Group 1

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1. *Theory of firm*

Suppose that production function is given by $Q = f(K,L) = \alpha\sqrt{K} + \beta L$ where K and L are the unit of capital installed and the number of employees hired, respectively. Assume that price of K and L are set equal to "r" and "w", respectively. Consider the following problems.

a) The firm wants to minimize cost and seek for combination of the two factor inputs to produce output level Q_0. Derive the factor inputs demand.

$$\min_{L,K} \quad \text{cost} = wL + rK$$

$$\text{s.t.} \quad Q = \alpha K^{1/2} + \beta L$$

$$\mathcal{L} = wL + rK + \lambda(Q_0 - \alpha K^{1/2} - \beta L)$$

F.O.C.

$$[L];\ \frac{\partial \mathcal{L}}{\partial L} = w - \lambda\beta = 0 \text{ ——— (1)} \longrightarrow w = \lambda\beta \text{ ——— (4)}$$

$$[K];\ \frac{\partial \mathcal{L}}{\partial K} = r - \frac{1}{2}\lambda\alpha K^{-1/2} = 0 \text{ ——— (2)} \longrightarrow r = \frac{1}{2}\lambda\alpha K^{-1/2} \text{ ——— (5)}$$

$$[\lambda];\ \frac{\partial \mathcal{L}}{\partial \lambda} = Q_0 - \alpha K^{1/2} - \beta L = 0 \text{ ——— (3)}$$

$$\frac{(4)}{(5)};\ \frac{w}{r} = \frac{2\beta}{\alpha} K^{1/2}$$

$$K^{1/2} = \frac{w}{r}\cdot\frac{\alpha}{2\beta} \text{ ——— (6)} \Rightarrow K^* = \left(\frac{\alpha w}{2\beta r}\right)^2 \quad \#$$

plug (6) in (3); $Q_0 - \alpha \frac{w}{r}\cdot\frac{\alpha}{2\beta} = \beta L$

$$L^* = \frac{2\beta r Q_0 - \alpha^2 w}{2\beta^2 r} \quad \#$$

b) Confirm your result with the second order derivative test.

S.O.C. (λ, L, K) 2 variable ⇒ check only last N-1 = 1

$$H = \begin{bmatrix} \mathcal{L}_{\lambda\lambda} & \mathcal{L}_{\lambda L} & \mathcal{L}_{\lambda K} \\ \mathcal{L}_{L\lambda} & \mathcal{L}_{LL} & \mathcal{L}_{LK} \\ \mathcal{L}_{K\lambda} & \mathcal{L}_{KL} & \mathcal{L}_{KK} \end{bmatrix} = \begin{bmatrix} 0 & -\beta & -\frac{1}{2}\alpha K^{-1/2} \\ -\beta & 0 & 0 \\ -\frac{1}{2}\alpha K^{-1/2} & 0 & \frac{1}{4}\lambda\alpha K^{-3/2} \end{bmatrix}$$

$|H_3| = (-\beta)(-1)\left(-\frac{1}{4}\lambda\alpha\beta K^{-3/2}\right) = -\frac{1}{4}\lambda\alpha\beta^2 K^{-3/2} < 0$; convex function along the constraint set

\# so (K^*, L^*) is cost minimization solution

c) State the condition under which demand for capital and labor are both strictly positive.

$K^* = \left(\frac{\alpha w}{2\beta r}\right)^2$ always positive ; $\beta \neq 0$ and $r \neq 0$ #

$L^* = \frac{2\beta r Q_0 - \alpha^2 w}{2\beta^2 r}$ positive when $2\beta r Q_0 - \alpha^2 w > 0$

$2\beta r Q_0 > \alpha^2 w$

$\beta \neq 0$, $r \neq 0$

$\beta > 0$, $r > 0$ #

d) Suppose that the condition required for strictly positive solution holds, derive the long-run optimal cost function, and show that marginal cost function is equal to the LaGrange multiplier of your cost minimization problem.

$\text{cost} = w \cdot L + r \cdot K$

$$C(Q) = w\left(\frac{2\beta r Q_0 - \alpha^2 w}{2\beta^2 r}\right) + r\left(\frac{\alpha w}{2\beta r}\right)^2$$

$$MC(Q) = \frac{dC(Q)}{dQ} = \frac{w}{\beta}$$

from (1) ; $w - \lambda\beta = 0$

$$\lambda = \frac{w}{\beta}$$

so $MC = \lambda = \frac{w}{\beta}$ #

2. *Theory of consumer*

Consider a household with the utility function given by,

$U(x, y) = [x^2 + y^2]^{\frac{1}{2}}$

where x *and* y are two different consumption goods, i.e. good x and good y. Suppose that (i) *the prices for each of the two consumption goods are* p_x *and* p_y *respectively*, and (ii) *household's income is equal to M.* Consider the following problems

a) Calculate the total differential of the utility function.

$$dU = \frac{dU}{dx} \cdot dx + \frac{dU}{dy} \cdot dy$$

$$= \frac{1}{2}(x^2+y^2)^{-1/2} \cdot 2x \, dx + \frac{1}{2}(x^2+y^2)^{-1/2} \cdot 2y \, dy$$

$$= x(x^2+y^2)^{-1/2} dx + y(x^2+y^2)^{-1/2} dy \quad \#$$

b) Set up the constrained optimization problem and derive the Marshallian demand function.

$$\max_{(x,y)} \; U(x,y) = (x^2+y^2)^{1/2}$$

$$\text{s.t.} \quad P_x \cdot x + P_y \cdot y = M$$

$$\mathcal{L} = (x^2+y^2)^{1/2} + \lambda(M - P_x \cdot x - P_y \cdot y)$$

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{1}{2}(x^2+y^2)^{-1/2} \cdot 2x - \lambda P_x = 0 \quad \text{---} (1)$$

$$\frac{\partial \mathcal{L}}{\partial y} = \frac{1}{2}(x^2+y^2)^{-1/2} \cdot 2y - \lambda P_y = 0 \quad \text{---} (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = M - P_x \cdot x - P_y \cdot y = 0 \quad \text{---} (3)$$

from (1) and (2)

$$\frac{1}{2}(x^2+y^2)^{-1/2} \cdot 2x = \lambda P_x \quad \text{---} (4)$$

$$\frac{1}{2}(x^2+y^2)^{-1/2} \cdot 2y = \lambda P_y \quad \text{---} (5)$$

$$\frac{(5)}{(4)}; \quad \frac{y}{x} = \frac{P_y}{P_x} \Rightarrow y = \frac{P_y}{P_x} \cdot x \quad \text{---} (6)$$

plug (6) in (3);

$$M - P_x \cdot x - P_y \cdot \frac{P_y}{P_x} \cdot x = 0$$

$$x^* = \frac{M}{P_x^2 + P_y^2} \cdot P_x$$

similarly; $y^* = \dfrac{M}{P_x^2 + P_y^2} \cdot P_y$

Marshallian Demand Function

$$(x^*, y^*) = \left(\frac{M}{P_x^2 + P_y^2} \cdot P_x \,,\, \frac{M}{P_x^2 + P_y^2} \cdot P_y \right) \quad \#$$

c) Does the demand function satisfy *the law of demand*? Mathematically, how do you know that?

Under law of demand, if x and y are normal goods, Price increase quantity will decrease

$$\frac{\partial y}{\partial P_y} : \frac{M(P_x^2+P_y^2) - 2P_y(P_y M)}{(P_x^2+P_y^2)^2}$$

$$= \frac{MP_x^2+MP_y^2-2P_y^2 M}{(P_x^2+P_y^2)^2}$$

$$= \frac{MP_x^2-MP_y^2}{(P_x^2+P_y^2)^2}$$

$P_x^2 > P_y^2 : \frac{+}{+}$ P↑Q↑ : not satisfy

$P_x^2 < P_y^2 : \frac{-}{+}$ P↑Q↓ : satisfy

$$\frac{\partial x}{\partial P_x} = \frac{M(P_x^2+P_y^2) - 2P_x(P_x M)}{(P_x^2+P_y^2)^2}$$

$$= \frac{MP_x^2+MP_y^2-2P_x^2 M}{(P_x^2+P_y^2)^2}$$

$$= \frac{MP_y^2-MP_x^2}{(P_x^2+P_y^2)^2}$$

$P_x^2 > P_y^2 : \frac{-}{+} = -$ P↑Q↓ : satisfy

$P_x^2 < P_y^2 : \frac{+}{+} = +$ P↑Q↑ : not satisfy

d) How does the demand for good y respond to price of good x?

$$\frac{dy^*}{dP_x} = \frac{d\,\frac{MP_y}{P_x^2+P_y^2}}{dP_x} = \frac{(P_x^2+P_y^2)\frac{d\,MP_y}{dP_x} - MP_y\frac{d}{dP_x}(P_x^2+P_y^2)}{(P_x^2+P_y^2)^2}$$

$$= \frac{-2MP_xP_y}{(P_x^2+P_y^2)^2}$$

when price of good x increases by 1 \$, demand for good y decreases by $\frac{2MP_xP_y}{(P_x^2+P_y^2)^2}$ units

good x and y are complement products

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e) What is the numerical value of λ^* when $M = \$300,\ p_x = 1,\ p_y = 1$?

$$x^* = \frac{M}{p_x^2 + p_y^2} \cdot p_x = \frac{300}{1^2+1^2} \cdot 1 = 150$$

$$y^* = \frac{M}{p_x^2 + p_y^2} \cdot p_y = \frac{300}{1^2+1^2} \cdot 1 = 150$$

from (4) ; $\frac{1}{2}(x^2+y^2)^{-1/2} \cdot 2x = \lambda p_x$

$$\lambda = \frac{\frac{1}{2}(x^2+y^2)^{-1/2} \cdot 2x}{p_x} = \frac{x \cdot (x^2+y^2)^{-1/2}}{p_x}$$

$$= \frac{150\,(150^2+150^2)^{-1/2}}{1}$$

$$= \frac{150}{150\sqrt{2}} = 0.707 \quad \#$$

f) Without redoing the optimization problem, what would be the new optimized level of maximum utility when income increases to \$310.

when $M = 300$; $U^*(x,y) = (x^{*2} + y^{*2})^{1/2}$

$x^* = 150,\ y^* = 150$ $\qquad = 212.1$

$$\lambda = \frac{\partial U}{\partial M}$$

$$\Delta U^* = \lambda^* \cdot \Delta M$$

$$= 0.707 \times (310 - 300)$$

$$= 7.07$$

As income increase by 10 \$, level of utility increases by 7.07

So, the new optimized level of maximum utility = 212.1 + 7.07

= 219.17 #

3. Suppose that a monopolist has its marginal cost function given by $MC = 16 + 6q^2$ where q is the amount of output produced. The monopolist faces the market demand function given by $P = 160 - 10q^2$ where P is the price per unit of output. Consider the following problem.

a) Suppose that fixed cost is equal to $240. Calculate the total and the average cost when q = 9 units.

$MC = 16 + 6q^2$

$\int MC\, dq = TC = \int (16 + 6q^2)\, dq$

$= 16q + 2q^3 + C$

Fixed cost = 240 = C

$TC(q) = 2q^3 + 16q + 240$

$TC(9) = 1,842$ #

$AC(9) = \frac{1,842}{9} = 204.67$ #

b) Determine the profit-maximizing level of output for the monopolist. Also, confirm your result by using the second derivative test.

$\pi = P \cdot q - TC$

$= (160 - 10q^2) \cdot q - (2q^3 + 16q + 240)$

F.O.C.

$\frac{d\pi}{dq} = 160 - 30q^2 - 6q^2 - 16 = 0$

$144 = 36q^2$

$q^* = 2$ units #

$P^* = 120$ \$/unit

S.O.C

$\frac{d^2\pi}{dq^2} = -60q - 12q$

$q^* = 2$; $\frac{d^2\pi}{dq^2} = -144 < 0$

π is concave , $q^* = 2$ is maximize solution #

c) Calculate the social welfare under the monopoly environment.

$MC = 16 + 6q^2$

$TR = 160q - 10q^3$

$MR = \frac{dTR}{dq} = 160 - 30q^2$

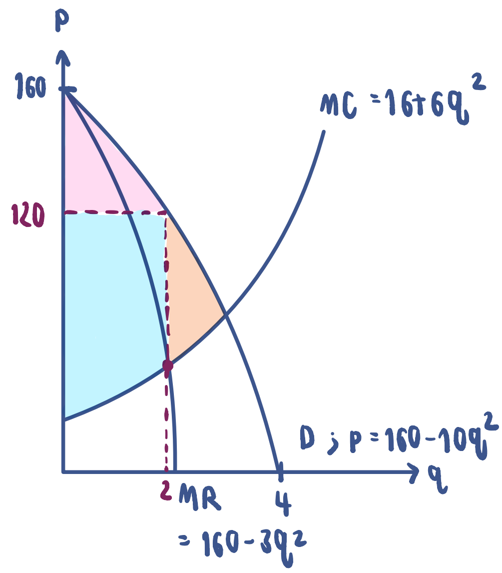
Demand curve ; $P = 160 - 10q^2$

Q-intercept ; $P = 0$; $0 = 160 - 10q^2$

$0 = 16 - q^2$

$q = -4, 4$

P-intercept ; $q = 0$; $P = 160$



Equilibrium under monopoly

$MR = MC$

$160 - 30q^2 = 16 + 6q^2$

$144 = 36q^2$

$q^M = 2$

$p^M = 160 - 10q^2 = 120$

$$CS^M = \int_0^2 (160 - 10q^2)\,dq - (120)(2)$$

$$= \left[160q - \frac{10}{3}q^3\right]_0^2 - 240$$

$$= 320 - \frac{80}{3} - 240$$

$$= 53.33$$

$$PS^M = (120)(2) - \int_0^2 (16 + 6q^2)\,dq$$

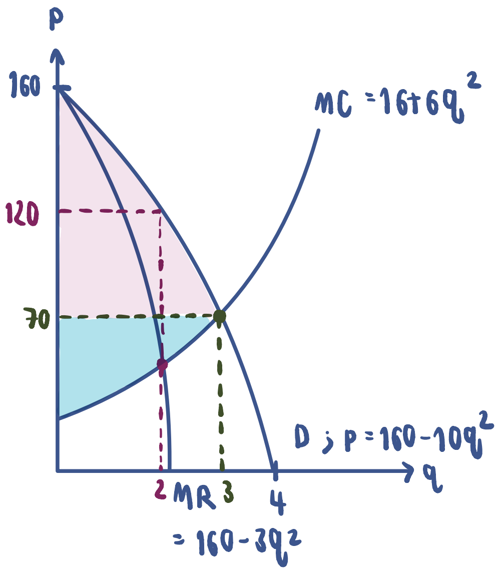
$$= 240 - \left[16q + 2q^3\right]_0^2$$

$$= 240 - (32 + 16)$$

$$= 192$$

Total welfare^M $= 53.33 + 192 = 245.33$ #

d) Calculate the social welfare loss under the monopoly environment.



Equilibrium under Perfect Competitive Market

$$P = MC$$

$$160 - 10q^2 = 16 + 6q^2$$

$$q^{PC} = 3$$

$$p^{PC} = 160 - 10(3^2) = 70$$

$$CS^{PC} = \int_0^3 (160 - 10q^2)\,dq - (70)(3)$$

$$= \left[160q - \frac{10q^3}{3}\right]_0^3 - 210$$

$$= 480 - 90 - 210$$

$$= 180$$

$$PS^{PC} = (70)(3) - \int_0^3 (16 + 6q^2)\,dq$$

$$= 210 - \left[16q + 2q^3\right]_0^3$$

$$= 210 - (48 + 54)$$

$$= 108$$

$$\text{Total welfare}^{PC} = 180 + 108 = 288$$

$\therefore$ Welfare loss under monopoly $=$ Total welfare^{PC} $-$ Total welfare^{M}

$= 288 - 245.33$

$= 42.67$ #

$=$ Deadweight Loss

4. Suppose the demand and supply curves are $P = \frac{6000}{Q+50}$ and $P = Q + 10$. Find the equilibrium price and quantity, and compute the consumer and producer surplus.

Equilibrium $Q^d = Q^s$; $P^d = P^s$

$$\frac{6000}{Q+50} = Q+10$$

$$6000 = Q^2 + 60Q + 500$$

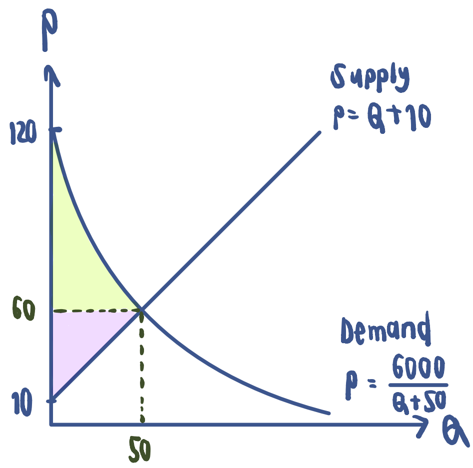
$$0 = Q^2 + 60Q - 5500$$

$$0 = (Q+110)(Q-50)$$

$$Q = -110, 50$$

$Q^* = 50$ units #

$P^* = 60$ \$ per unit #



$$CS = \int_0^{50} \frac{6000}{Q+50}\,dQ - (60)(50)$$

$$= 6000\Big[\ln|Q+50|\Big]_0^{50} - 3000$$

$$= 6000(0.693) - 3000$$

$$= 1{,}158.883$$ #

$$PS = (60)(50) - \int_0^{50}(Q+10)\,dQ$$

$$= 3000 - \left[\frac{Q^2}{2} + 10Q\right]_0^{50}$$

$$= 3000 - (1250 + 500)$$

$$= 1250$$ #

Total welfare $= 1{,}158.883 + 1{,}250$

$$= 2{,}408.883$$

5. Let $MR = 25 - 5x - 2x^2$ and $MC = 10 - 3x - x^2$, where x is the unit of output. Assume that fixed cost is \$7. Determine the level of production that contributes to maximum profit and determine the level of maximized profit.

$MR = 25 - 5x - 2x^2$

$TR = \int MR \, dx$

$= \int (25 - 5x - 2x^2) \, dx$

$TR = 25x - \frac{5}{2}x^2 - \frac{2}{3}x^3 + c$

$MC = 10 - 3x - x^2$

$TC = \int MC \, dx$

$= \int (10 - 3x - x^2) \, dx$

$= 10x - \frac{3}{2}x^2 - \frac{x^3}{3} + c$

fixed cost = 7 \$

$TC = 10x - \frac{3}{2}x^2 - \frac{1}{3}x^3 + 7$

$\pi = TR - TC$

$\pi = 25x - \frac{5}{2}x^2 - \frac{2}{3}x^3 + c - \left[10x - \frac{3}{2}x^2 - \frac{1}{3}x^3 + 7\right]$

F.O.C. $\pi' = TR' - TC' = MR - MC = 0 \Rightarrow MR = MC$

$\frac{d\pi}{dx} = 25 - 5x - 2x^2 - 10 + 3x + x^2 = 0$

$15 - 2x - x^2 = 0$

$x = -5, 3$

$x^* = 3$ #

$\pi(3) = 75 - \frac{45}{2} - 18 + c - \left(30 - \frac{27}{2} - 9 + 7\right)$

$= 20 + c$ #