY FUNCTIONS, INDIFFERENCE CURVES, AND PCC

- CASE 1 WHAT DOES A PRICE CONSUMPTION CURVE LOOK LIKE FOR COBB-DOUGLAS PREFERENCES?

TAKE $U(x_1, x_2) = x_1^a \cdot x_2^b$. [GIVEN $P_1x_1 + P_2x_2 = M$]

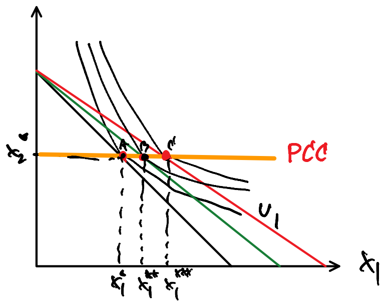
SO THE ORDINARY DEMAND FUNCTIONS FOR GOOD 1 AND GOOD 2 ARE

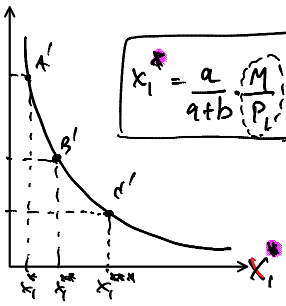
$$x_1^*(P_1, P_2, M) = \frac{a}{a+b} \cdot \frac{M}{P_1}.$$

$$x_2^*(P_1, P_2, M) = \frac{b}{a+b} \cdot \frac{M}{P_2}.$$

NOTE: x_2^* DOES NOT VARY W/ P_1 AND SO PPC IS FLAT.

x_2 P_1





$$\boxed{x_1^* = \frac{a}{a+b}\frac{M}{P_1}}$$

Ex: $a = 0.5$
$b = 0.5$
$a+b = 1$

$$\boxed{x_1^* = \frac{1}{2}\frac{M}{P_1}}$$

$$\boxed{x_2^* = \frac{1}{2}\frac{M}{P_2}}$$

PTY: $M = 1000$
$P_1 = 20$, $P_1' = 10$, $P_1'' = 5$
$P_2 = 10$

FIND
$x_1^* (P_1 = 20, P_2 = 10, M = 1000) = ?$
$x_1^{**} (P_1' = 10, P_2 = 10, M = 1000) = ?$
$x_1^{***} (P_1'' = 5, P_2 = 10, M = 1000) = ?$

CASE 2 WHAT DOES PCC LOOK LIKE FOR A PERFECT-SUBSTITUTE UTILITY FUNCTION ?

TAKE $U(x_1, x_2) = x_1 + x_2$

Ex:
$$\left.\begin{array}{l} x_1 = 10,\ x_2 = 0 \\ x_1 = 9,\ x_2 = 1 \\ x_1 = 8,\ x_2 = 2 \end{array}\right\} \rightarrow U(x_1, x_2) = x_1 + x_2 = 10$$

$$\left.\begin{array}{l} x_1 = 5,\ x_2 = 0 \\ x_1 = 4,\ x_2 = 1 \\ x_1 = 3,\ x_2 = 2 \end{array}\right\} \rightarrow U(x_1, x_2) = x_1 + x_2 = 5$$

$$x_1^*(P_1, P_2, M) = \begin{cases} 0 & \text{IF } P_1 > P_2 \\ \frac{M}{P_1} & \text{IF } P_1 < P_2 \end{cases}$$

AND
$$x_2^*(P_1, P_2, M) = \begin{cases} 0 & \text{IF } P_1 < P_2 \\ \frac{M}{P_2} & \text{IF } P_1 > P_2. \end{cases}$$