SUBJECT: EE320 (Semester 1/2013) NO: 1 DATE: / /

Homework 4 Solutions.

① $U = 25x_1 + 27x_2 - 3x_1^2 - 7x_1x_2 - 4x_2^2.$

(a) $MU_1 = \dfrac{\partial U}{\partial x_1} = 25 - 6x_1 - 7x_2$

$MU_2 = \dfrac{\partial U}{\partial x_2} = 27 - 7x_1 - 8x_2$

(b) $MRS_{12} = \dfrac{dx_2}{dx_1} = -\dfrac{MU_1}{MU_2}$

At $(x_1, x_2) = (2,1)$, $MRS_{12} = -\dfrac{[25-6(2)-7(1)]}{[27-7(2)-8(1)]} = -\dfrac{6}{5} = -1.2$

② (a) $U(x,y) = 6x^2 + 5xy + 9y^2$

$$H = \begin{bmatrix} U_{xx} & U_{xy} \\ U_{yx} & U_{yy} \end{bmatrix} = \begin{bmatrix} 12 & 5 \\ 5 & 18 \end{bmatrix}$$

(b) $Z(x,y) = 3(11x - 6y)^2$

$Z_x = 66(11x - 6y) \Rightarrow Z_{xx} = 726$; $Z_{xy} = -396$

$Z_y = -36(11x - 6y) \Rightarrow Z_{yy} = 216$; $Z_{yx} = -396.$

$$H = \begin{bmatrix} 726 & -396 \\ -396 & 216 \end{bmatrix}.$$

③ (a) $z = 4x^3 - 13xy - 6y^5$

$dz = (12x^2 - 13y)\,dx + (-13x - 30y^4)\,dy$

(b) $z = (2x^2 - y)(3x - 4y^3)$

Let $u = 2x^2 - y$ and $v = 3x - 4y^3$

$\Rightarrow du = 4x\,dx - dy$; $dv = 3\,dx - 12y^2\,dy.$

$dz = u\,dv + v\,du = (2x^2 - y)[3\,dx - 12y^2\,dy] + (3x - 4y^3)[4x\,dx - dy]$

$\therefore dz = (18x^2 - 16xy^3 - 3y)\,dx + (-24x^2y^2 + 16y^3 - 3x)\,dy$

(c) $Z = \dfrac{7y^3}{(5x-2y)}$

Let $u \equiv 7y^3$ and $v \equiv 5x-2y$

$\Rightarrow du = 21y^2 dy$; $dv = 5dx - 2dy$.

$$dz = \frac{v\,du - u\,dv}{v^2} = \frac{(5x-2y)(21y^2dy) - (7y^3)[5dx-2dy]}{(5x-2y)^2}$$

$$= \frac{[105xy^2 - 42y^3 + 14y^3]dy - 35y^3dx}{(5x-2y)^2}$$

$$\therefore\ dz = \frac{(-35y^3)dx + (105xy^2 - 28y^3)dy}{(5x-2y)^2}$$

(d) $z = 8x^{1/2}y^{1/4}w^{1/4}$

$dz = z_x dx + z_y dy + z_w dw$

$$\therefore\ dz = (4x^{-1/2}y^{1/4}w^{1/4})dx + (2x^{1/2}y^{-3/4}w^{1/4})dy + (2x^{1/2}y^{1/4}w^{-3/4})dw.$$

4 Let $F(K,L) \equiv 18[0.2K^{-0.4} + 0.8L^{-0.4}]^{-2.5} - 1936 = 0$.

Using implicit function rule, $\dfrac{dK}{dL} = -\dfrac{F_L}{F_K}$.

$$F_L = (18)(-2.5)[0.2K^{-0.4} + 0.8L^{-0.4}]^{-3.5}(-0.32)L^{-1.4}$$
$$= 14.4[0.2K^{-0.4} + 0.8L^{-0.4}]^{-3.5} \times L^{-1.4}$$

$$F_K = (18)(-2.5)[0.2K^{-0.4} + 0.8L^{-0.4}]^{-3.5}(-0.08)K^{-1.4}$$
$$= 3.6[0.2K^{-0.4} + 0.8L^{-0.4}]^{-3.5} \times K^{-1.4}$$

$$\Rightarrow \underbrace{\frac{dK}{dL}}_{MRTS} = -\frac{F_L}{F_K} = -\frac{14.4\,\cancel{[0.2K^{-0.4} + 0.8L^{-0.4}]^{-3.5}} \cdot L^{-1.4}}{3.6\,\cancel{[0.2K^{-0.4} + 0.8L^{-0.4}]^{-3.5}} \cdot K^{-1.4}} = -4\left(\frac{K}{L}\right)^{1.4}.$$

⑤ $Q_d = D(P, Y_0, t_0)$; $\frac{\partial D}{\partial P} < 0$, $\frac{\partial D}{\partial Y_0} > 0$, $\frac{\partial D}{\partial t_0} > 0$.

$Q_s = S(P, T_0)$; $\frac{\partial S}{\partial P} > 0$, $\frac{\partial S}{\partial T_0} < 0$.

Let P^* be the equilibrium price.

$P^* = P^*(Y_0, t_0, T_0)$.

At Eqm, $D(P^*, Y_0, t_0) = S(P^*, T_0)$.

$\Rightarrow F \equiv D(P^*, Y_0, t_0) - S(P^*, T_0) = 0$. : Implicit function.

- $$\frac{\partial P^*}{\partial t_0} = -\frac{F_{t_0}}{F_{P^*}} = -\frac{\overset{\oplus}{\partial D/\partial t_0}}{\underbrace{\underset{\ominus}{\partial D/\partial P^*} - \underset{\oplus}{\partial S/\partial P^*}}_{\ominus}} > 0.$$

$\Rightarrow$ As the taste increases, the equilibrium price rises.

- $$\frac{\partial P^*}{\partial T_0} = -\frac{F_{T_0}}{F_{P^*}} = -\frac{\overset{\oplus}{\boxed{-\overset{\ominus}{\partial S/\partial T_0}}}}{\underbrace{\frac{\partial D}{\partial P^*} - \frac{\partial S}{\partial P^*}}_{\ominus}} > 0$$

$\Rightarrow$ As the tax imposed on producer increases, the equilibrium price increases.

- $$\frac{\partial Q^*}{\partial Y_0} = \underset{\oplus}{\frac{\partial S}{\partial P^*}} \cdot \underset{\oplus}{\frac{\partial P^*}{\partial Y_0}}$$ [$\because Q^* = S(P^*, T_0) = D(P^*, Y_0, t_0)$ and $P^* = P^*(Y_0, t_0, T_0)$].

$$\frac{\partial P^*}{\partial Y_0} = -\frac{F_{Y_0}}{F_{P^*}}$$

$$= -\frac{\overset{\ominus}{\partial D/\partial Y_0}}{\underset{\ominus}{\frac{\partial D}{\partial P^*}} - \underset{\oplus}{\frac{\partial S}{\partial P^*}}} > 0.$$

$$\therefore \frac{\partial Q^*}{\partial Y_0} = \frac{\partial S}{\partial P^*} \cdot \frac{\partial P^*}{\partial Y_0} > 0.$$

$\Rightarrow$ As income increases, the equilibrium quantity increases.

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<u>Note</u>: For $\frac{\partial Q^*}{\partial Y_0}$, students can also use $Q^* = D(P^*, Y_0, t_0)$.

$$\Rightarrow \frac{\partial Q^*}{\partial Y_0} = \frac{\partial D(P^*, Y_0, t_0)}{\partial Y_0}$$

$$= \underbrace{\frac{\partial D}{\partial P^*} \cdot \frac{\partial P^*}{\partial Y_0}}_{\text{indirect effect of } Y_0} + \underbrace{\frac{\partial D}{\partial Y_0}}_{\text{direct effect of } Y_0} > 0 \text{ if direct effect is greater than indirect effect.}$$

(Hence, I chose $Q^* = S(P^*, T_0)$ in this analysis because there is only an indirect effect of Y_0 on Q^* via price, P^*.)

- $$\frac{\partial Q^*}{\partial T_0} = \frac{\partial D(P^*, Y_0, t_0)}{\partial T_0}$$

$$= \underset{\ominus}{\frac{\partial D}{\partial P^*}} \cdot \underset{\oplus}{\frac{\partial P^*}{\partial T_0}} < 0.$$

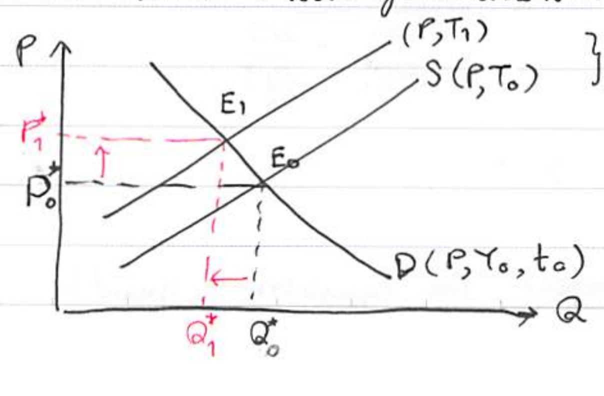
Proved previously

$\therefore$ As tax increases, the equilibrium quantity decrease.

<u>Comments</u>:

- For $\frac{\partial P^*}{\partial t_0}$ & $\frac{\partial P^*}{\partial T_0}$, use implicit function rule.
- For $\frac{\partial Q^*}{\partial Y_0}$ & $\frac{\partial Q^*}{\partial T_0}$, use partial total derivative (with some help of implicit f^n rule).
- You can double check your answers by using graph.

<u>Ex</u>



$\Rightarrow \frac{\partial Q^*}{\partial T_0} < 0$ and $\frac{\partial P^*}{\partial T_0} > 0$.