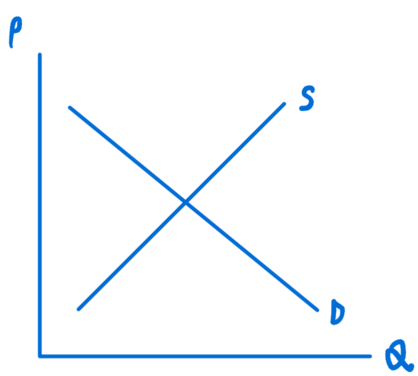
EE320 Placement test

1. Attempt all.
2. Submit your work (in .pdf) on the Moodle. The required format of your filename is studentID_PT
3. You will get **TWO** bonus points if you submit this placement test by the deadline.
4. **This placement test is due on Friday 14th, at 11 AM. Late submission will not be accepted.**

1. Suppose that market demand is given by $P = 10 - Q^2$ and the market supply is given by $Q = a + P$, where P is the unit price, Q is the quantity of output, and a is the coefficient in the supply equation.
   1.1) Graph the market demand and market supply curve in a P-Q diagram. Set the value of a equal to – 14.
   1.2) Solve for the market equilibrium quantity (Q*) and price (P*) when a = -14. Show your work.
   1.3) If “a” increases to -12, what would happen to the market equilibrium quantity and price? State the qualitative predictions without redoing the algebra.

1



Demand , $P = 10 - Q^2$

Supply , $P = Q - a$

$= Q - (-14)$

$= Q + 14$

2 At equilibrium Demand = Supply

$10 - Q^2 = Q + 14$

$Q^2 + Q + 4 = 0$

$$Q = \frac{-1 \pm \sqrt{1 - 2(1)(4)}}{2}$$

3 $Q = P - 12$

Market equilibrium price and quantity will increase.

2. Suppose that the revenue function is given by $R(Q) = \ln(Q^2 + 1) + 3\left(\frac{Q}{Q+1}\right), Q \geq 0$. Use the derivative technique and calculate the marginal revenue function. Is the revenue function an increasing or decreasing function?

$$\frac{dR(Q)}{dQ} = \frac{1}{Q^2+1}(2Q) + 3\left(\frac{(Q+1)(1) - (Q)(1)}{(Q+1)^2}\right)$$

$$= \frac{2Q}{Q^2+1} + \frac{3}{(Q+1)^2}$$

$$MR = \frac{2Q}{Q^2+1} + \frac{3}{(Q+1)^2}$$

$$\frac{dMR}{dQ} = \frac{(Q^2+1)(2) - (2Q+3)(2Q)}{(Q^2+1)^2} - \frac{(3)(2(Q+1))}{(Q+1)^4}$$

$$= \frac{2Q^2+2-4Q^2-6Q}{(Q^2+1)^2} - \frac{6Q+6}{(Q+1)^4}$$

if $Q=0$; $\frac{2}{1} - \frac{6}{1} = -4 < 0$

Hence, revenue function is a decreasing function

3. Suppose that the profit function is given by $\pi(Q) = -\frac{1}{3}Q^3 - Q^2 + 8Q - 1$ where Q is the level of output. Use the calculus and solve for the level of profit-maximizing output. Confirm your answer with the second derivative.

$$\frac{d\pi(Q)}{dQ} = -Q^2 - 2Q + 8 = 0$$

$$Q^2 + 2Q - 8 = 0$$

$$(Q+4)(Q-2) = 0$$

$$Q = 2$$

$Q = 2$

$$-\frac{8}{3} - 4 + 16 - 1$$

$$\pi = \frac{25}{3} = 8.33$$

$$\frac{d\pi'(Q)}{dQ} = 2Q + 2 = 0$$

$$Q = -1 < 0 \quad \therefore \text{Max}$$

4. Suppose that $A = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, calculate the following object. Show your work.

4.1 A + B

$\emptyset$

4.2 A*B

$$\begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 8+36 & 16+45 & 24+54 \\ 10+44 & 20+55 & 30+66 \end{bmatrix}$$

$$= \begin{bmatrix} 44 & 61 & 78 \\ 55 & 75 & 96 \end{bmatrix}$$

4.3 det(A)

$(8 \times 11) - (10 \times 9) = -2$

4.4 det(B)

$((5 \times 1) + (6 \times 2)) - ((4 \times 2) + (5 \times 3)) = -6$

4.5 det(C)

$((1 \times 5 \times 9) + (2 \times 6 \times 7) + (3 \times 4 \times 8)) - ((7 \times 5 \times 3) + (8 \times 6 \times 1) + (9 \times 4 \times 2))$

$= 225 - 225$

$= 0$

5. Suppose that $U(x, y) = x^a y^b + \ln\left(\frac{x}{x+y}\right)$. Use the partial derivative technique, calculate $\frac{\partial U}{\partial x}$ and $\frac{\partial U}{\partial y}$.

$$\frac{\partial V}{\partial x} = a x^{a-1} y^b + \frac{x+y}{x}\left(\frac{(x+y) - (x)}{(x+y)^2}\right)$$

$$= a x^{a-1} y^b + \frac{x+y}{x}\left(\frac{x}{(x+y)^{\cancel{2}}}\right)$$

$$= a x^{a-1} y^b + \frac{y}{x(x+y)}$$

$$\frac{\partial V}{\partial y} = x^a b y^{b-1} + \frac{x+y}{x}\left(\frac{(x+y)(0) - (x)(1)}{(x+y)^2}\right)$$

$$= x^a b y^{b-1} - \frac{1}{(x+y)}$$