Bachelor of Economics
THAMMASAT UNIVERSITY

FN 211 Financial Markets

Class 3:
Interest Rates and Security Valuation

Win Phromphaet, CFA

Today's Outline

1. Equity Valuation
2. Bond Valuation
   - Price-Yield Relationship
   - Duration
   - Convexity

Equity Valuation:
How much is an Apple worth?

Apple Inc. (AAPL) - NasdaqGS

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694.09 ↑ 11.11(1.63%) 9:48AM EDT - Nasdaq Real Time Price

Prev Close:	682.98	Day's Range:	687.89 - 694.10
Open:	689.97	52wk Range:	354.24 - 685.50
Bid:	688.75 x 100	Volume:	3,309,402
Ask:	688.91 x 200	Avg Vol (3m):	13,869,200
1y Target Est:	756.07	Market Cap:	645.75B
Beta:	0.88	P/E (ttm):	16.19
Next Earnings Date:	15-Oct-12	EPS (ttm):	42.55
		Div & Yield:	10.60 (1.60%)

1d 5d 1m 3m 6m 1y 2y 5y max

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People viewing **AAPL** also viewed:
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Apple (NASDAQ: AAPL) became the most valuable company in history in terms of market capitalization. The company has a market-cap of over $648 billion.

That is twice the size of Thailand's GDP!

Equity Valuation:
The Basics

Estimated Value vs. Market Price

- Estimated value > market price => undervalued
- Estimated value = market price => fairly valued
- Estimated value < market price = > overvalued

Equity Valuation Models

1. **Present Value** – PV of expected future cash flows
   - *DDM, FCFE*
2. **Multiplier Model** – intrinsic value based on multiple of some fundamental variable:
   - *P/E, P/S, EV/EBITDA*
3. **Asset-based Valuation** – based on estimated value of assets - liabilities
   - *Book Value*

Equity Valuation:
Dividend Discount Model (DDM)

DDM

- Intrinsic value of a share is the PV of expected future dividends/cash flows.
- The issuing company is assumed to be a going concern.

$$P_0 = \frac{D_1}{(1+k)} + \frac{D_2}{(1+k)^2} + \ldots + \frac{D_\infty}{(1+k)^\infty} = \sum_{t=1}^{n} \frac{D_t}{(1+k)^t}$$

$$P_0 = \frac{FCFE_1}{(1+k)} + \frac{FCFE_2}{(1+k)^2} + \ldots + \frac{FCFE_\infty}{(1+k)^\infty} = \sum_{t=1}^{n} \frac{FCFE_t}{(1+k)^t}$$

where:

P_0 = value of common stock

D_t = dividend during time period t

k = required rate of return on stock

FCFE = Free Cash Flow for Equity = CFO – FCInv + Net borrowing

Equity Valuation:
Dividend Discount Model (DDM)

The N-Period Model

- If the stock is held for only N period, e.g. 2 years, and a sale at the end of year 2 would imply:

$$P_0 = \frac{D_1}{(1+k)} + \frac{D_2}{(1+k)^2} + \frac{SP_2}{(1+k)^2}$$

- The expected selling price, SP_2, of stock at the end of Year 2 is crucial, which is in fact the present value of future expected dividends

Example - *A stock just paid $10 dividend. dividend will grow 10% each year. The stock is expected to be sold at $150 at the end of year 2. k = 20%.*

$$P_0 = \frac{10(1+0.1)^1}{(1+0.2)^1} + \frac{10(1+0.1)^2}{(1+0.2)^2} + \frac{150}{(1+0.2)^2}$$

Equity Valuation:
Gordon Growth Model

- Given the constant growth rate, the earlier formula can be reduced to:

$$P_0 = \frac{D_1}{k-g}$$

$$\frac{P_0}{E_1} = \frac{D_1 / E_1}{k-g} = \frac{Payout}{k-g}$$

Assumptions of DDM:

- Dividends grow at a constant rate
- The constant growth rate will continue for an infinite period
- The required rate of return (k) is greater than the infinite growth rate (g)

Example: *A stock has a current dividend of \$5. Dividends will rise at 10% a year. The required rate of return on the stock is 15%.*

$$P_0 = \frac{D_0(1+g)}{k-g} = \frac{5(1+0.1)}{0.15-0.10} = \$110$$

Equity Valuation: Multistage Dividend Discount Model

- **Information:** A stock has just paid $10 dividend.
  - Dividends will grow at 30% for year 1 and year 2, 15% for years 3 to 5, and then 5% from year 6.
- **Solution:**
  - The present value of the 1st stage supernormal growth:

$$V_1 = + \frac{\$10(1+.3)}{(1+.1)^1} + \frac{\$10(1+.3)^2}{(1+.1)^2} = \$25.8$$

  - The present value of the 2nd stage supernormal growth:

$$V_2 = \frac{\$10(1+.3)^2(1+.15)}{(1+.1)^3} + \frac{\$10(1+.3)^2(1+.15)^2}{(1+.1)^4} + \frac{\$10(1+.3)^2(1+.15)^3}{(1+.1)^5} = \$45.9$$

Equity Valuation:
Multistage Dividend Discount Model

- The terminal value of constant growth at the end of the 5th year:

$$\text{Terminal Value} = \frac{\$10(1+.3)^2(1+.15)^3(1+0.05)}{10\% - 5\%} = \$539.8$$

- The present value of constant growth:

$$V = \frac{\$539.8}{(1+0.1)^5} = \$335.3$$

- The total value of the stock: \$25.8 + \$45.9 + \$335.3 = \$407

Equity Valuation: Multiplier Models

- **Price to earnings (P/E)**
  - Better use forward P/E (predict future earnings) than trailing P/E
  - can only be used to value stocks with positive EPS
  - EPS can be manipulated
- **Price to Book Value (P/BV)**
  - typically positive even if EPS is negative, more stable than P/E
  - P/BV can be used to value firms that are going out of business.
  - It is inappropriate to use the P/BV ratio to compare firms with different levels of fixed assets or firms with different accounting choices
- **Price to sales (P/S)**
  - typically positive even if EPS is negative
  - sales are not sensitive to choices of accounting alternatives
  - However, sales cannot reflect the firm's profitability
- **Price to cash flow (P/CF)**
  - More stable than P/E
  - cash flow is not easily manipulated by management

Equity Valuation:
Preferred Stock Valuation

- The value is simply the stated annual dividend divided by the required rate of return on preferred stock (k_p)

$$P_0 = \frac{\text{Dividend}}{k_p}$$

- Assume a preferred stock has a $100 par value and a dividend of $8 a year and a required rate of return of 9 percent

$$P_0 = \frac{\$8}{0.09} = \$88.89$$

Today's Outline

1. Equity Valuation
2. Bond Valuation
   - Price-Yield Relationship
   - Duration
   - Convexity

Bond Valuation:
The Basics

- Maturity: the length of time until the loan agreement (or the bond contract) expires, essentially the remaining life of the bond.

- Par value: the amount that the borrower promises to pay on or before the maturity date on the issue, also known as the redemption or face value.
  - If the market interest rates move above the coupon rate, the bond will sell below par value (at discount).
  - If the rates move below the coupon rate, the bond will sell above par value (at premium).

Bond Valuation:
The Basics

Interest Rates for Bonds

- The *coupon rate* of a bond is the promised interest rate that the security issuer agrees to pay at the time the security is issued.
  - *Example: A bond with a par value of $1000 and a coupon rate of 9% pays an annual coupon of $90.*

- The *current yield* of a bond is the ratio of the annual coupon interest generated by the asset to its market value.
  - *Example: The current yield of bond selling for 900 Baht in the market and paying an annual coupon of 30 Baht to the bondholder is 30/900 = 3.33%.*

Bond Valuation:
The Basics

Interest Rates for Bonds

- The *yield to maturity* (YTM) is the rate of interest that the market is prepared to pay today for a bond.
- It is the rate that equates the purchase price (P) with the present value of all the expected annual net cash flows (CF) from the bond.

$$P = \frac{CF_1}{(1+YTM)^1} + \frac{CF_2}{(1+YTM)^2} + \cdots + \frac{CF_n}{(1+YTM)^n}$$

Bond Valuation:
The Basics

Interest Rates for Bonds

- A bond trades at a *discount from par* if its price is less than its par value, i.e. if its current yield to maturity is higher than its coupon rate.
- A bond trades at a *premium over par* if its price is more than its par value, i.e. if its current yield to maturity is lower than its coupon rate.
- A bond trades *at par* if its price equals its par value, i.e. if the current market interest rate on comparable securities equals its coupon rate.

Bond Valuation:
The Basics

Interest Rates for Bonds

- 'YTM' or 'Yield' represents market interest rate for bonds.
- Bond Dealers usually quote *two* yields for a bond. *(Dealers in the US quote 'price' instead of 'yield'.)*
  - *The lower ask yield (higher price) is the dealer's selling price, while the higher bid yield (lower price) is the dealer's buying price.*
- The difference between the bid and ask prices – known as the *spread* – provides the dealer's return for creating a market for the security.

Bond Valuation:
The Basics

- All bonds are priced according to the present value of their future cash flows. There are 3 steps in bond valuation.

  (Throughout our course, bonds are assumed to be non-callable, which means the issuer has no right to redeem the bond before its maturity. Therefore, it is assumed that the amount and timing of a bond's future cash flow are known with certainty.)

1. Estimate the cash flows: coupon payments and repayment of principal at maturity
2. Determine the appropriate discount rate which is the bond's yield to maturity
3. Calculate the present value:

Bond Valuation:
Pure Discount Bonds

Pure Discount Bonds

- Also called Zero Coupon Bonds
- are bonds that do not pay interest and are sold at discount from their par values.
- They then increase in value over time, and appreciate to their par values at maturity.

Bond Valuation:
Pure Discount Bonds

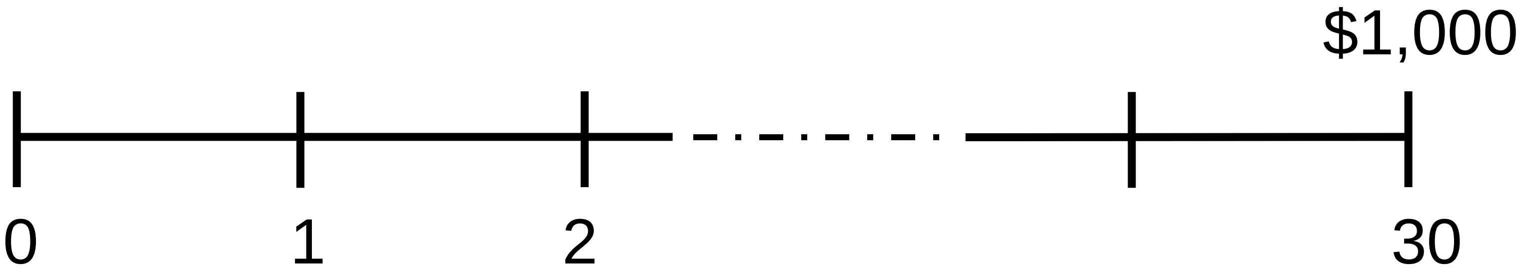
Present value of a pure discount bond at time 0:

$$PV = \frac{F}{(1+r)^T}$$

Bond Valuation:
Pure Discount Bonds

Pure Discount Bonds

Find the value of a 30-year zero-coupon bond with a $1,000 par value and a YTM of 6%.

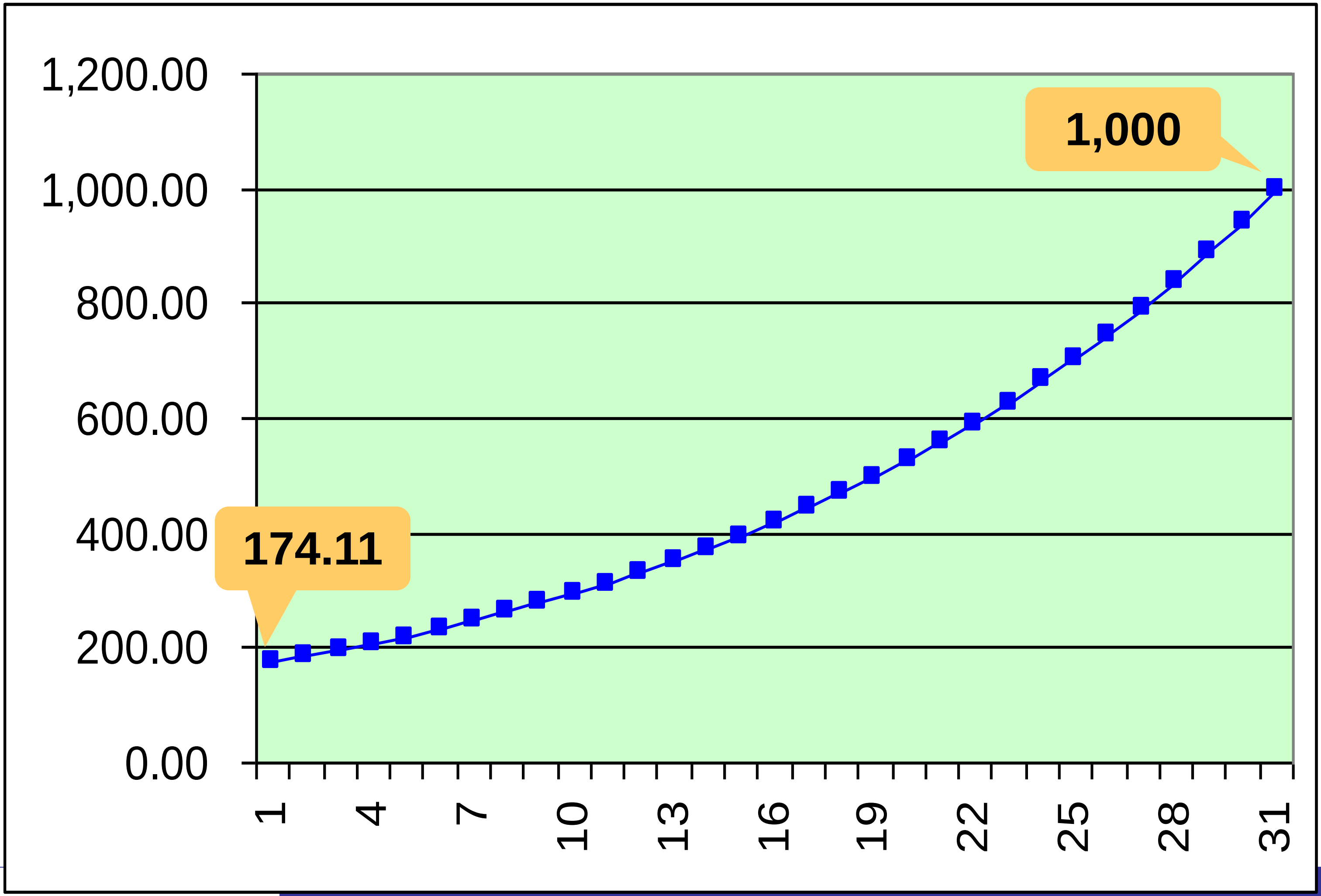


$$PV = \frac{F}{(1+r)^T} = \frac{\$1,000}{(1.06)^{30}} = \$174.11$$

INPUT	**30**	**6**		**0**	**1,000**
	N	**I/Y**	**PV**	**PMT**	**FV**
OUTPUT			**-174.11**		

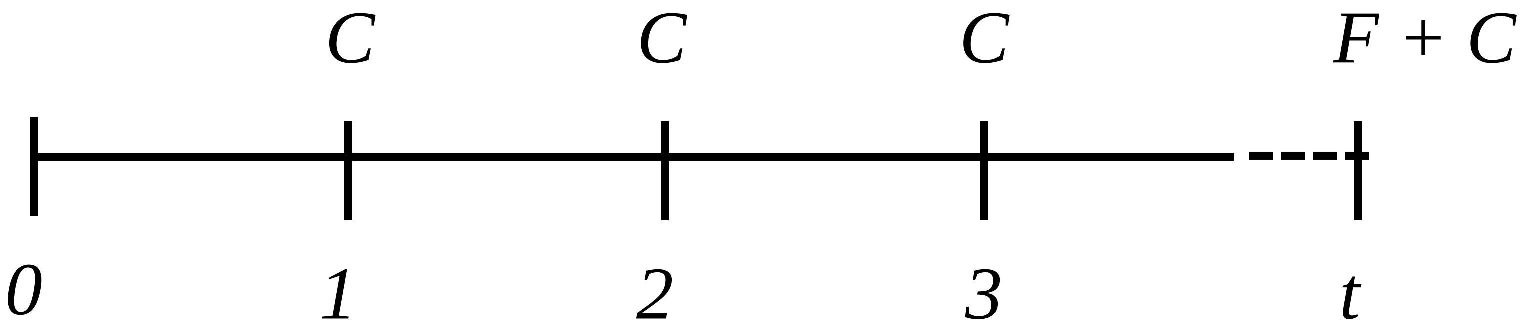
Bond Valuation: Pure Discount Bonds

Pure Discount Bonds: Value increases over time



Bond Valuation:
Coupon Bonds

Coupon Bonds



$$PV = PV \text{ of coupons} + PV \text{ of par}$$

$$PV = C * \left(1 - \frac{1}{(1+r)^t}\right)\left(\frac{1}{r}\right) + \frac{F}{(1+r)^t}$$

Where: F = par value

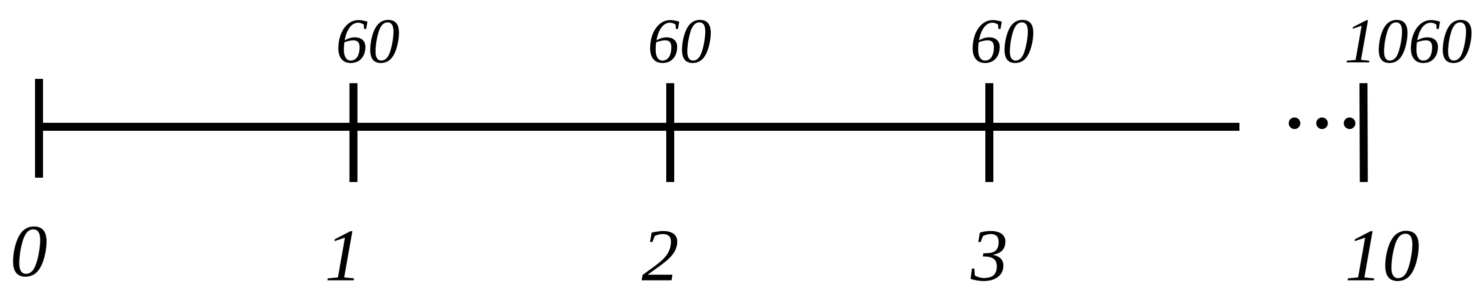
C = coupon payment for each period

r = the bond's Yield to Maturity

t = number of time periods to maturity

Bond Valuation:
Coupon Bonds

Suppose there is a 10-year, B1,000 par value, 6% annual-pay coupon bond. What is the bond value, given YTM =8%?

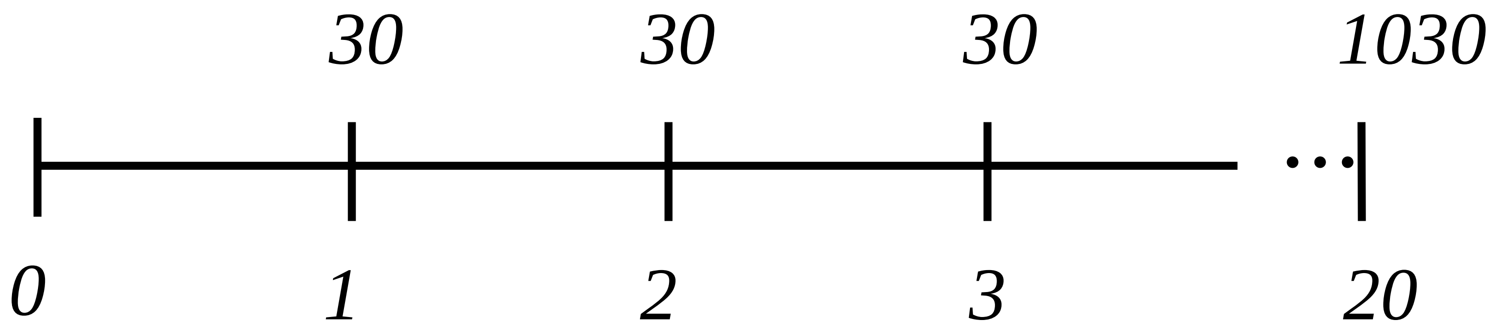


$$PV = 60 * \left(1 - \frac{1}{(1+0.08)^{10}}\right)\left(\frac{1}{0.08}\right) + \frac{1000}{(1+0.08)^{10}} = 865.80$$

INPUT	**10**	**8**		**60**	**1,000**
	N	**I/Y**	**PV**	**PMT**	**FV**
OUTPUT			**-865.80**		

Bond Valuation:
Coupon Bonds

Suppose there is a 10-year, B1,000 par value, 6% coupon bond. The coupons are paid <u>semiannually</u>. What is the bond value, given YTM =8%?



INPUT

N **I/Y** **PV** **PMT** **FV**

OUTPUT

Bond Valuation: Price-Yield Relationship

- Over the life of a bond, cash flows (coupons and face value) will not change while market yield (YTM) can change dramatically.

- Therefore, the market price of the bond must change to reflect any change in market yield.

- Basically, the price of a bond will move inversely with its market yield: bond prices go up when the yield goes down, and vice versa.

Bond Valuation:
Price-Yield Relationship

Suppose there is a 30-year bond issued 20 years ago and now has 10 years to maturity. The bond pays 10% coupon semiannually. Calculate the bond price today given YTM of 7%, 10% and 13%

YTM = 7%

INPUT

N	I/Y	PV	PMT	FV

OUTPUT

YTM = 10%

INPUT

N	I/Y	PV	PMT	FV

OUTPUT

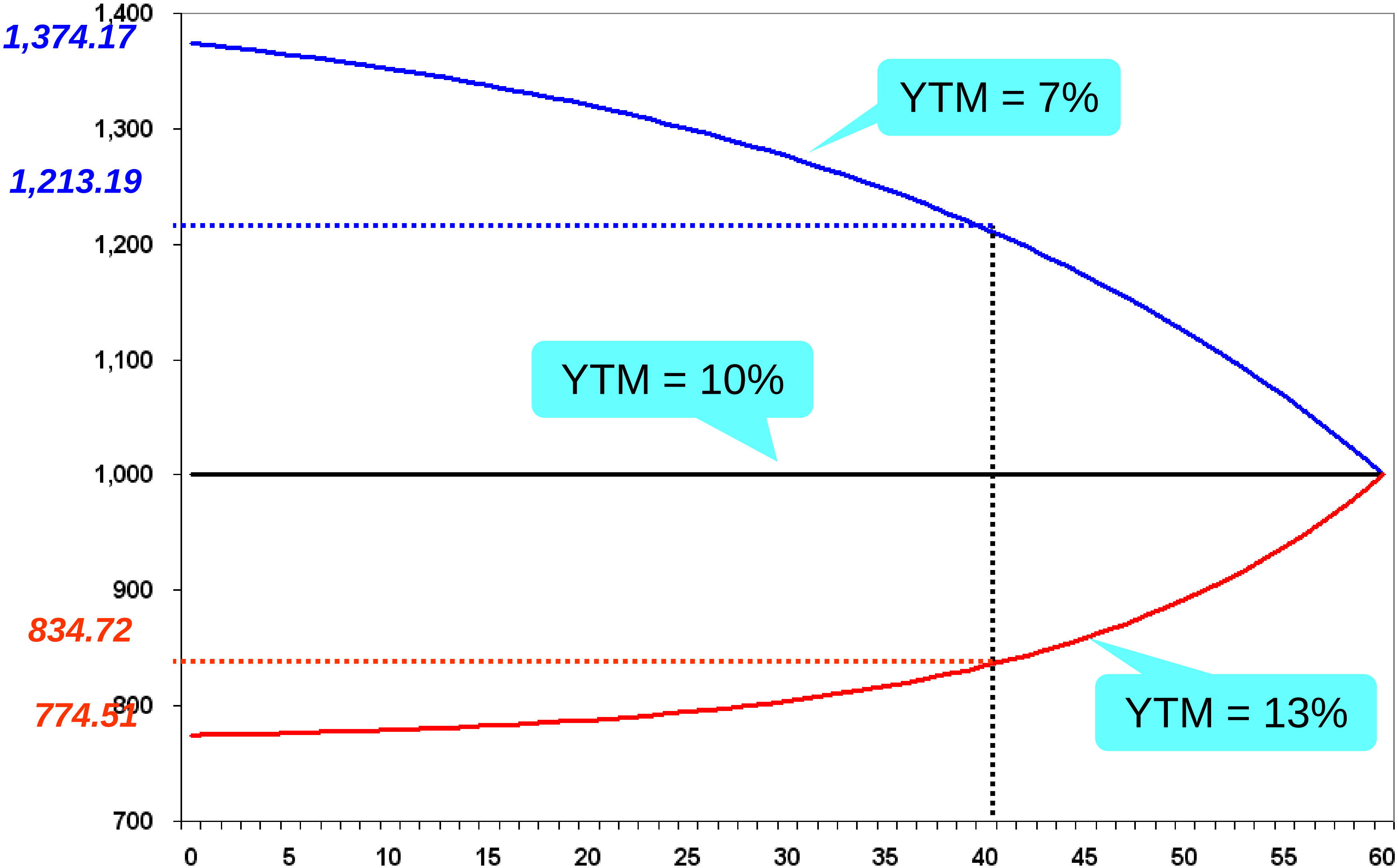
YTM = 13%

INPUT

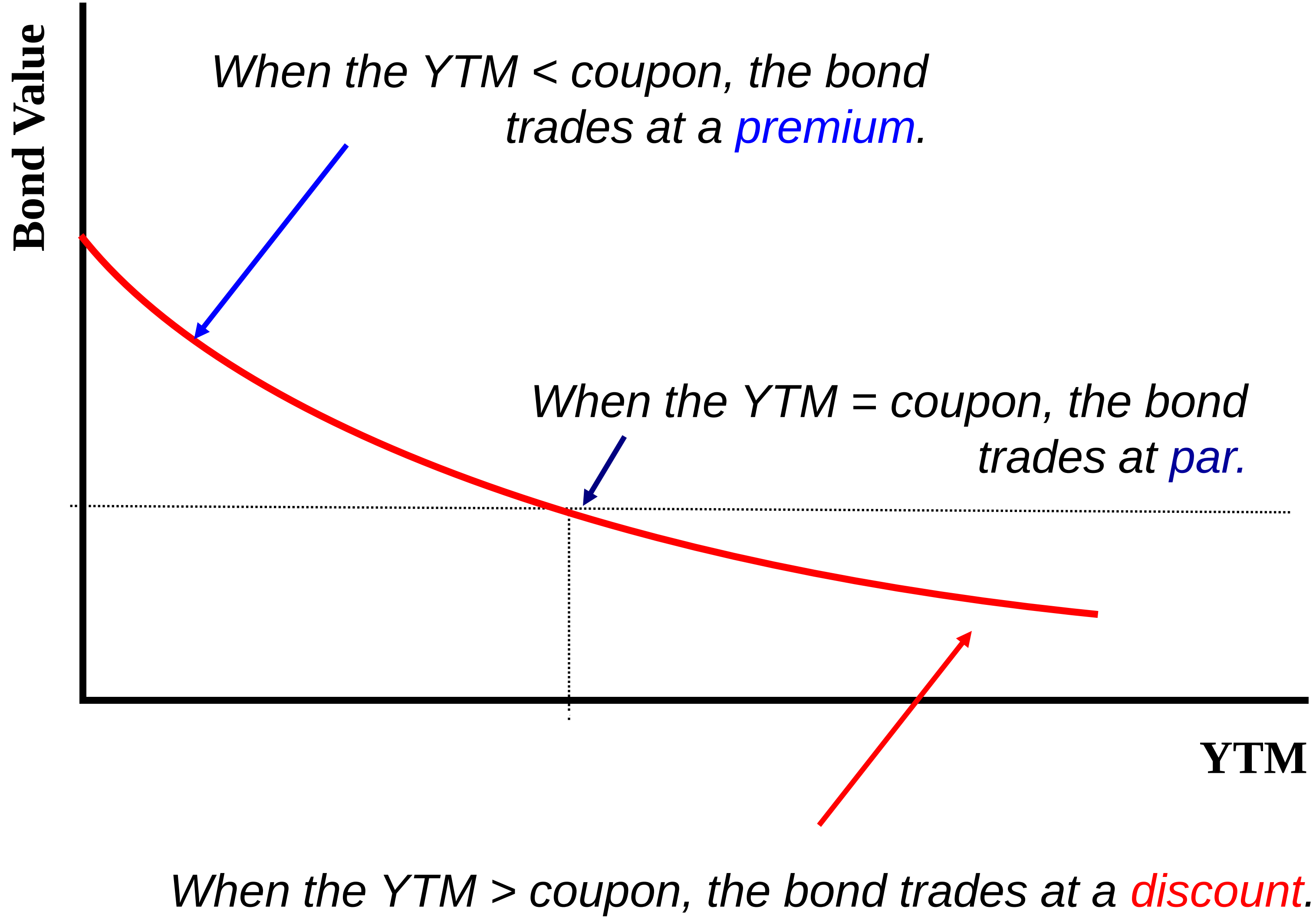
N	I/Y	PV	PMT	FV

OUTPUT

Bond Valuation:
Price-Yield Relationship



Bond Valuation:
Price-Yield Relationship



Bond Valuation: Maturity and Bond Price Volatility

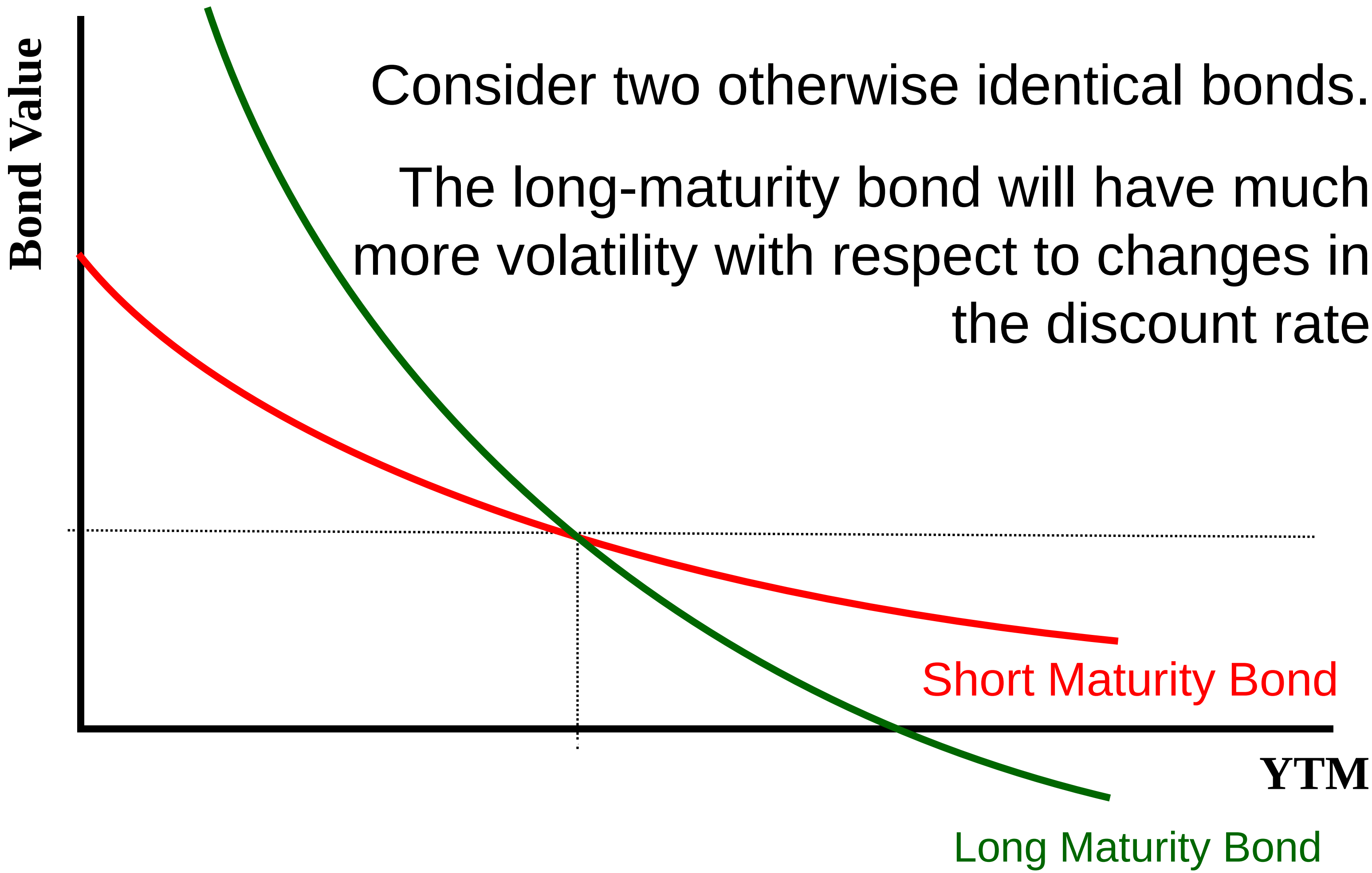
Compute the value of a B1,000 par value bond, with 3-year, 2-year and 1-year life, paying 6% semiannual coupons to an investor with a required rate of return of 5%, 6% and 7%.

Notice that a bond with longer maturity has higher relative (%) price change than one with shorter maturity when interest rate (YTM) changes. So, the prices of long-term bonds are <u>more volatile</u> than the prices of short-term bonds. All other features are identical.

Bond Valuation:
Maturity and Bond Price Volatility

Consider two otherwise identical bonds.

The long-maturity bond will have much more volatility with respect to changes in the discount rate



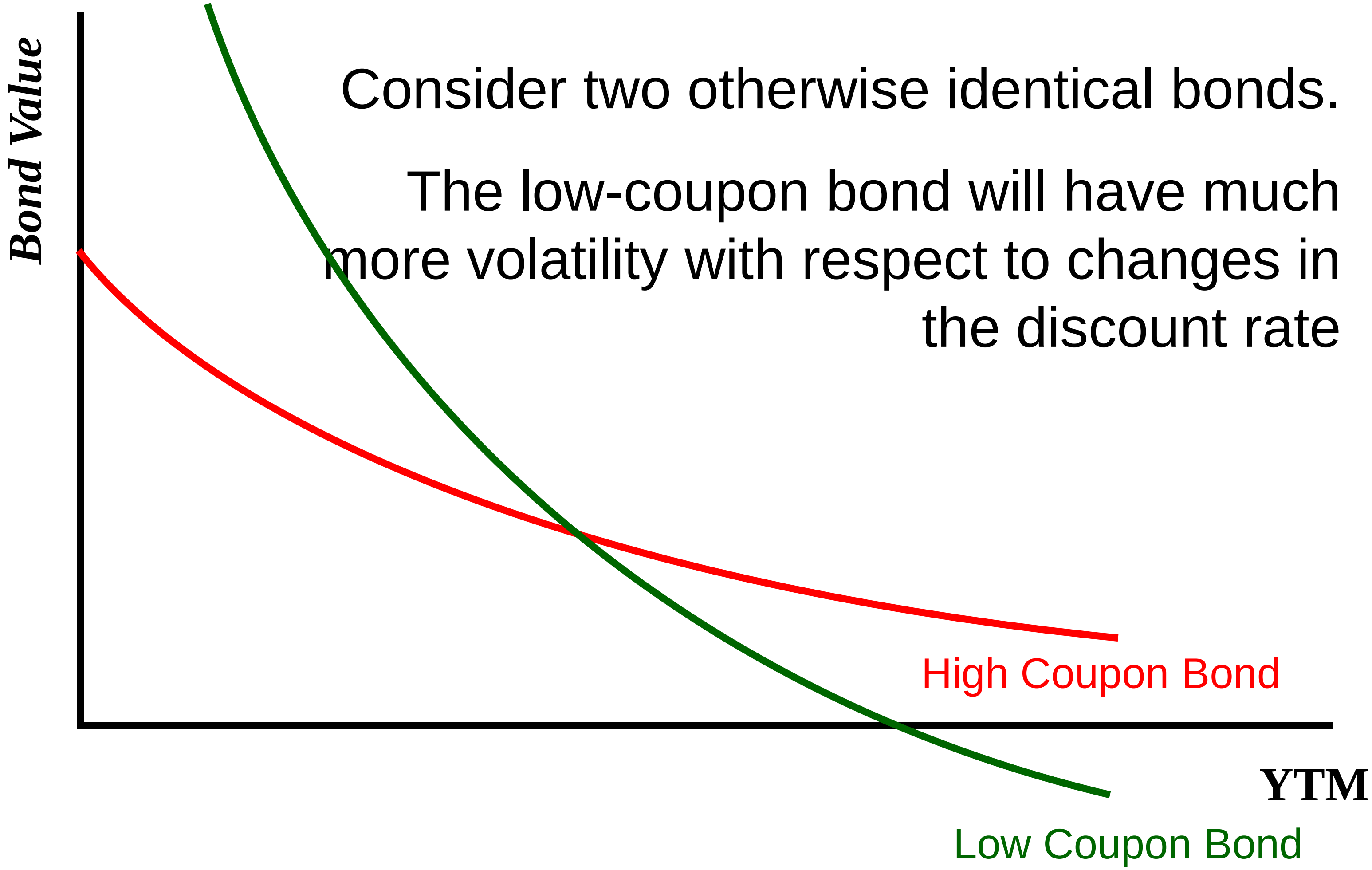
Bond Valuation:
Coupon Rate and Bond Price Volatility

Compute the value of a B1,000 par value bond, with a 3-year life, paying 8%, 6% and 4% semiannual coupons to an investor with a required rate of return of 5%, 6% and 7%.

Bond Valuation:
Coupon Rate and Bond Price Volatility

Notice that a lower coupon bond has a higher relative price change than a higher coupon bond when YTM changes. So, prices of bonds that pay low coupon are more volatile than those that pay high coupon. All other features are identical.

Bond Valuation: Coupon Rate and Bond Price Volatility



Consider two otherwise identical bonds.

The low-coupon bond will have much more volatility with respect to changes in the discount rate

Bond Valuation:
Macaulay Duration

- Macaulay Duration measures the average maturity of a bond's total cash flows – coupons as well as principal.
- The average is weighted by the present value dollars paid in each period.

$$\text{Macaulay Duration} = \frac{\sum_{t=1}^{n} \frac{C_t(t)}{(1+r)^t}}{\sum_{t=1}^{n} \frac{C_t}{(1+r)^t}}$$

- t = time in which the cash flow (coupon or par value) occurs
- C_t = cash flows (interest or principal payment) that occurs in period t
- r = the yield to maturity (YTM) on the bond

Bond Valuation: Macaulay Duration

$$\text{Macaulay Duration} = \frac{\displaystyle\sum_{t=1}^{n} \frac{C_t(t)}{(1+r)^t}}{\displaystyle\sum_{t=1}^{n} \frac{C_t}{(1+r)^t}}$$

- *The denominator is the PV of all cash flows, which is essentially the price of bond.*
- *The numerator is the time to maturity of each cash flow weighted according to the PV of the cash flow.*

Bond Valuation:
Macaulay Duration

1. Estimate the cash flows: coupon payments and repayment of principal at maturity
2. Calculate the present value of the cash flows series
3. Calculate the sum of the above PVs
4. Multiply each of PVs in #2 with time period
5. Calculate the sum of #4 PVs.
6. Duration is the result of #5 divided by that of #3

Notice: #2 and #3 - the denominator of the formula
#4 and #5 - the numerator of the formula

Bond Valuation: Macaulay Duration

Suppose there is a 5-year, B1,000 par value, 9% annual pay bond. What is the bond's duration, given YTM = 9%?

Period	Cash Flow	PV of CF	t x PV
1	90	82.57	82.57
2	90	75.75	151.50
3	90	69.50	208.49
4	90	63.76	255.03
5	1,090	708.43	3,542.13
		1,000.00	4,239.72

Duration **4.2397 years**

Notice that… Time to maturity = 5 years

Macaulay Duration = 4.2397 years

Bond Valuation:
Macaulay Duration

Suppose the 5-year bond above pays semi-annually. What is the bond's duration, given YTM = 9%?

Notice that…	Time to maturity	= 5 years
	Macaulay Duration	= …… years

Bond Valuation:
Macaulay Duration

Duration and coupon interest

- Macaulay duration of a bond with coupon payments always will be less than its term to maturity because duration gives weight to these interim interest payments.
- The higher the coupon payment, the lower the bond’s duration
- Duration of discount (or zero-coupon) bond is equal to its time to maturity.

Duration and yield to maturity

- the higher the yield to maturity, the lower the bond’s duration

Bond Valuation: Macaulay Duration

Duration and maturity

- duration increases with maturity at a decreasing rate
- Given the same maturity, a callable bond has <u>shorter</u> duration than a non-callable bond because there is a chance that the issuer exercise the right to call the bond (repay the loan) early.

Bond Valuation: Modified Duration

- For option-free bond, modified duration can provide an approximation to the interest rate sensitivity of a bond.
- Modified Duration equals Macaulay Duration divided by 1 plus the current yield to maturity divided by the number of payments in a year.

$$\text{Modified Duration} = \frac{\text{Macaulay Duration}}{\left(1 + \frac{YTM}{m}\right)}$$

- From previous example, a 5-year, 9% semiannual-pay coupon bond with a Macaulay duration of 4.1344 years, YTM of 9% …

$$\text{Modified Duration} = \frac{4.1344}{\left(1 + \frac{0.09}{2}\right)} = 3.96$$

Bond Valuation: Modified Duration

Modified duration can be used to see how the price of a bond will respond to a specific change in yield.

Approximate change in price due to duration

= (-) (Modified duration) (Change in yield in decimal term)

Bond Valuation: Modified Duration

Suppose there is a 5-year, B1,000 par value, 9% semiannual pay bond. The bond's YTM = 9% and we have calculated that modified duration is 3.96.

If YTM falls by 1% (or 100 basis points) from 9% to 8%, the approximate change in price of the 5-year bond is

= (-)(3.96)(-0.01)

= 0.0396

Current bond price, not par value!!!

Estimated price

= (1+0.0396)(1,000)

= 1,039.60

Actual Price (can be calculated using financial calculator)

= 1,040.55

Bond Valuation: Modified Duration

Suppose there is a 5-year, B1,000 par value, 9% semiannual pay bond. The bond's YTM = 9% and we have calculated that modified duration is 3.96.

If YTM rises by 1% (or 100 basis points) from 9% to 10%, the approximate change in price of the 5-year bond is

Bond Valuation: Effective Duration

- Modified Duration has one limitation. It assumes that, when interest rates change, cash flows of bond do not change.
- This is a problem when measuring the exposure of bonds with embedded options (e.g., callable bonds, mortgage-back securities, etc.) to changes in interest rates.
- Effective Duration, in contrast, takes changes in cash flows into account and thus, is the appropriate measure for bonds with embedded options.
- For bonds without embedded options, Modified Duration and Effective Duration produce the same duration value so both can be used.

Bond Valuation: Effective Duration

$$\text{Effective Duration} = \frac{BV_{-} - BV_{+}}{2(BV_0)(\Delta y)}$$

BV_{-} is bond value when yield falls

BV_{+} is bond value when yield rises

BV_0 is bond value when yield stays the same

Δy is change in yield in decimal term

$\Delta y = 0.01$ for 1% change in yield

$\Delta y = 0.005$ for 0.5% change in yield

Notice that, since BV_{-} is always greater than BV_{+} for option-free bonds, you will always get duration in positive term.

Bond Valuation:
Effective Duration

Suppose there is a 5-year, B1,000 par value, 9% semiannual pay bond. The bond's YTM = 9%. Determine effective duration given 1% change in yield.

$$\text{Effective Duration} = \frac{1040.55 - 961.39}{2(1{,}000)(0.01)}$$

$$= \frac{79.1587}{20}$$

$$= 3.96$$

Notice that Modified Duration and Effective Duration produce the same duration value of 3.96. Since our example is a bond without embedded option, both duration measures will produce the same duration number and both can be used to estimate bond price.

Bond Valuation: Duration of a Portfolio

Duration of a portfolio is simply the weighted average of the duration of the bonds in the portfolio. The average is weighted by market value of bonds within the portfolio.

For example, if a portfolio worth 5 million Baht has 2 bonds

– The previous 5-year bonds, B1,000 par value, 9% semiannual pay bond. The bond's YTM = 9%. It's modified duration is 3.96 with total market value of 2,000,000 Baht

– 15-year, B1,000 par value, 7% annual pay bonds, YTM = 7% with modified duration of 9.11 and total market value of 3,000,000

$$\text{Portfolio Duration} = \frac{3.96(2,000,000) + 9.11(3,000,000)}{5,000,000}$$

$$= 7.05$$

Bond Valuation: Duration of a Portfolio

Given portfolio's modified duration of 7.05, If YTM rises by 1% across the board, the approximate change in price of the portfolio is

= (-)(7.05)(+0.01)

= -0.0705

Estimated portfolio value after YTM rises by 1%

= (1-0.0705)(5,000,000)

= 4,647,500

Bond Valuation:
Duration of a Portfolio

Given portfolio's modified duration of 7.05, If YTM falls by 1% across the board, the approximate change in price of the portfolio is

Estimated portfolio value after YTM rises by 1%

Bond Valuation:
PV01

PV01 = Price Value of One Basis Point Change. It is the change in value (in Baht or Dollar terms) per one million nominal value, given 1 basis point change in yield

Example: Suppose our previous 5-year bond has 9% semiannual coupon, YTM of 9%, market price of 1,000, modified duration of 3.96.

The estimated bond price change given 1 bpΔ in yield
= (MD) x (Change in Yield Decimal Term) x (Bond Price)
= (3.96) x (0.0001) x (1,000)
= 0.396 Baht.

So, bond price will change by 0.396 Baht given 1bpΔ in yield.

Bond Valuation:
PV01

Notice that the price change of 0.396 Baht above is <u>per one unit of bond</u>.

Since PV01 is defined as ‘change in value per one million Baht nominal value and since Thai Baht Bond par value is B1,000; one million Baht nominal value translate to 1,000 units of bonds. Therefore, we need to multiply 1,000 to the above calculation to achieve PV01 formula…

$$PV01 = MD \times 0.0001 \times \text{Bond Price} \times 1{,}000$$

Bond Valuation:
PV01

Example: Suppose our previous 5-year bond has 9% semiannual coupon, YTM of 9%, market price of 1,000, modified duration of 3.96. Calculate PV01

PV01 = MD x 0.0001 x Bond Price x 1,000
= 3.96 x 0.0001 x 1,000 x 1,000
= 396 Baht

- *So, we can predict that, if yield change by 1 basis point, the value of this bond, with 1 million Baht nominal, will change by 396 Baht.*
- *You can check this prediction by calculating that, if yield change by 1 basis point from 9% to 9.01%, the actual value will change by 395.54 Baht*

Bond Valuation:
PV01 : Exercise

A 5-year bond has 5% annual coupon, YTM of 4%, market price of 1,044.52, modified duration of 4.38. Calculate PV01

Bond Valuation: PV01

- Position PV01 = PV01 x nominal value in million Baht
- Position PV01 of a portfolio is the sum of total Position PV01 of the bonds in the portfolio.
- For example, if a portfolio with 50 million nominal has 2 bonds

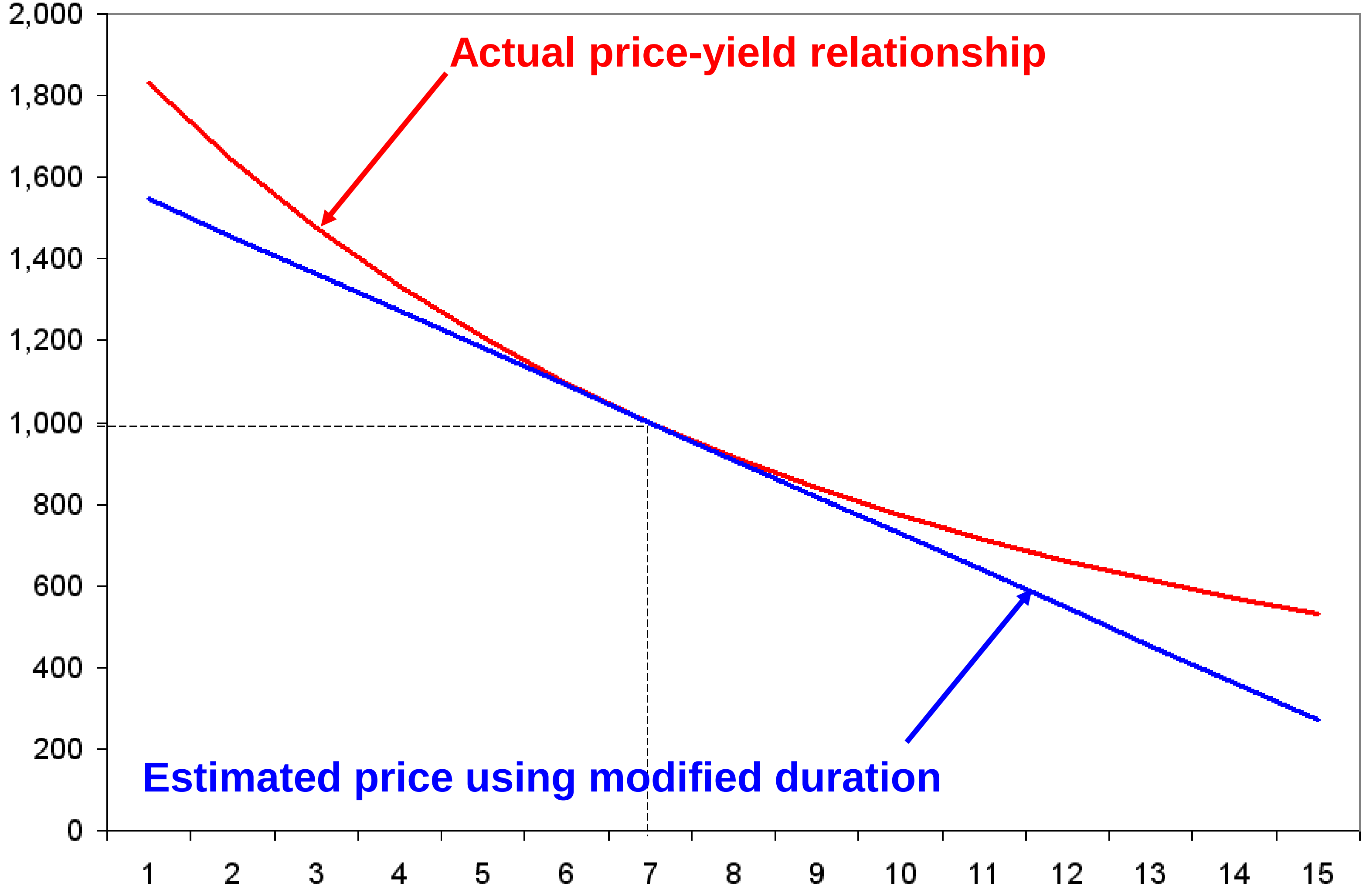
Issue	Bond Price	Modified Duration	PV01	Nominal (million Baht)	Position PV01
5 years	1,000.00	3.96	396.00	15.00	5,940
15 years	1,000.00	9.11	911.20	35.00	31,892
		Position PV01 of Portfolio =			37,832

This means, given 1 basis point change in yield, the value of this portfolio will change by 37,832 baht

Bond Valuation:
Convexity

- Modified duration allows us to estimate bond price changes for a change in interest rates.
- However, estimated price using duration is accurate only for very small changes in market yields.
- This is because modified duration is a linear approximation of a bond price.
- However, the actual price-yield relationship is a convex function.

Bond Valuation: Convexity



Bond Valuation: Convexity

- Convexity is a measure of how much a bond's price-yield curve deviates from the linear approximation of that curve.
- Convexity is always a positive number, implying that the price-yield curve lies above the modified-duration line.
- Mathematically, convexity is the second derivative of price with respect to yield. (Duration is the first derivative)

- *Modified duration is the percentage change in price for a given change in yield.*
- *Convexity is the percentage change in duration for a given change in yield.*

Bond Valuation:
Convexity

- As yield increases, the rate at which the price of the bond declines becomes slower.
- When yield declines, the rate at which the price of the bond increases becomes faster.
- The estimate using only modified duration will underestimate the actual price increase caused by yield decline and overestimate the actual price decline caused by an increase in yields.

Bond Valuation:
Convexity

$$\text{Convexity} = \frac{BV_{-} + BV_{+} - 2(BV_0)}{2(BV_0)(\Delta y)^2}$$

BV_{-} is bond value when yield falls

BV_{+} is bond value when yield rises

BV_0 is bond value when yield stays the same

Δy is change in yield in decimal term

$\Delta y = 0.01$ for 1% change in yield

$\Delta y = 0.005$ for 0.5% change in yield

Bond Valuation:
Convexity

Continued with the previous 5-year, B1,000 par value, 9% semiannual pay bond. The bond's YTM = 9%. We have calculated that modified duration = 3.96. Determine the bond's convexity given 1% change in yield.

$$\text{Convexity} = \frac{BV_{-} + BV_{+} - 2(BV_0)}{2(BV_0)(\Delta y)^2}$$

$$= \frac{1{,}040.55 + 961.39 - 2(1{,}000)}{2(1{,}000)(0.01)^2}$$

$$= \frac{1.94}{0.2}$$

$$= 9.7$$

Bond Valuation: Convexity

Convexity can be used to improve the estimation by modified duration to see how the price of a bond will respond to a specific change in yield.

Approximate change in price using convexity

$= (\text{convexity}) \times (\text{change in yield in decimal term})^2$

$= (9.7)(0.01)^2$

$= 0.00097$

Bond Valuation: Using Modified Duration Only

Suppose there is a 5-year, B1,000 par value, 9% semiannual pay bond. The bond's YTM = 9% and we have calculated that modified duration is 3.96.

If YTM <u>falls</u> by 1% (or 100 basis points) from 9% to 8%, the approximate change in price of the 5-year bond is

= (-)(3.96)(-0.01)

= 0.0396

Estimated price

= (1+0.0396)(1,000)

= 1,039.60

Current bond price, not par value!!!

Actual Price (can be calculated using financial calculator)

= 1,040.55

Bond Valuation:
Using Both Modified Duration + Convexity

Suppose there is a 5-year, B1,000 par value, 9% semiannual pay bond. The bond's YTM = 9%. We have calculated that modified duration is 3.96 and convexity is 9.7

If YTM <u>falls</u> by 1%, the approximate change in price is

$$= (-)(3.96)(-0.01) + (9.7)(-0.01)^2$$

$$= 0.0396+0.00097$$

$$= 0.04057$$

Estimated price

$$= (1+0.04057)(1{,}000)$$

$$= 1{,}040.57$$

Actual Price (can be calculated using financial calculator)

$$= 1{,}040.55$$

Bond Valuation:
Using Both Modified Duration + Convexity

Suppose there is a 5-year, B1,000 par value, 9% semiannual pay bond. The bond's YTM = 9%. We have calculated that modified duration is 3.96 and convexity is 9.7

If YTM rises by 1%, the approximate change in price is