EE431 Economics of Financial Markets and Institutions
Problem Set 2 : Debt Market and Structure of Interest Rate

Solution

1. Write the Fisher Equation, relating the nominal interest rate i, the real interest rate r, and expected (or anticipated) inflation π^e :

   ANSWER.

   Fisher Equation shows the relationship between nominal interest rate, real interest rate and expected inflation rate. The Fisher equation is as follows.

   $(1 + i) = (1 + r)(1 + \pi^e)$

   The well-known simplified version is $i = r + \pi^e$.

   - π^e is the expected inflation rate. It represents the anticipated change in a broad price index that incorporates the prices of all assets (new and used), commodities and services. It measures the change in the general purchasing power of money over some specified time period. Since this price index can move up, down or remain unchanged, the associated expected rate of inflation can be positive, negative or zero.
   - r is the real interest rate. It is the relative price paid for earlier availability of goods and services in real terms—that is, in terms of the goods and services themselves. Stockman (1996, p. 705) notes: "Economists define the real interest rate as the relative price of a single 2 good at two points in time." Fisher (1930, pp. 61-2) stated that real interest rates, in the world we inhabit, must be positive for two reasons: time preference and investment opportunities.
   - i is the nominal interes rate. It is rate of interest on credit instruments.

2. A = the last digit of your student ID. **If the last digit of your student ID = 0, use A = 5**.

   Fill in the table below.

A	=..........................

   Example.

   A student with ID 540464031**9**, A = 9.

   A student with ID 550464024**3**, A = 3

   A student with ID 550464011**0**, A = 5.

   Suppose you earn A% nominal interest from your deposit account. If inflation rate is 2%, what is the real rate of return?

   ANSWER.

   The Fisher equation is $(1 + i) = (1 + r)(1 + \pi^e)$ or the well-known simplified relationship is $i = r + \pi^e$.

   Nominal interest rate is A% and inflation rate is 2%.

   Applying the fisher equation, $1 + r = \dfrac{1 + i}{1 + \pi} = \dfrac{1 + A\%}{1 + 0.02}$. Real interest rate $= \dfrac{1 + A\%}{1 + 0.02} - 1$.

   Applying the well-known simplified relationship, $i = r + \pi^e$. Real interest rate $= i - \pi^e = A\% - 2\%$.

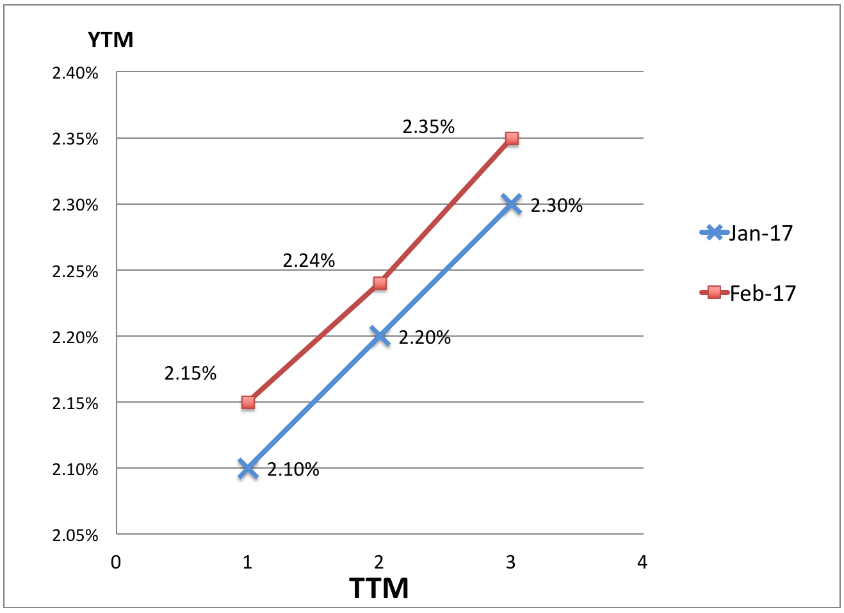
A	real interest rate: $(1+i)=(1+r)(1+\pi^e)$	real interest rate: $i=r+\pi^e$
1	-0.98%	-1%
2	0%	0%
3	0.98%	1%
4	1.96%	2%
5	2.94%	3%
6	3.92%	4%
7	4.90%	5%
8	5.88%	6%
9	6.86%	7%

3. Use the following information. Answer all parts of this question.

Government Bond Yield

TTM (Time To Maturity)	January 2017	February 2017
1	2.1%	2.15%
2	2.2%	2.24%
3	2.3%	2.35%

(a) Sketch the yield curves in January 2017 and February 2017 in the same graph.



(b) Use the government bond yield data in **January 2017**. Suppose the expectation theory of the yield curve holds, what is the expected one-year government bond rate for 2018 and 2019? Would you be able answer this question without assuming that the expectations hypothesis is true?

ANSWER. According to expectation theory, $i_{n,t} = \dfrac{i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2} + i^e_{1,t+3} + ... + i^e_{1,t+n-1}}{n}$.
In words, interest rate on long bond is equal to the average of short rates expected to occur over life of long bond.

$$
\begin{aligned}
i_{2,t} &= \frac{i_{1,t} + i^e_{1,t+1}}{2} \\
i_{2,2016} &= \frac{i_{1,2016} + i^e_{1,2017}}{2} \\
2.2\% &= \frac{2.1\% + i^e_{1,2017}}{2} \\
i^e_{1,2016} &= 2.3\%
\end{aligned}
$$

$$
\begin{aligned}
i_{2,t} &= \frac{i_{1,t} + i^e_{1,t+1}}{2} \\
i_{3,2016} &= \frac{i_{1,2016} + i^e_{1,2017} + i^e_{1,2018}}{3} \\
2.3\% &= \frac{2.1\% + 2.3\% + i^e_{1,2017}}{3} \\
i^e_{1,2017} &= 2.5\%
\end{aligned}
$$

ANS1. The expected one-year government bond rate for 2017 and 2018 are 2.3% and 2.5%, respectively. The market expects interest rate to increase.
ANS2. It is not possible to calculate the expected one-year government bond rate for 2017 and 2018 from the given information without assuming that the expectations hypothesis holds true. The given information is insufficient without the assumption.

(c) Use the government bond yield in **January 2017**. Suppose liquidity premium for 1 year and 2 year bonds are 0% and 0.5% respectively. According to the liquidity premium theory of the yield curve, what is your prediction for the interest rate on one-year bonds in 2018?

.According to liquidity premium theory, $i_{n.t} = \frac{i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2} + ... + +i^e_{1,t+n-1}}{n} + \eta_{n,t}$. In words, interest rate on long bond is equal to the average of short rates expected to occur over life of long bond plus term premium.

$$
\begin{aligned}
i_{2,t} &= \frac{i_{1,t} + i^e_{1,t+1}}{2} + \eta_{2,t} \\
i_{2,2016} &= \frac{i_{1,2016} + i^e_{1,2017}}{2} + \eta_{2,2016} \\
2.2\% &= \frac{2.1\% + i^e_{1,2016}}{2} + 0.5\% \\
2.2 - 0.5 &= \frac{2.1\% + i^e_{1,2016}}{2} \\
i^e_{1,2016} &= 3.4 - 2.1 \\
i^e_{1,2016} &= 1.3
\end{aligned}
$$

The interest rate on one-year bonds in 2016 is expected to be equal to 1.3%. The market expects interest rate to decrease.

4. Suppose one is examining the term structure of a 3 year discount bond, and the expectations hypothesis of the term structure holds.

$i_{3,t} = \dfrac{i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2}}{3}$

Suppose yesterday, $i_{1,t}$= 0.03 and $i_{3,t} = 0.06$

(a) Calculate the average value of $i^e_{1,t+1}$ and $i^e_{1,t+2}$.

ANSWER.

$i_{3,t} = \dfrac{i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2}}{3}$

multiply both sides by 3,

$3i_{3,t} = i_{1.t} + i^e_{1,t+1} + i^e_{1,t+2}$

$3 \times 0.06 = 0.03 + i^e_{1,t+1} + i^e_{1,t+2}$

subtract the current one year bond yield $= i_{1,t}$= 0.03 from both sides:

$0.15 = i^e_{1,t+1} + i^e_{1,t+2}$.

To get the average value, divide both sides by two.

$\dfrac{0.15}{2} = \dfrac{i^e_{1,t+1} + i^e_{1,t+2}}{2}$.

$0.075 = \dfrac{i^e_{1,t+1} + i^e_{1,t+2}}{2}$.

Hence, Calculate the average value of $i^e_{1,t+1}$ and $i^e_{1,t+2}$ is equal to **0.075 = 7.5%. ANS.**

(b) Suppose today the yield to maturity on the 3 year discount bond has increased by $\Delta i_{3,t}$,

- while the yield to maturity on a one year bond is unchanged from yesterday [$\Delta i_{1,t} = 0$] and
- the short term expected yield on the one year bond in period t+2 is unchanged [$\Delta i^e_{1,t+2} = 0$].
- Derive the **algebraic expression** for the implied change in the expected one year bond in period t+1[Find $\Delta i^e_{1,t+1}$].

ANSWER

$i_{3,t} = \dfrac{i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2}}{3}$

Take total differential and get

$$\begin{aligned}\Delta i_{3,t} &= \frac{\Delta i_{1,t} + \Delta i^e_{1,t+1} + \Delta i^e_{1,t+2}}{3} \\ \Delta i_{3,t} &= \frac{0 + \Delta i^e_{1,t+1} + 0}{3} \\ 3\Delta i_{3,t} &= \Delta i^e_{1,t+1}\end{aligned}$$

Therefore, $\Delta i^e_{1,t+1} = 3\Delta i_{3,t}$. **ANS.**

(c) Assume the 3 year bond yield is given by:

$i_{3,t} = \dfrac{i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2}}{3} + \eta_{3,t}$

And going from one day to the next day the yield on the 3 year discount bond has increased by $\Delta i_{3,t}$. Can one say for certain whether the increase is due to change in expected future rates, or due to a change in the risk premium? Why or why not?

ANSWER

$i_{3,t} = \dfrac{i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2}}{3}$

Take total differential and get

$$\Delta i_{3,t} = \frac{\Delta i_{1,t} + \Delta i^e_{1,t+1} + \Delta i^e_{1,t+2}}{3} + \Delta \eta_{3,t}$$

It is not possible to distinguish between the future interest rate effect and the risk premium effect. ANS.

5. Consider a group of investors who want to lend money out in financial markets for a two-year period and who must therefore choose between one of the following two strategies.

   Strategy 1: Buy a one-year bond today and when it matures, buy another one-year bond.
   Strategy 2: Buy a two-year bond today and hold it to maturity.
   Suppose, too, that the expected return on strategy 1 is higher than the expected return on strategy 2.

   (a) If these investors behave according to the assumptions of the expectations hypothesis, which strategy all investors will follow? Why?

   ANSWER.

   All investors will follow strategy 1 (buying one-year bond) since it gives higher expected return. The expecations hypothesis assumes that investors are indifferent between short-term bonds and long-term bonds. They pay attention on the expected return on the differences in the expected returns.

   (b) Suppose investors behave according to the assumptions of the expectation hypothesis and where the expected return on strategy 1 is higher than the expected return on strategy 2. Investors follow strategy in question (a). What are the effect of investors' action on interest rate on one-year bond and interest rate on two-year bond?

   ANSWER.

   From the answer in question (a), All investors will follow strategy 1, buying one-year bond.

   - The price of one-year bonds will increase and the interest rate on one-year bond to decrease. As a result, the expected return on strategy 1 falls.
   - Two year bonds cannot be sold at the original price. The price of two-year bonds will decrease and the interest rate on two-year bond to increase. As a result, the expected return on strategy 2 rises.
   - Originally, he expected return on strategy 1 is higher than the expected return on strategy 2. Investors' actions cause the expected return on strategy 1 to fall and the expected return on strategy 2 rise until the expected return on the two strategy equal.

   (c) Suppose investors behave according to the assumptions of the expectations hypothesis, how do expected return on the two strategy compare in equilibrium?

   ANSWER.

   Equilibrium is achieved when expected return on the two strategy are equal. If not, investors will follow the strategy which gives a higher return, causing the expected return to fall until the gap between the expected return on the two strategy is equal to zero.

   (d) Suppose instead that investors behave according to the assumptions of segmented markets theory. In this case, does investors' action will necessarily cause interest rates on one-year bonds to fall and interest rates on two-year bonds to rise until the expected returns on the two strategies are equal?

   ANSWER.

   According to assumptions of segmented market theory, there is no relationship between the markets for bonds of different maturity lengths. Bonds of different maturities are not substitutes at all. Investos may follow either strategy 1 by buying one-year bonds or strategy 2 by buying two-year bonds, depending on whether they prefer one-year bonds or two-year bonds and paying no attention to differences in expected returns. Hence, investors' action will **not** necessarily cause interest rates on one-year bonds to fall and interest rates on two-year bonds to rise until the expected returns on the two strategies are equal.

6. A lender forecasts inflation to be 4% over the upcoming year, 2% in following year, and 5% in the year after that. If she wants a real return (real interest rate) of 3% every year, determine the term structure of bonds with maturities of 1, 2, and 3 years implied by the expectations hypothesis and the Fisher relation.

   ANSWER.

   From the Fisher relation $i = r + \pi^e$, this implies the current and expected future one period nominal interest rates are

   - $i_{1,t} = 3\% + 4\% = 7\%$.
   - $i_{1,t+1} = 3\% + 2\% = 5\%$
   - $i_{1,t+2} = 3\% + 5\% = 8\%$

   Since, according to the expectations hypothesis, the long rates are an average of the current and expected future short rates, we have:

$$
\begin{aligned}
i_{2,t} &= \frac{i_{1,t} + i^e_{1,t+1}}{2} \\
i_{2,t} &= \frac{7\% + 5\%}{2} \\
&= 6\%
\end{aligned}
$$

$$
\begin{aligned}
i_{3,t} &= \frac{i_{1,t} + i^e_{1,t+1} + i^e_{1,t+2}}{3} \\
i_{3,t} &= \frac{7\% + 5\% + 8\%}{3} \\
&= 6.67\%
\end{aligned}
$$

   the term structure of bonds with maturities of 1, 2, and 3 years are equal to 7%, 6% and 6.67%, respectively. **ANS**.