

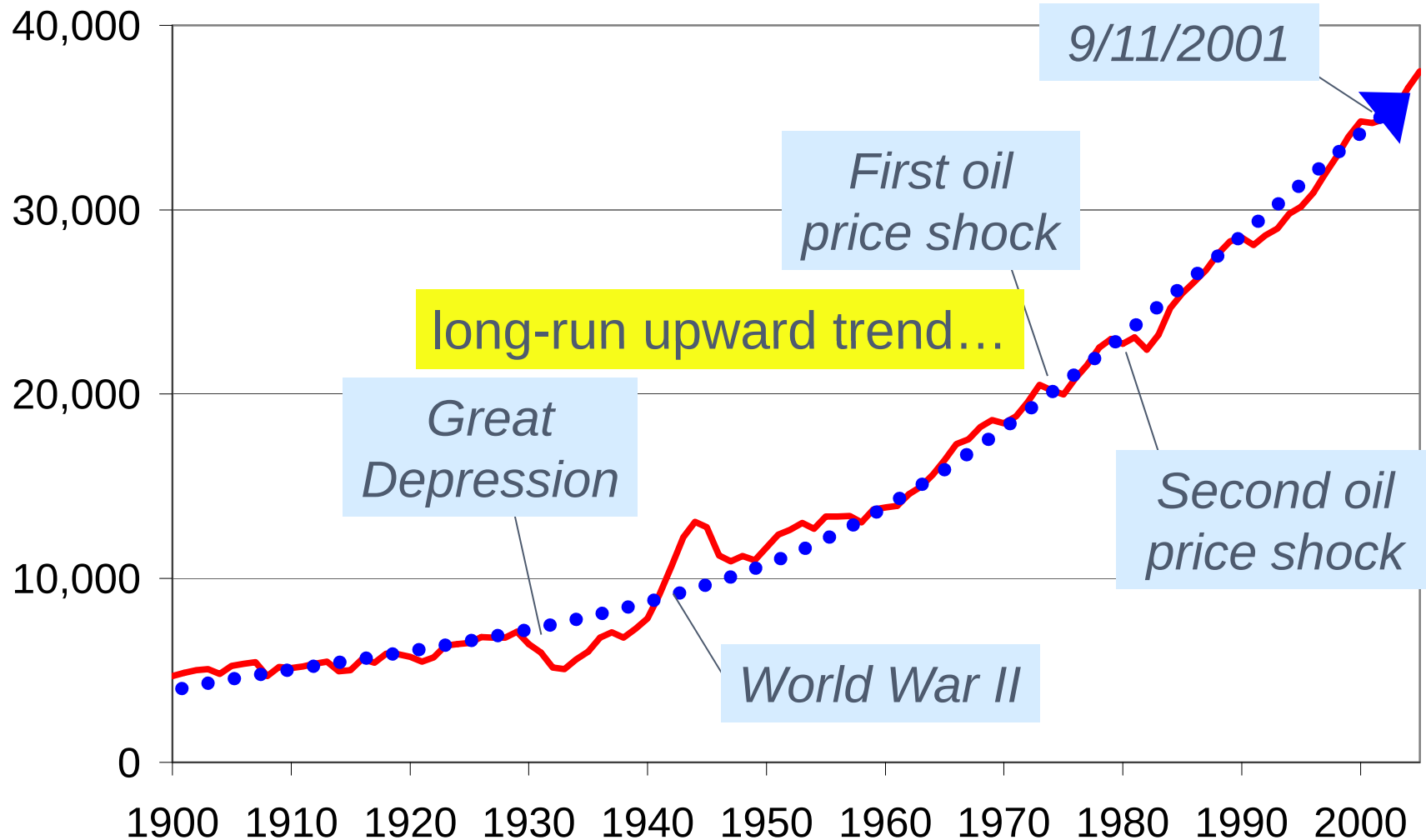
Macroeconomic Analysis (EE412)

Introduction to the framework of dynamic macroeconomic analysis

Dynamic Macroeconomic Model

- Dynamic macroeconomic model is an analytical framework that economists use to explain certain long-term and short-term macroeconomic facts,
- (1) Persistent long term growth in modern economic history,
- (2) There are both expansion and recession periods in the short run (known as “Business cycle”).
- (3) sustained diversity in income levels among different countries.

U.S. Real GDP per capita (2000 dollars)



Persistent Long Term Growth of Modern Economic Growth v.s. Malthusian Dilemma

- Sir Thomas Malthus wrote in 1798 the famous “Essay on the Principle of Population”
- He predicted that the world would experience exponential growth but only arithmetical food growth.
- As soon as the population levels exceed food levels by some critical amount, a Malthusian dilemma (doom day) occurs.

Sustained diversity in income levels

- World Bank's World Development Report 1983
 - The diversity across countries measured in per capita income levels is quite large
 - (US. \$10,000, India \$240) (1980)
 - This is a difference of a factor of 40 in living standard.

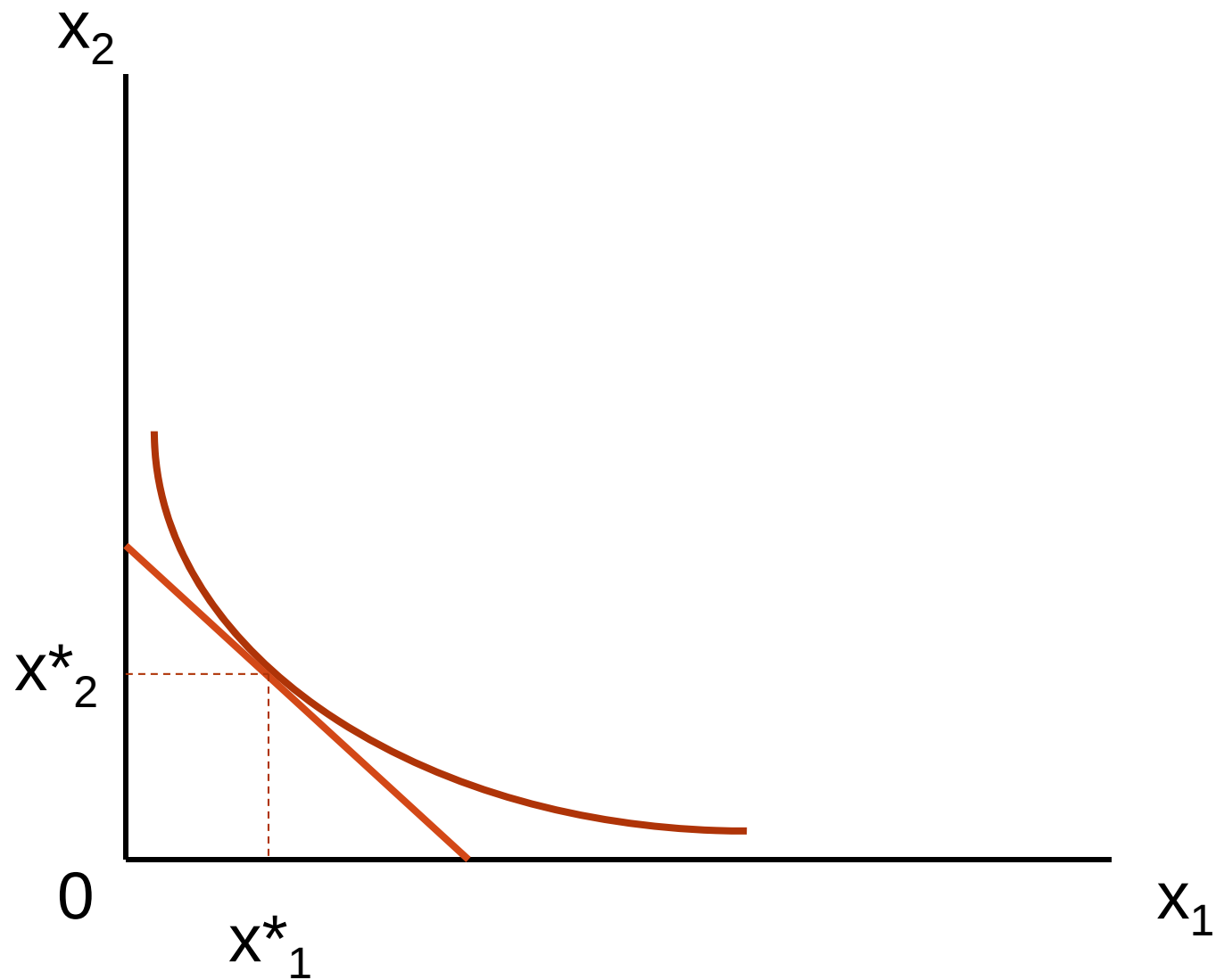
Static vs. Dynamic Problems

Static problem:
single period Problem

$$\underset{x_1, x_2}{Max} \quad U(x_1, x_2)$$

s.t.

$$p_{x_1} \cdot x_1 + p_{x_2} \cdot x_2 = Y$$



$$L = U(x_1, x_2) + \lambda \cdot [Y - p_{x_1} x_1 - p_{x_2} x_2],$$

first – order conditions,

$$\frac{\partial L}{\partial x_1} = 0,$$

$$\frac{\partial L}{\partial x_2} = 0,$$

$$\frac{\partial L}{\partial \lambda} = 0.$$

How do we solve a 2-period problem
(without uncertainty)?

$$\max_{\{x_1(t), x_2(t)\}_{t=0}^1} \sum_{t=0}^1 \left\{ (1+r)^{-t} \cdot U_t(x_1(t), x_2(t)) \right\}, \quad 0 < r < 1$$

s.t.

$$p_{x_1(0)} \cdot x_1(0) + p_{x_2(0)} \cdot x_2(0) \leq Y(0),$$

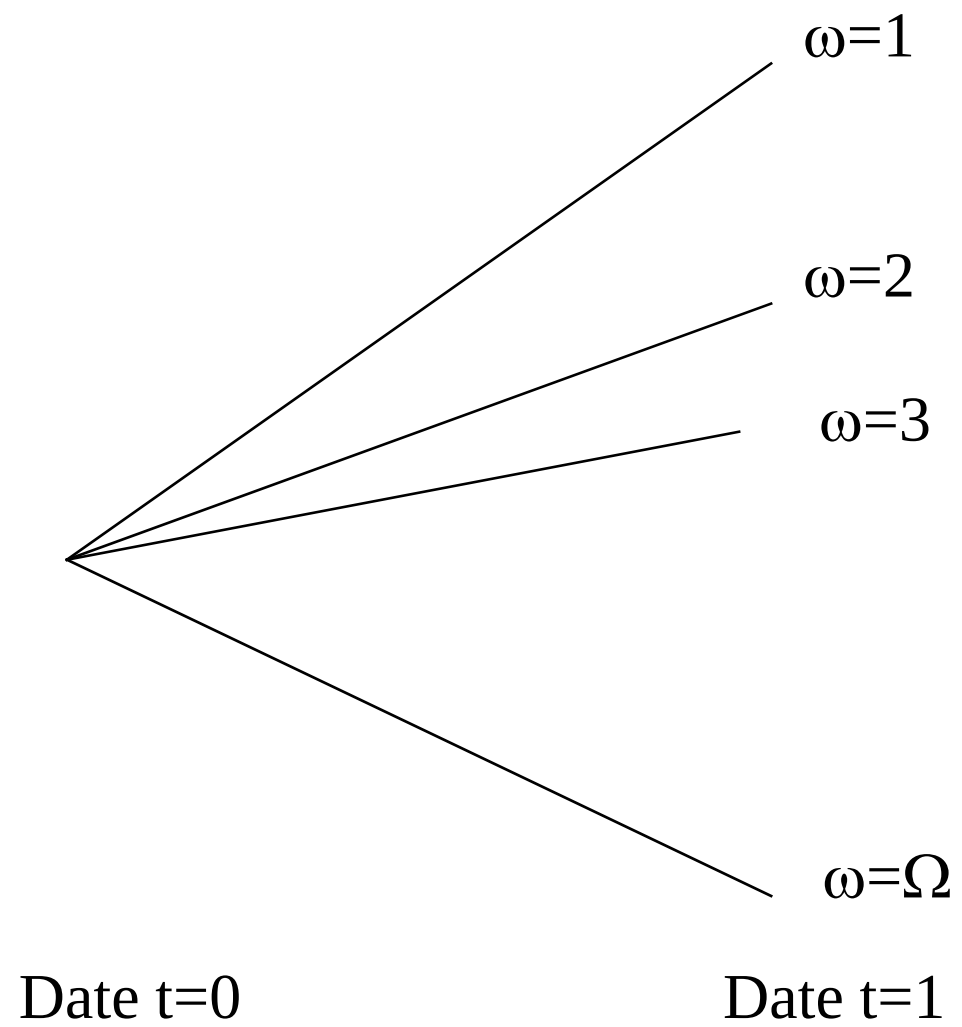
$$(1+r)^{-1} \cdot \left[p_{x_1(1)} \cdot x_1(1) + p_{x_2(1)} \cdot x_2(1) \right] = \left[Y(0) - p_{x_1(0)} \cdot x_1(0) - p_{x_2(0)} \cdot x_2(0) \right]$$

Should we solve each period problem separately?

Why?

Dynamic Model, Two-Date Model: Complete Markets

- Consider two dates: $t=0$ and $t=1$.
- There is uncertainty at $t=1$,
with $\omega = 1, 2, 3, \dots, \Omega$ states of the world.
- Let assume that there is only 1 physical commodity, at each date/state.
- In this case, complete markets means the number of markets for trading this commodity must be $\Omega+1$.



Complete Market

- In a complete markets, there exists an “Arrow-Debreu security” for each possible state of the world.
- An Arrow-Debreu security for state ω pays one unit of consumption only in state ω and nothing otherwise.
- The price of these Arrow-Debreu securities, $p(\omega)$, are called state prices.

Two-Date Model: Complete Markets

- Under these conditions, we can use the standard tools of price theory (in Microeconomics) under certainty to analyze an economy with contingent consumption.
- Each consumer $i=1,2,3,\dots,I$, has the problem:

Two-Date Model: Complete Markets

$$\begin{aligned} & \underset{\{x_i \in X_i\}}{\text{Max}} U_i(c_{0i}, c_{1i}, \dots, c_{\Omega i}) \\ \text{s.t.} \quad & p_0 c_{0i} + \sum_{\omega=1}^{\Omega} p(\omega) \cdot c_{\omega i} \equiv Y_{0i} \end{aligned}$$

where

p_0 is the price at $t=0$ of the commodity;

$p(\omega)$ is the price at $t=0$ of the contingent commodity ω ;

c_{0i} is the consumer i 's demand for commodity at $t=0$;

$c_{\omega i}$ is the consumer i 's demand for contingent commodity ω ;

Y_{0i} is $t=0$ wealth of the consumer i .

A Competitive Equilibrium

- A competitive equilibrium for the contingent claims economy is a price vector
 $(p_0^*, p^*(1), \dots, p^*(\omega), \dots, p^*(\Omega))$
consumer demands
 $c_i^* \equiv (c_{0i}^*, c_{1i}^*, \dots, c_{\Omega i}^*), i \in I$
such that :

$$c_i^* \text{ solves : } \text{Max } U_i(c_i)$$

(Note: this definition can also be extended to cover the case of an economy with production sector)

Multiple dates Model (More than two periods; i.e., $t=0,1,2,\dots,T$): Complete Market

- Problem: The number of all markets that must be cleared simultaneously (at $t=0$) will increase with the number of time periods.
- So, we may have to solve simultaneously a very large consumer problem.
- To avoid such problem, we may limit our analysis of consumer problems to those cases that have some special utility functions (to assume a much more restrictive type of preference).

Special Utility Functions

- For example, assuming for additive and separable utility over time (as well as over states)

$$U_i(c_{0i}, c_{1i}, \dots, c_{\Omega i}) = u_i(c_{0i}) + \sum_{\omega=1}^{\Omega} u_i(c_{\omega i}) \pi_{\omega},$$

where $u_i'(\bullet) > 0$, $u_i''(\bullet) \leq 0$.

$$0 < \pi_{\omega} < 1, \quad \sum_{\omega=1}^{\Omega} \pi_{\omega} = 1$$