



Lecture 4



The Risk and Term Structure of Interest Rates

Roadmap

- ▶ Previous lecture: study determinant and behavior of just one interest rates
- ▶ This lecture: examine relationship of various interest rates to one another
 - : Understand why interest rates differ from bond to bond
 - : Understand source and cause of fluctuation in interest rate relative to one another



2 terminology to explore

- ▶ **1. Risk structure of interest rates:**

- ▶ Explain why bonds with same term to maturity have different interest rates.

(i.e. due to different risk, liquidity, income tax rule)

- ▶ **2. Term structure of interest rates:**

- ▶ Explain why bonds with same risk, liquidity, tax characteristics have different interest rates

(i.e. due to different time remaining to maturity)

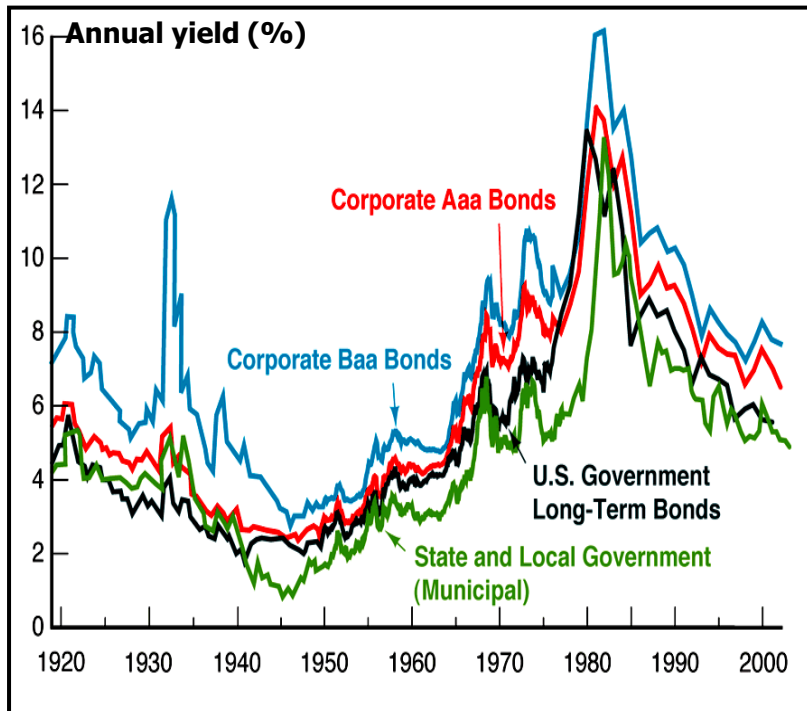


I.

Risk structure of interest rates



Risk Structure of Long-Term Bonds in the United States



2 important features of interest rate behavior for bonds of same maturity

1. Different bond category pay different rates

Due to default risk and risk premium:

- Interest rate on corporate bond always higher than US Treasury-Bond
- Interest rate on Baa bond is greater than Aaa bond

2. Spread between interest rates varies over time

- Spread between interest rate on Baa bond and US government bond is very large during great depression (1930-33) due to \uparrow default risk and smaller during 40s-60s and widen again
- Interest rate on municipal bond is greater than US government bond in late 1930s but lower afterwards

3 Factors influence interest rate on bond

▶ **1. Default risk**

- Occurs when issuer of bond is unable/unwilling to make promised interest payment/pay off face value when bond matures
- Default free bond = US treasury (no default risk)
- Risk premium = spread between interest on bond with default risk and default free bond, both are at same maturity.
- Bond with default risk has positive risk premium
- Bond with larger default risk has larger risk premium

▶ 2. Liquidity

▶ 3. Income tax consideration



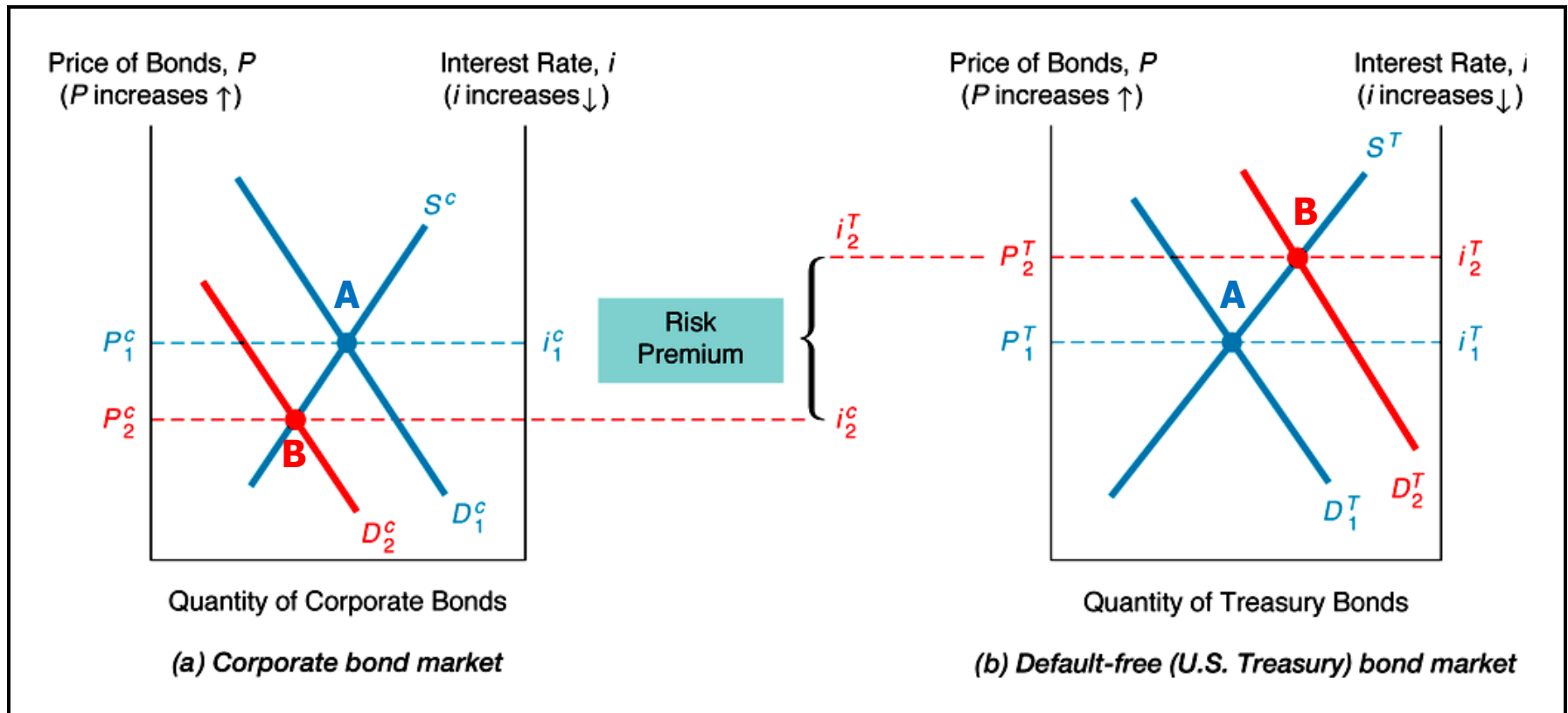
Increase in Default Risk on Corporate Bonds

Initial equilibrium at A:

Assume identical risk, maturity, zero risk premium on both

Possibility of default \uparrow equilibrium move to B:

default risk of corp. bond \uparrow , expected relative return \downarrow , uncertainty or return \uparrow



Bond Ratings

Table 1 Bond Ratings by Moody's and Standard and Poor's

Rating		Descriptions	Examples of Corporations with Bonds Outstanding in 2003
Moody's	Standard and Poor's		
Aaa	AAA	Highest quality (lowest default risk)	General Electric, Pfizer Inc., North Carolina State, Mobil Oil
Aa	AA	High quality	Wal-Mart, McDonald's, Credit Suisse First Boston
A	A	Upper medium grade	Hewlett-Packard, Anheuser-Busch, Ford, Household Finance
Baa	BBB	Medium grade	Motorola, Albertson's, Pennzoil, Weyerhaeuser Co., Tommy Hilfiger
Ba	BB	Lower medium grade	Royal Caribbean, Levi Strauss
B	B	Speculative	Rite Aid, Northwest Airlines Inc., Six Flags
Caa	CCC, CC	Poor (high default risk)	Revlon, United Airlines
Ca	C	Highly speculative	US Airways, Polaroid
C	D	Lowest grade	Enron, Oakwood Homes

3 Factors influence interest rate on bond

▶ 1. Default risk

▶ **2. Liquidity**

- Liquid asset means it can be quickly and cheaply converted into cash if need arises.
 - The more liquid asset is, the more desirable it is
 - US T-bond is the most liquid on all long term bonds (widely traded, easiest to sell quickly, low cost of selling)
 - Corporate bond are not as liquid (fewer bond are traded, costly to sell in emergency, hard to find buyer quickly)
- ▶ 3. Income tax consideration
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Corporate Bonds Become Less Liquid

Corporate Bond Market

1. Less liquid corporate bonds $D^c \downarrow$, D^c shifts left
2. $P^c \downarrow$, $i^c \uparrow$

Treasury Bond Market

1. Relatively more liquid Treasury bonds, $D^T \uparrow$, D^T shifts right
2. $P^T \uparrow$, $i^T \downarrow$

Outcome:

Risk premium, $i^c - i^T$, rises

Risk premium reflects not only corporate bonds' default risk, but also lower liquidity

Hence risk premium consist of both default risk and liquidity risk

3 Factors influence interest rate on bond

▶ 1. Default risk

▶ 2. Liquidity

▶ **3. Income tax consideration**

- Municipal bonds (issue by state & local government) are not default free
- They are not as liquid as US T-bond

US T-bond (income tax 35%)

Face value = \$1000

Coupon payment \$100 ($i=10\%$)

Income after tax = \$65

Actual earning = 6.5%

Municipal bond (tax free)

Face value = \$1000

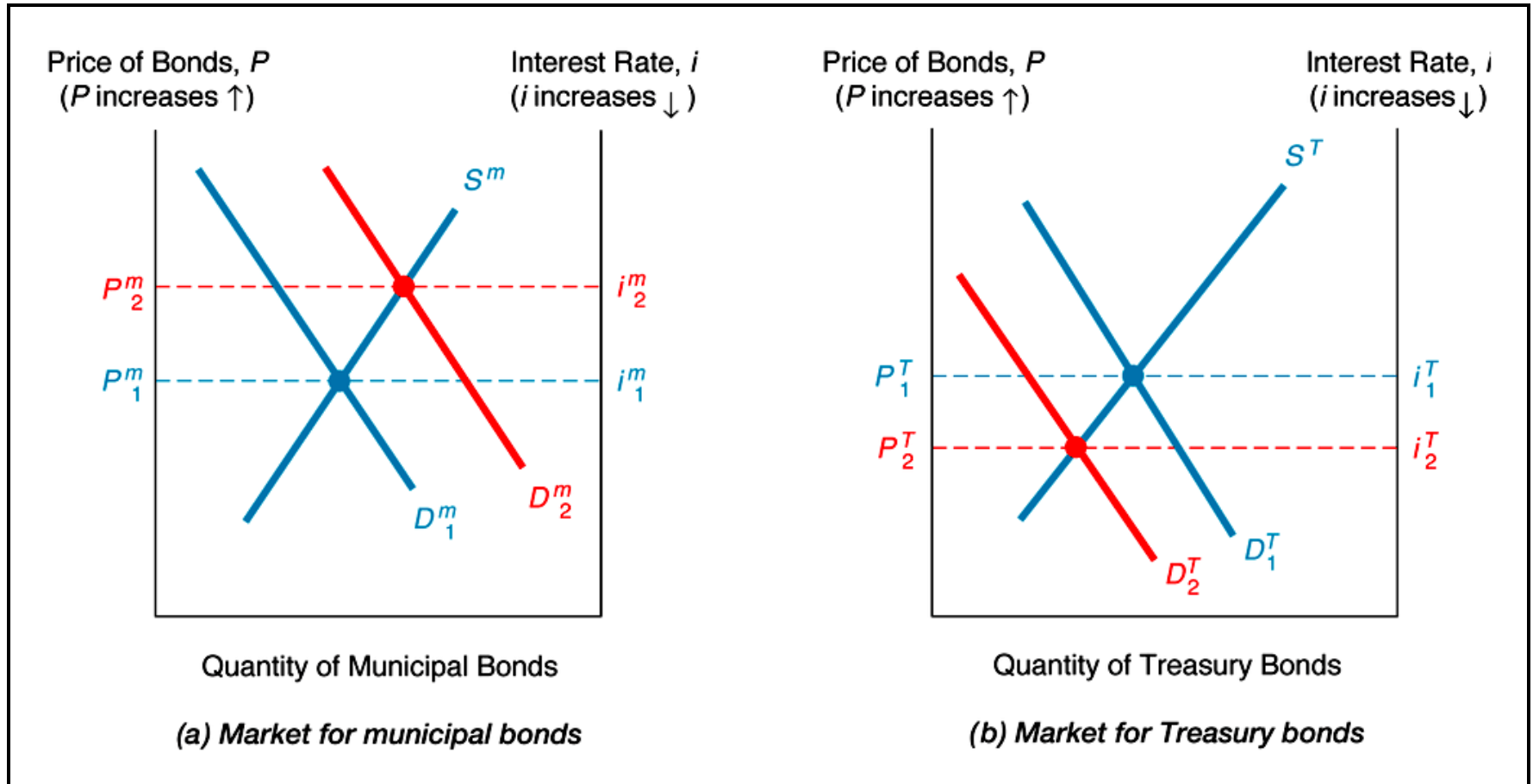
Coupon payment \$80 ($i=8\%$)

Income after tax = \$80

Actual earning = 8%

This explain why people willing to hold riskier and less liquid asset like municipal bond, even though it has lower interest rate than US T-bond

Tax Advantages of Municipal Bonds



Analysis of Figure 3: Tax Advantages of Municipal Bonds

Municipal Bond Market

1. Tax exemption raises relative RET^e on municipal bonds, $D^m \uparrow$, D^m shifts right
2. $P^m \uparrow$, $i^m \downarrow$

Treasury Bond Market

1. Relative RET^e on Treasury bonds \downarrow , $D^T \downarrow$, D^T shifts left
2. $P^T \downarrow$, $i^T \uparrow$

Outcome:

$$i^m < i^T$$



II.

Term structure of interest rates



Term Structure Facts to be Explained

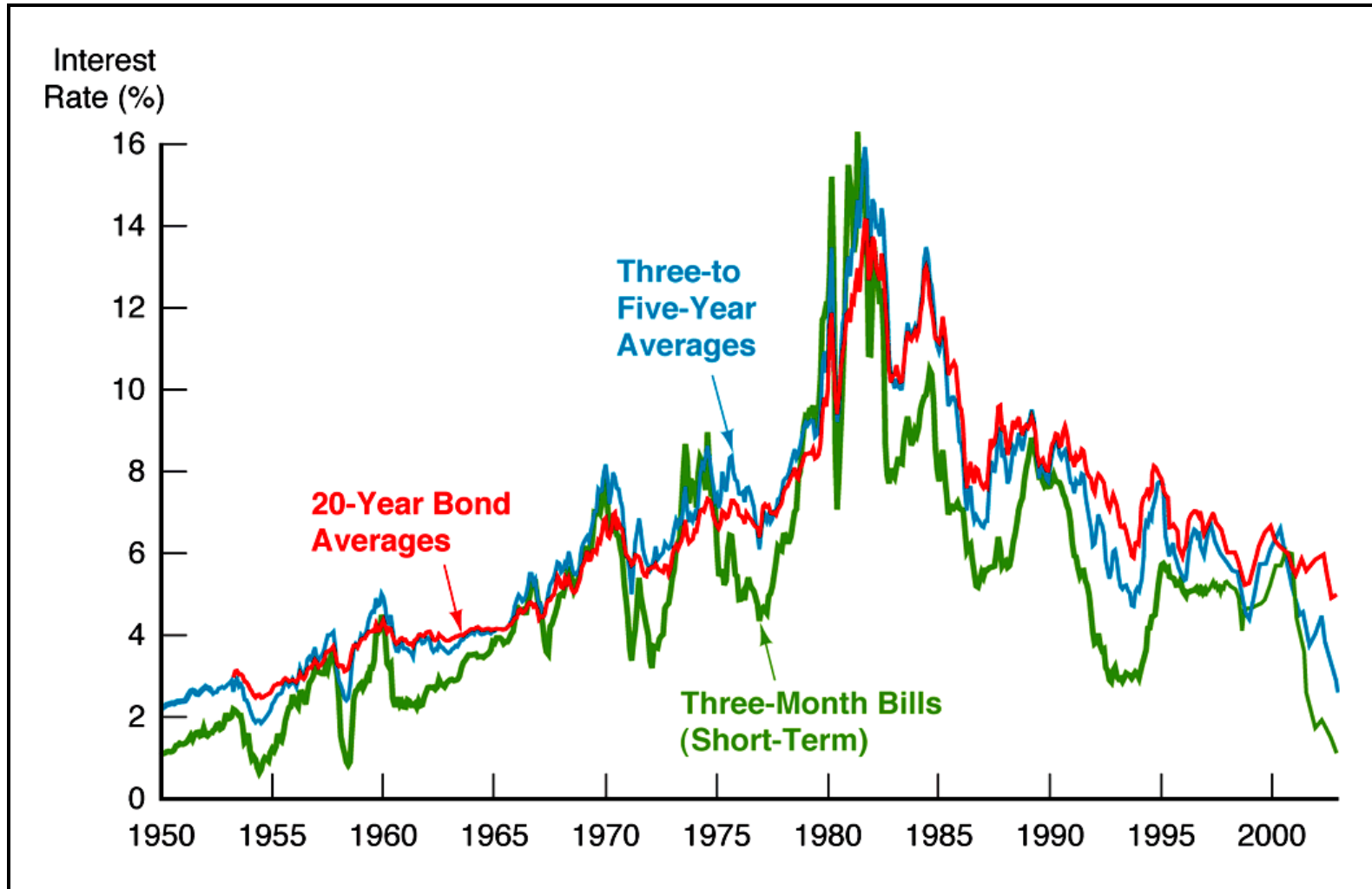
1. Interest rates for different maturities move together over time
2. Yield curves tend to have steep upward slope when short rates are low and downward slope when short rates are high
3. Yield curve is typically upward sloping

Three Theories of Term Structure

1. Expectations Theory
 2. Segmented Markets Theory
 3. Liquidity Premium (Preferred Habitat) Theory
 - A. Expectations Theory explains 1 and 2, but not 3
 - B. Segmented Markets explains 3, but not 1 and 2
 - C. Solution: Combine features of both Expectations Theory and Segmented Markets Theory to get Liquidity Premium (Preferred Habitat) Theory and explain all facts
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Interest Rates on Different Maturity Bonds Move Together



Yield Curves

Yield curve is a plot of yield with different term to maturity but same risk & liquidity & tax consideration
It describe term structure of interest rate for particular bond

Following the Financial News

Yield Curves

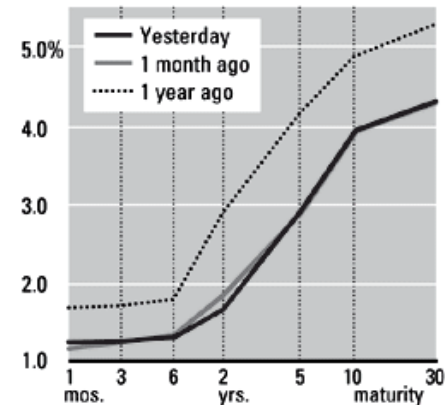
The *Wall Street Journal* publishes a daily plot of the yield curves for Treasury securities, an example of which is presented here. It is typically found on page 2 of the "Money and Investing" section.

The numbers on the vertical axis indicate the interest rate for the Treasury security, with the maturity given by the numbers on the horizontal axis. For example, the yield curve marked "Yesterday" indicates that the interest rate on the three-month Treasury bill yesterday was 1.25%, while the one-year bill had an interest rate of 1.35% and the ten-year bond had an interest rate of 4.0%. As you can see, the yield curves in the plot have the typical upward slope.

Source: *Wall Street Journal*, Wednesday, January 22, 2003, p. C2.

Treasury Yield Curve

Yield to maturity of current bills, notes and bonds.



Source: Reuters

Expectations Hypothesis

Key Assumption: Bonds of different maturities are perfect substitutes

Implication: RET^e on bonds of different maturities are equal

Investment strategies for two-period horizon

1. Buy \$1 of one-year bond and when it matures buy another one-year bond
2. Buy \$1 of two-year bond and hold it

Expected return from strategy 2

$$\frac{(1 + i_{2t})(1 + i_{2t}) - 1}{1} = \frac{1 + 2(i_{2t}) + (i_{2t})^2 - 1}{1}$$

Since $(i_{2t})^2$ is extremely small, expected return is approximately $2(i_{2t})$



Expected Return from Strategy 1

$$\frac{(1 + i_t)(1 + i_{t+1}^e) - 1}{1} = \frac{1 + i_t + i_{t+1}^e + i_t(i_{t+1}^e) - 1}{1}$$

Since $i_t(i_{t+1}^e)$ is also extremely small, expected return is approximately

$$i_t + i_{t+1}^e$$

From implication above expected returns of two strategies are equal: Therefore

$$2(i_{2t}) = i_t + i_{t+1}^e$$

Solving for i_{2t}

$$i_{2t} = \frac{i_t + i_{t+1}^e}{2}$$



Expected Return from Strategy 1

More generally for n -period bond:

$$i_{nt} = \frac{i_t + i_{t+1}^e + i_{t+2}^e + \dots + i_{t+(n-1)}^e}{n}$$

In words: Interest rate on long bond = average short rates expected to occur over life of long bond

Numerical example:

One-year interest rate over the next five years 5%, 6%, 7%, 8% and 9%:

Interest rate on two-year bond:


$$(5\% + 6\%) / 2 = 5.5\%$$

Interest rate for five-year bond:

$$(5\% + 6\% + 7\% + 8\% + 9\%) / 5 = 7\%$$

Interest rate for one to five year bonds:

5%, 5.5%, 6%, 6.5% and 7%.



Expectations Hypothesis and Term Structure Facts

Explains why yield curve has different slopes:

1. When short rates expected to rise in future, average of future short rates = i_{nt} is above today's short rate: therefore yield curve is upward sloping
2. When short rates expected to stay same in future, average of future short rates are same as today's, and yield curve is flat
3. Only when short rates expected to fall will yield curve be downward sloping

Expectations Hypothesis explains Fact 1 that short and long rates move together

1. Short rate rises are persistent
2. If $i_t \uparrow$ today, i_{t+1}^e, i_{t+2}^e etc. $\uparrow \Rightarrow$ average of future rates $\uparrow \Rightarrow i_{nt} \uparrow$
3. Therefore: $i_t \uparrow \Rightarrow i_{nt} \uparrow$, i.e., short and long rates move together



Explains Fact 2 that yield curves tend to have steep slope when short rates are low and downward slope when short rates are high

1. When short rates are low, they are expected to rise to normal level, and long rate = average of future short rates will be well above today's short rate: yield curve will have steep upward slope
2. When short rates are high, they will be expected to fall in future, and long rate will be below current short rate: yield curve will have downward slope

Doesn't explain Fact 3 that yield curve usually has upward slope

Short rates as likely to fall in future as rise, so average of future short rates will not usually be higher than current short rate: therefore, yield curve will not usually slope upward



Segmented Markets Theory

Key Assumption: Bonds of different maturities are not substitutes at all

Implication: Markets are completely segmented: interest rate at each maturity determined separately

Explains Fact 3 that yield curve is usually upward sloping

People typically prefer short holding periods and thus have higher demand for short-term bonds, which have higher price and lower interest rates than long bonds

Does not explain Fact 1 or Fact 2 because assumes long and short rates determined independently



Liquidity Premium (Preferred Habitat) Theories

Key Assumption: Bonds of different maturities are substitutes, but are not perfect substitutes

Implication: Modifies Expectations Theory with features of Segmented Markets Theory

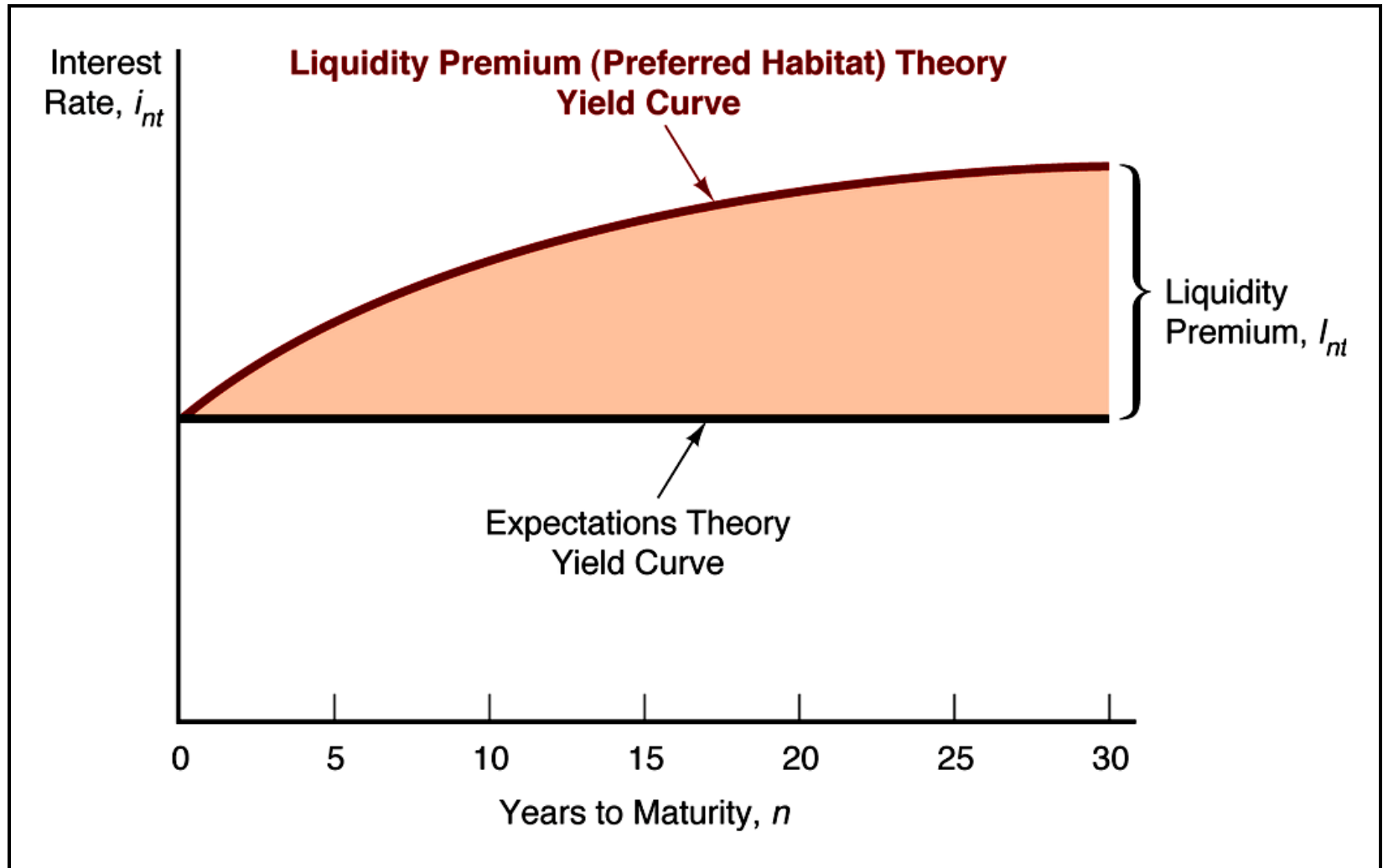
Investors prefer short rather than long bonds \Rightarrow must be paid positive liquidity (term) premium, l_{nt} , to hold long-term bonds

Results in following modification of Expectations Theory

$$i_{nt} = \frac{i_t + i^e_{t+1} + i^e_{t+2} + \dots + i^e_{t+(n-1)}}{n} + l_{nt}$$



Relationship Between the Liquidity Premium (Preferred Habitat) and Expectations Theories



Numerical Example

1. One-year interest rate over the next five years:
5%, 6%, 7%, 8% and 9%
2. Investors' preferences for holding short-term bonds, liquidity premiums for one to five-year bonds:
0%, 0.25%, 0.5%, 0.75% and 1.0%.

Interest rate on the two-year bond:

$$(5\% + 6\%)/2 + 0.25\% = 5.75\%$$

Interest rate on the five-year bond:

$$(5\% + 6\% + 7\% + 8\% + 9\%)/5 + 1.0\% = 8\%$$

Interest rates on one to five-year bonds:

$$5\%, 5.75\%, 6.5\%, 7.25\% \text{ and } 8\%.$$

Comparing with those for the expectations theory, liquidity premium (preferred habitat) theories produce yield curves more steeply upward sloped

Liquidity Premium (Preferred Habitat) Theories: Term Structure Facts

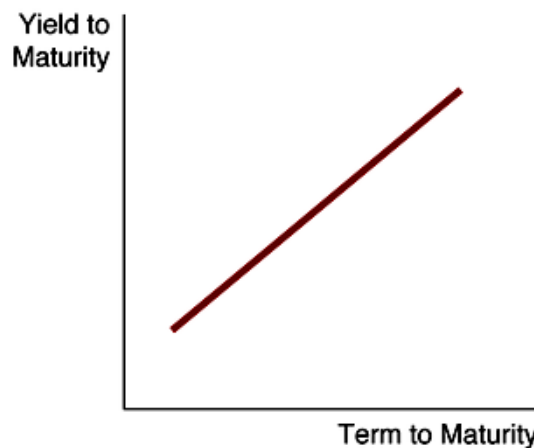
Explains all 3 Facts

Explains Fact 3 of usual upward sloped yield curve by investors' preferences for short-term bonds

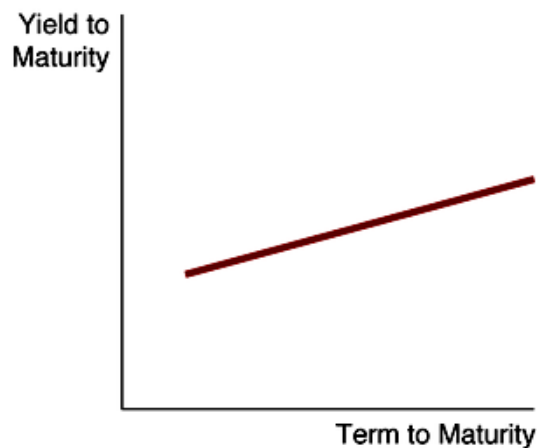
Explains Fact 1 and Fact 2 using same explanations as expectations hypothesis because it has average of future short rates as determinant of long rate



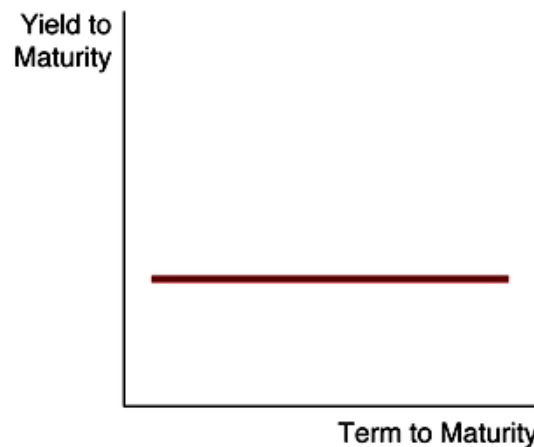
Market Predictions of Future Short Rates



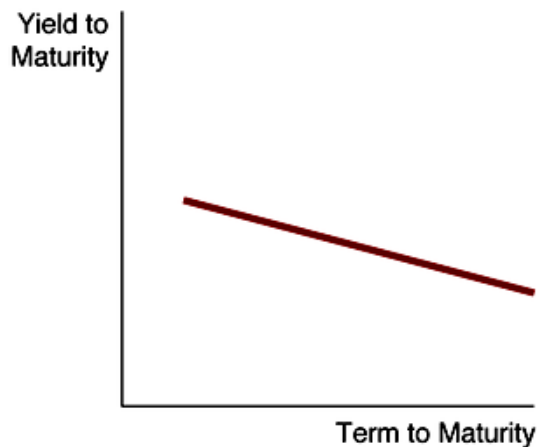
(a) *Future short-term interest rates expected to rise*



(b) *Future short-term interest rates expected to stay the same*



(c) *Future short-term interest rates expected to fall moderately*



(d) *Future short-term interest rates expected to fall sharply*



Interpreting Yield Curves 1980–2000

