

EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.), 2020

Use 4 decimal places for numerical answers

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

Table 1.a

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

29.7 621

$$\text{var}(X) = \sum (X_i - \bar{X})^2$$

$X_i - \bar{X}$	$(X_i - \bar{X})^2$
① -14.625	213.89
② -5.625	31.64
③ 0.375	0.140625
④ 3.375	11.3906
⑤ 4.375	19.1406
⑥ -2.625	6.89
⑦ -2.625	6.89
⑧ 12.375	153.14

511.87

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim \text{NIID}(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1 . (Note: NIID = Normally, Identically, and Independently Distributed).

$$\hat{\beta}_1 = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2} = \frac{(-0.4125)(-14.625) + (0.1875)(-5.625) + (-0.2125)(0.375) + (0.2875)(3.375) + (0.3875)(4.375) + (-0.2125)(-2.625) + (-0.2125)(-2.625) + (0.4875)(12.375)}{511.87} = 0.034$$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} = 2.97 - 0.034(77.625) = 0.57325$$

1.2 For each observation i , find $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum_{i=0}^N \hat{u}_i = 0$.

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i = 0.5733 + 0.034 X_i$$

$$\hat{u}_i = Y_i - \hat{Y}_i$$

$\hat{Y}_i$	$\hat{u}_i$
① 2.7153	① 0.0847
② 3.0213	② 0.3787
③ 3.1253	③ -0.2253
④ 3.3293	④ 0.1707
⑤ 3.5313	⑤ 0.0687
⑥ 3.1233	⑥ -0.1233
⑦ 3.1233	⑦ -0.4233
⑧ 3.6333	⑧ 0.0667

$\sum_{i=0}^N \hat{u}_i = -0.0006 \approx 0$
 -0.0693

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1.3 Find $var(\hat{u}_i)$, $var(\hat{\beta}_0)$, $var(\hat{\beta}_1)$

$$var(\hat{u}_i) = \hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{0.4346}{6} = 0.0724$$

$$var(\hat{\beta}_1) = \frac{var(\hat{u}_i)}{\sum (x_i - \bar{x})^2} = \frac{0.0724}{511.87} = 0.0014$$

$$var(\hat{\beta}_0) = \frac{\hat{\sigma}^2}{\sum x_i^2} = \frac{0.0724}{48717} = 0.00000148613$$

2. Data is listed in the table

X_i	Y_i	$\bar{x} = \frac{200}{10} = 20$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$y_i - \bar{y}$
10	0		-10	100	-9.1
12	2	$\bar{y} = \frac{91}{10} = 9.1$	-8	64	-7.1
14	5		-6	36	-4.1
16	6		-4	16	-3.1
18	7		-2	4	-2.1
22	10		2	4	0.9
24	10		4	16	0.9
26	15		6	36	5.9
28	16		8	64	6.9
30	20		10	100	10.9

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning. 394

$$\hat{\beta}_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{(-10)(-9.1) + (-8)(-7.1) + (-6)(-4.1) + (-4)(-3.1) + (-2)(-2.1) + (2)(0.9) + (4)(0.9) + (6)(5.9) + (8)(6.9) + (10)(10.9)}{440} = 0.8955$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 9.1 - (0.8955)(20) = -8.81$$

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum_{i=0}^N \hat{u}_i = 0$.

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i = -8.81 + 0.8955 X_i$$

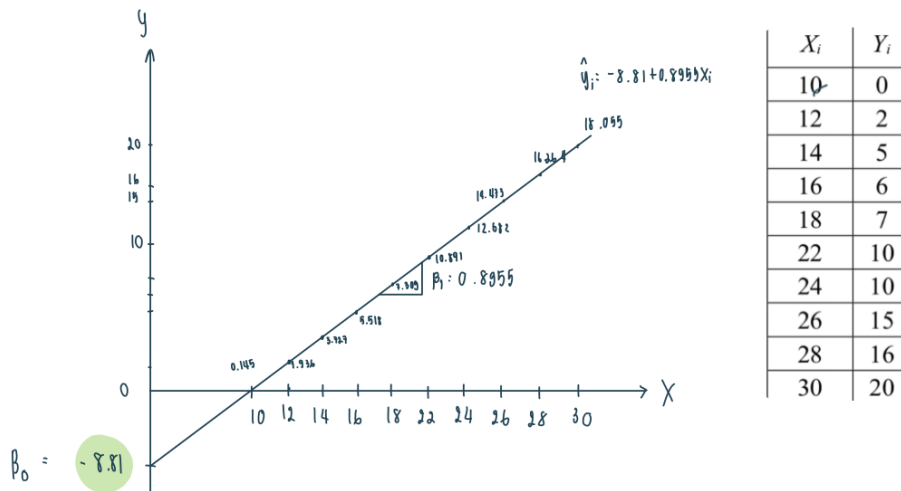
$$\hat{u}_i = Y_i - \hat{Y}_i$$

i	X_i	Y_i	$\hat{Y}_i$	$\hat{u}_i$
1	10	0	0.145	-0.145
2	12	2	1.936	0.064
3	14	5	3.727	1.273
4	16	6	5.518	0.482
5	18	7	7.309	-0.309
6	20	10	9.091	-0.891
7	22	10	10.882	-0.882
8	24	15	12.673	2.327
9	26	16	14.464	-0.264
10	28	20	16.255	3.745

$\sum \hat{u}_i = 0$

14.09091

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?



2.4 If $X_i = 16$, what is the predicted Y ?

$$\hat{Y}_4 = 5.518$$

2.5 Find $var(\hat{u}_i)$, $var(\hat{\beta}_0)$, $var(\hat{\beta}_1)$

$$var(\hat{u}_i) = \hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-2} = \frac{14.0909}{8} = 1.76136$$

$$var(\hat{\beta}_1) = \frac{var(\hat{u}_i)}{\sum (x_i - \bar{x})^2} = \frac{1.76136}{440} = 0.004$$

$$var(\hat{\beta}_0) = \frac{\hat{\sigma}^2}{\sum x_i^2} = \frac{1.76136}{440} = 0.00398 \approx 0.004$$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i,$$

Where $u_i \sim \text{NIID}(0, \sigma^2)$. Find an OLS estimator of β_1 . Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$$\hat{\beta}_1 = \frac{\sum (y_i - \bar{y})(x_i - \bar{x})}{\sum (x_i - \bar{x})^2}$$

$$k_i = x_i - \bar{x} \quad k = \frac{u_i}{\sum u_i^2}$$

$$\hat{\beta}_1 = \sum (y_i - \bar{y}) k$$

$$= \sum (\beta_0 + \beta_1 x_i + u_i - \beta_0 - \beta_1 \bar{x}) k$$

$$= \beta_1 \sum (x_i - \bar{x}) k + u_i k$$

$$= \beta_1 \sum u_i \frac{u_i}{\sum u_i^2} + u_i k$$

$$= \beta_1 + u_i k$$

$$E(\hat{\beta}_1) = E(\beta_1 + \sum u_i k)$$

$$= \beta_1 + E(\sum u_i k) \rightarrow \text{SLR 4: } E(u_i | x_i) = 0$$

$$= \beta_1 + \sum k E(u_i)$$

$$E(\hat{\beta}_1) = \beta_1$$