CHAPTER 3

Static and Comparative Static Equilibrium Analysis

Topics: Static and Comparative Static Equilibrium Analysis, PART1

Outline:

- Linear models in economics
- Simultaneous system of equations
- * A partial-equilibrium market model: A model of price determination in an isolated market
- * Excise Tax and Market Equilibrium
- What is elasticity ? How can we derive elasticity?
- * Tax incidence and Elasticity

"Equilibrium" means

→ a situation that selected interrelated variables so adjusted to one another that no tendency to change

→ All variables in the model must be simultaneously endogenous

in a state of rest

"Static" means

statics = equilibrium

Nongoal type of equilibrium: ⇒ Ch3, Ch4 (use matrix to solve equilibrium), Ch 6, 8 (optimization) → goal-type equilibrium

→ The equilibrium that is not a result of any particular objective, but is derived from an impersonal process of interaction

⇒ we could have underemployment equilibrium level of national income

Linear economic model vs. Nonlinear economic model

Linear		Nonlinear
→ $Q^D = Q^S$	①	$Q^D = Q^S$
→ $Q^D = a - bP$	②	$Q^D = a - bP^2$
→ $Q^S = -c + dP$	③	$Q^S = -c + dP^2$

"Simultaneous system of equations" means

system of equations in which each equation is considered to be related

→ e.g. finding equilibrium = finding solution to the system of equation

The solution to system of equations must make every equations true simultaneously

3.1

A partial-equilibrium market model: A model of price determination in an isolated market

Suppose we are interested in one commodity, energy drink. Since only one commodity is being considered, the economic model for this market is comprised of:

1. Q^d
2. Q^s
3. P

After having chosen the variables, we next make certain assumptions regarding the working of the market. First, we must specify an equilibrium condition- something indispensable in an equilibrium model. The standard assumption is that equilibrium occurs in the market if and only if the excess demand is zero, that is, if and only if the market is cleared. The Clearing Market Condition can be written as:

$$\boxed{Q^D = Q^S} \quad (1)$$

to note: $\boxed{P^d = P^s}$

Equation (1.) is conditional equation of the market. The model requires that we specify behavioral equations to explain how exactly the demand and supply are each determined.

use (*) (*) to solve for Q^*, P^*

$Q^D = Q^S$

Demand for energy drink

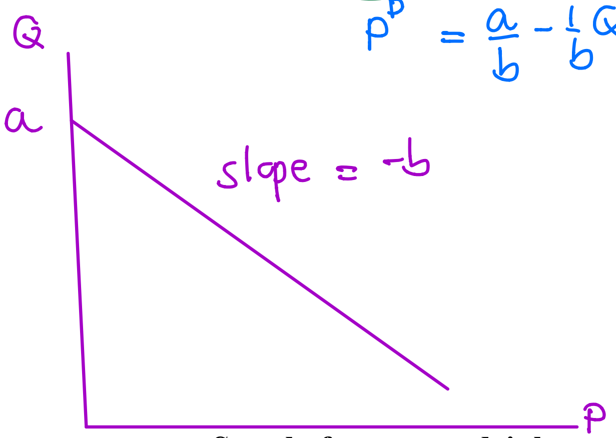
We assume that Q_d is a decreasing linear function of P.

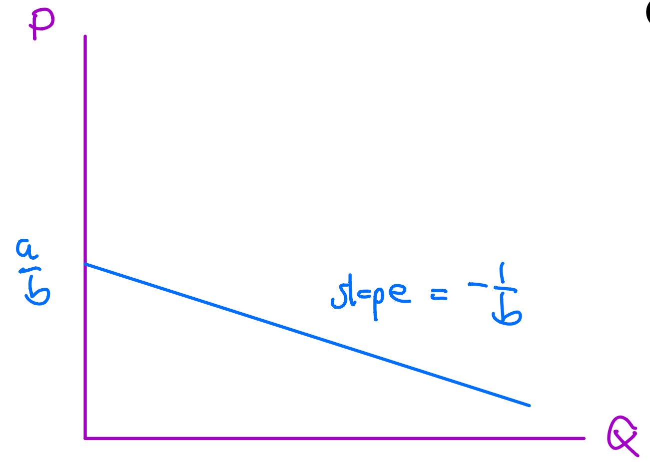
$Q^D = a - bP$, $b > 0$ —— (*)

$P^D = \frac{a}{b} - \frac{1}{b}Q$

indicate restriction on a,b,c,d such that Q^*, P^* can happen

(2)

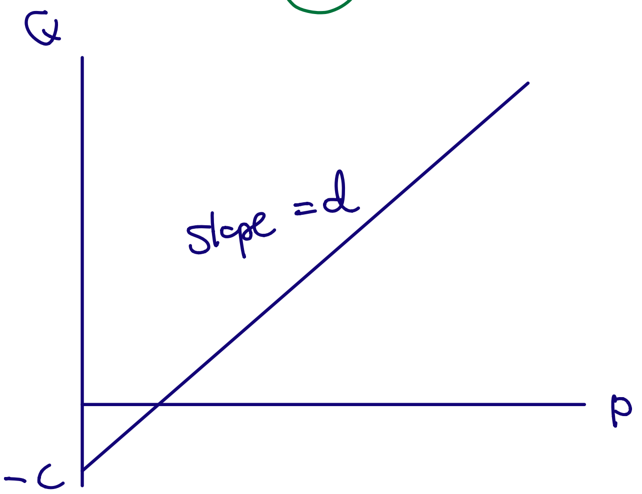


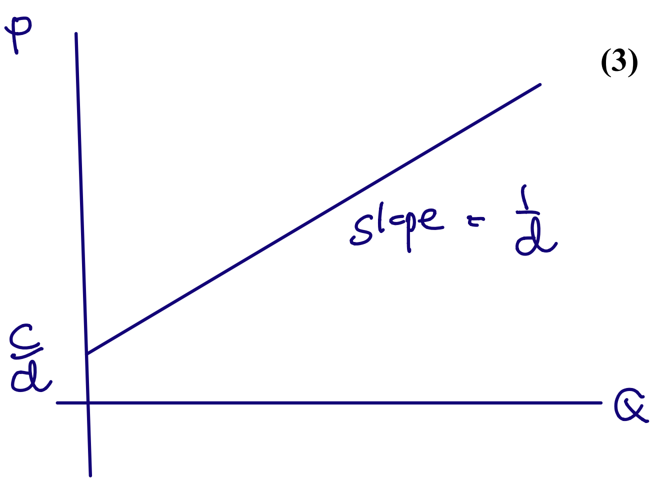


Supply for energy drink

We assume that Q_S is an increasing linear function of P.

$Q^S = -c + dP$, $d > 0$, $c > 0$ (*)





(3)

In all, the model will contain one equilibrium condition plus two behavioral equations which govern the demand and supply sides of the market.

① $\boxed{Q_d = Q_s} \Rightarrow$ 1 Conditional equation

② $\boxed{Q_d = a - bP}$ $(a, b > 0)$ } 2 Behavioral equations

③ $\boxed{Q_s = -c + dP}$ **(4)**

The next step is to obtain the solution values of the three endogenous variables, Q_d, Q_s, P.

The solution values are those values that satisfy the three equations in (4) simultaneously.

We usually denote the solution value of an endogenous variable with an asterisk. Thus, the solution values of Q_d, Q_s, P are denoted by Q_d^*, Q_s^*, P^*.

Since $\boxed{Q_d^* = Q_s^*}$ they can be replaced by a single symbol Q^*. $Q_d^* = Q_s^* = Q^*$

Hence, an equilibrium solution of the model may simply be denoted by an ordered pair (P^*, Q^*).

☆ That is, we equate quantity demanded with quantity supplied & solve for market price☆ at equilibrium

Solution by Elimination of Variables: From equilibrium mkt. price, we can further solve for related market quantity at equilibrium

From mkt. equilibrium condition:

$$Q^D = Q^S$$

$$a - bP = -c + dP$$

$$a + c = (b+d)P$$

$$\boxed{P^* = \frac{a+c}{b+d}}$$

the solution that we are looking for

$\Rightarrow$ endo var $= f$(parameters, exo var.)

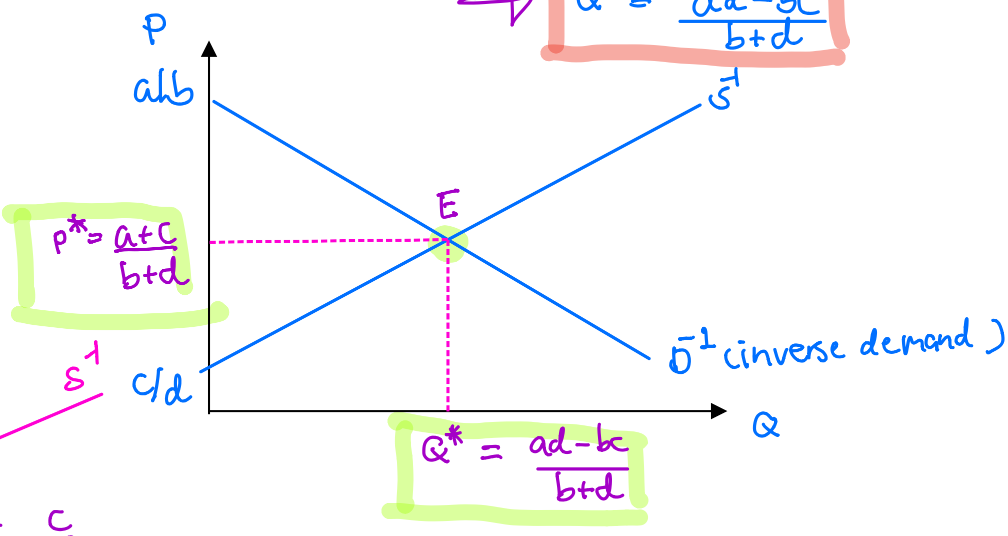
$$Q^D = Q^S = Q^*$$

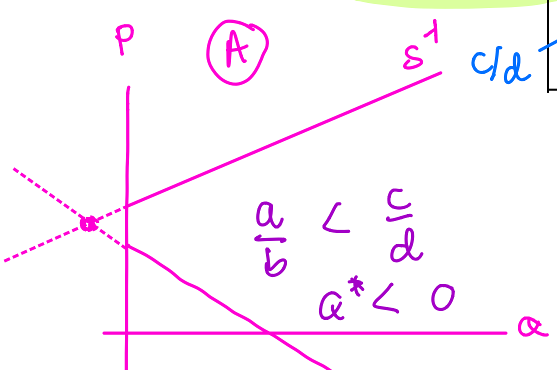
$$a - b\left(\frac{a+c}{b+d}\right) = -c + d\left(\frac{a+c}{b+d}\right) = Q^*$$

$\Rightarrow$

$$Q^* = \frac{\cancel{ab} + ad - \cancel{ab} - bc}{b+d}$$

$$\boxed{Q^* = \frac{ad-bc}{b+d}}$$





Additional restriction for an economically meaningful solution:

If we would like to have $Q^* > 0$

$$\frac{ad-bc}{b+d} > 0$$

$$ad - bc > 0$$

$$ad > bc$$

$$\frac{a}{b} > \frac{c}{d}$$

Note: Compare between partial equilibrium and general equilibrium model

two mkts. : X, Y

① Partial equilibrium

$$Q_d^X = a - bP_X + cP_Y$$

$$Q_s^X = d + eP_X$$

$$Q_d^X = Q_s^X$$

take as given

P_X^*, $Q_X^* = Q_d^X = Q_s^X$

Solving for 2 solutions (P_X^*, Q_X^*)

P_Y is exogenous var

P_X, Q_d^X, Q_s^X endo var

② General equilibrium

$$Q_d^X = a - bP_X + cP_Y$$

$$Q_s^X = d + eP_X$$

$$Q_d^Y = f - gP_Y + hP_X$$

$$Q_s^Y = j + kP_Y$$

$$Q_d^X = Q_s^X$$

$$Q_d^Y = Q_s^Y$$

$Q_d^X, Q_s^X, Q_d^Y, Q_s^Y, P_X, P_Y$ are endogenous variables

Solving for 4 solutions (P_X^*, Q_X^*) (P_Y^*, Q_Y^*)

3.2 Excise Tax and Market Equilibrium

We will use a partial-equilibrium market model from previous section in an analysis of the impact of excise tax.

Definition of an excise tax:

Excise taxes are narrowly based taxes on consumption, levied on specific goods, services, and activities. They can be either a per unit tax (such as the per gallon tax on gasoline) or a percentage of price (such as the airline ticket tax). Generally, excise taxes are collected from producers or wholesalers, and are embedded in the price paid by final consumers.

"Statics" equilibrium before tax $\xleftarrow{\text{Compare}}\!\!\rightarrow$ "Statics" equilibrium after tax

"Comparative static analysis"

consumer
producer

(a.) **<u>Specific Tax</u>** or per unit tax or unit tax: is a tax that is defined as a fixed amount for each unit of a good or service sold, such as cents per kilogram.

......t baht per unit...... e.g. 3 baht per gallon of gasoline

consumer
producer

according to value

(b.) **<u>Ad Valorem Tax</u>** is a tax whose amount is based on the value of a transaction or of property. It is a charge based on a fixed percentage of the product value. It is typically imposed at the time of a transaction, as in the case of a sale or value-added tax (VAT).

levied on the price of a product

......t% of price......

7% × 10 baht, 7% × 20 baht

Tax can be collected from buyers or producers, depending on convenience and efficacy of tax collection. Either way, market equilibrium will change. We can compare *market equilibrium before tax to market equilibrium after tax*, and analyze how the change is. This is called "Comparative Static Analysis".

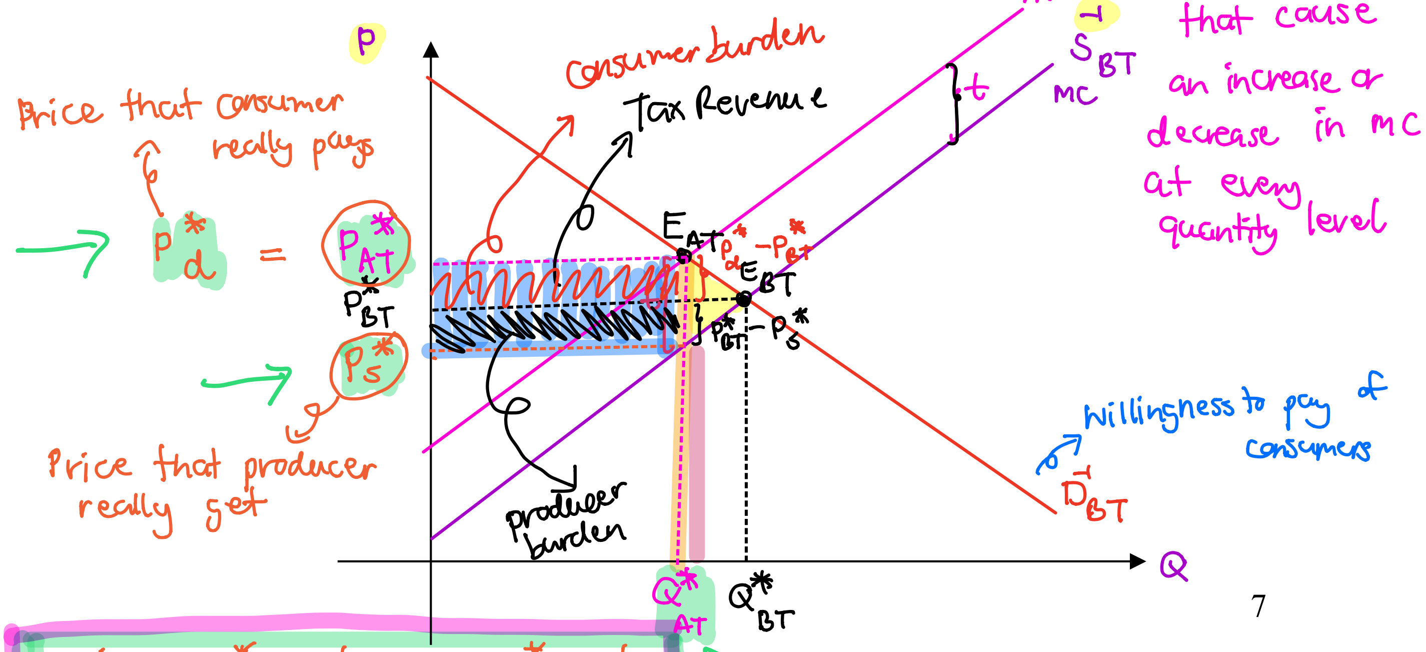
Comparative statics, as the name suggested, is concerned with the comparison of different equilibrium states that are associated with different sets of values of parameters and exogeneous variables.

CASE 1: Specific tax

Collect tax t baht per unit of energy drink, e.g. t baht per bottle

The supply curve is derived from firms' MC and can be shifted up or down by any changes in the market that cause an increase or decrease in MC at every quantity level

CASE 1.1: Collect from producers/suppliers



$$\boxed{P_s^* = P_d^* - t = P_{AT}^* - t}$$

Market equilibrium before tax

short version

$$Q^d = Q^s$$
$$Q^d = a - bP, \quad a, b > 0$$
$$Q^s = -c + dP, \quad c, d > 0$$
$$a - bP = -c + dP$$
$$P^* = \boxed{\frac{a+c}{b+d}}, \quad Q^* = \frac{ad - bc}{b+d}$$

long version

① $Q^d = Q^s$

② $Q^d = a - bP_d$: demand is a fn of the price really paid by consumers

③ $Q^s = -c + dP_s$: supply is a fn of the price really received by producers

At mkt equilibrium with no tax

④ $P_d = P_s = P$ mkt price that we are going to solve for

To solve, $Q^d = Q^s$

$$a - bP_d = -c + dP_s$$
$$a - bP = -c + dP$$

Market equilibrium after tax

① $Q^d = Q^s$

② $Q^d = a - bP_d$

③ $Q^s = -c + dP_s$

④ $P_s = P_d - t$

; $P_d = P$ market price to solve for

$P_s = P - t$ price received by producers is the price paid by consumer minus tax

To solve : $Q^d = Q^s$

$$a - bP_d = -c + dP_s$$
$$a - bP = -c + d(P - t)$$
$$a + c + dt = (b + d)P$$
$$P^*_{AT} = \boxed{\frac{a + c + dt}{b + d}} = P^*_d$$

since $d > 0$, $P^*_{AT} > P^*_{BT}$

$$Q^*_{AT} = a - bP^*_d$$
$$= a - b\left(\frac{a + c + dt}{b + d}\right)$$
$$Q^*_{AT} = \frac{ad - bc - bdt}{b + d}$$

, since $b, d > 0$

$Q^*_{AT} < Q^*_{BT}$

$$P_s^* = P_d^* - t = \frac{a+c+dt}{b+d} - t = \frac{a+c-bt}{b+d}$$

$P_s^* < P_{BT}^*$

Tax revenue is equal to:

$$\text{Tax Revenue} = t\,Q_{AT}^*$$

$$= t\left[\frac{ad - bc - bdt}{b+d}\right]$$

Tax burden on consumers/buyers

$$CB \text{ per unit} = P_d^* - P_{BT}^* = \frac{dt}{b+d}$$: tax burden per unit on consumer

$$CB = (P_d^* - P_{BT}^*)\,Q_{AT}^* = \frac{dt}{b+d}\left[\frac{ad - bc - bdt}{b+d}\right]$$

Tax burden on producers/sellers

$$PB \text{ per unit} = P_{BT}^* - P_s^* = \frac{bt}{b+d}$$: tax burden per unit on producer

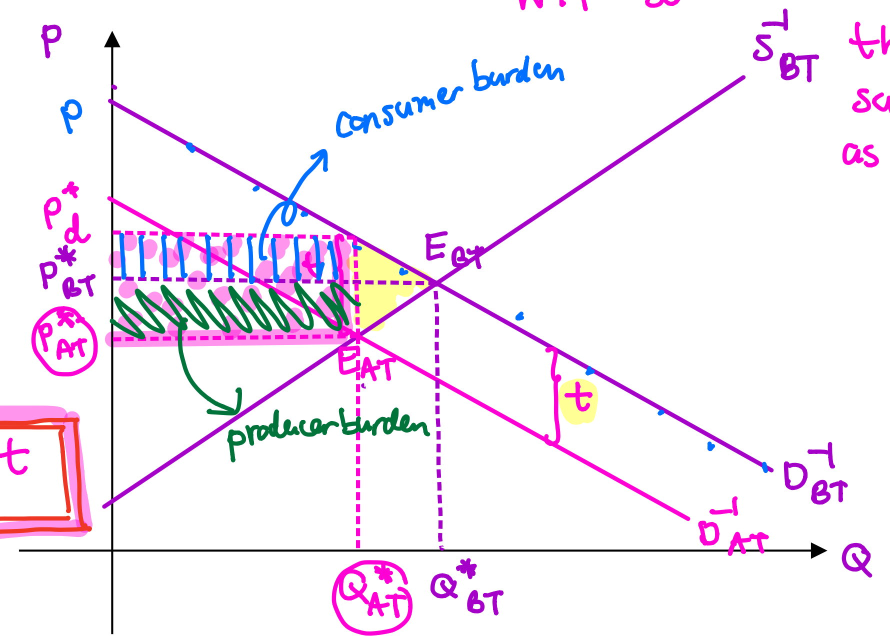
$$PB = (P_{BT}^* - P_s^*)\,Q_{AT}^* = \frac{bt}{b+d}\left[\frac{ad - bc - bdt}{b+d}\right]$$

$\boxed{\textbf{CASE 1.2: Collect from consumers/buyers}}$

consumer will have lower WTP so that after paying tax, they pay the same amount as before tax

$P_s^* = P_{AT}^*$

$\boxed{P_d^* = P_s^* + t}$



$$P_d^* = P_s^* + t$$

whether P_{AT}^*, Q_{AT}^*, Tax burden, Tax revenue are the same as the case of
P_d^*, P_s^*

$_d$, $_s$

Collecting tax from producer ?

Market equilibrium before tax

From mkt. equilibrium condition :

$$Q^D = Q^S$$

$$a - bP = -c + dP$$

$$a + c = (b+d)P$$

$$P^* = \frac{a+c}{b+d} \qquad \boxed{Q^* = \frac{ad-bc}{b+d}}$$

Market equilibrium after tax

1 $Q^D = Q^S$

2 $Q^D = a - bP_d$

3 $Q^S = -c + dP_s$

4 $P_s = P$: mkt. equilibriumprice after tax to solve for

5 $P_d = P_s + t = P + t$

Therefore, at mkt. equi. $Q^D = Q^S$

$$a - b(P+t) = -c + dP \quad : \text{1 equation, 1 endo var}$$

$$P^*_{AT} = \frac{a+c-bt}{b+d} = P^*_s < P^*_{BT}$$

$$Q^*_{AT} = Q^*_s = -c + dP^*_s$$

$$= -c + d\left(\frac{a+c-bt}{b+d}\right)$$

$$Q^*_{AT} = \frac{ad-bc-bdt}{b+d} < Q^*_{BT}$$

$$P^*_d = P^*_s + t$$

(P^*_{AT})

$$= \frac{a+c-bt}{b+d} + t$$

$$P^*_d = \frac{a+c+dt}{b+d} > P^*_{BT}$$

Note:

Collect tax from producer

$P^*_{AT} = P^*_d$

Collect tax from consumer

$P^*_{AT} = P^*_s$

Tax revenue is equal to:

the same as the previous case (collecting tax from producer)

Tax burden on consumers/buyers

the same as the previous case (collecting tax from producer)

Tax burden on producers/sellers

the same as the previous case (collecting tax from producer)

OBSERVATION:

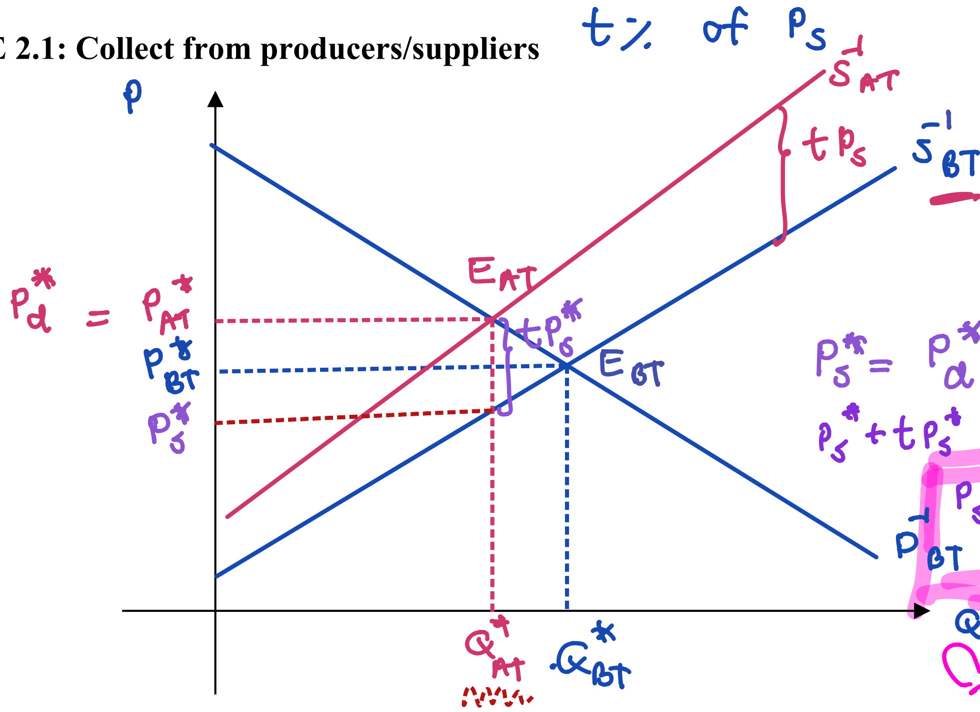
the same except p^*_{AT}

(p^*_d, p^*_s, Q^*_{AT}, tax revenue, CB, PB are the same)

CASE 2: Ad Valorem tax

Collect tax t% of price, e.g. 7% of price per bottle

CASE 2.1: Collect from producers/suppliers



$$P_s^* = P_d^* - tP_s^*$$

$$P_s^* + tP_s^* = P_d^*$$

$$\boxed{P_s^* = \frac{P_d^*}{1+t} = \frac{P_{AT}^*}{1+t}}$$

Market equilibrium before tax

Market equilibrium after tax

At mkt equilibrium $\quad Q_d = Q_s$

$$a - bP_d = -c + dP_s$$

$$\boxed{a - bP = -c + d\left(\frac{P}{1+t}\right)}$$

$$P_{AT}^* = \frac{(1+t)(a+c)}{(1+t)b + d} = P_d^*$$

$$Q_{AT}^* = a - bP_d^*$$

$$= \frac{ad - bc(1+t)}{b(1+t) + d}$$

$$P_s^* = \frac{P_d^*}{1+t}$$

Tax revenue is equal to:

$$\text{Tax Revenue} = t\,P_S^* Q_{AT}^*$$

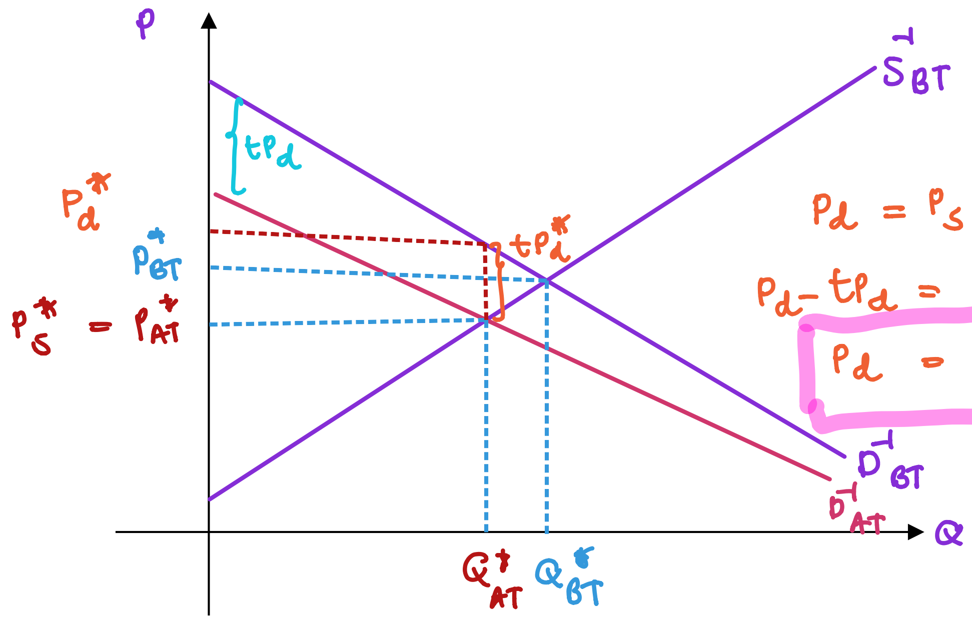
Tax burden on consumers/buyers

$$CB = (P_d^* - P_{BT}^*)\,Q_{AT}^*$$

Tax burden on producers/sellers

$$PB = (P_{BT}^* - P_S^*)\,Q_{AT}^*$$

CASE 2.2: Collect from consumers/buyers t% of P_d



$P_d = P_s + tP_d$

$P_d - tP_d = P_s$

$$\boxed{P_d = \frac{P_s}{1-t} = \frac{P^*}{1-t}}$$

mkt. equilibrium price after tax that we need to solve for

Market equilibrium before tax

Market equilibrium after tax

At mkt. equilibrium $Q_d = Q_s$

$$(*) \quad a - b\left(\frac{p}{1-t}\right) = -c + dp$$

$$P_{AT}^* = \frac{(a+c)(1-t)}{b+(1-t)d} = P_s^* < P_{BT}^*$$

$$Q_{AT}^* = -c + dP_s^*$$

$$= \frac{ad(1-t) - bc}{b+d(1-t)}$$

$$P_d^* = \frac{1}{1-t}P_s^* = \frac{a+c}{b+(1-t)d}$$

Tax revenue is equal to:

Tax burden on consumers/buyers

Tax burden on producers/sellers

OBSERVATION:

P_d^*, P_s^*, P^*_{AT}, Q^*_{AT}, tax revenue, CB, PB of

1.) collecting VAT from producers

2) consumers

will be different.

Tax incidence (or incidence of tax) is an economic term for understanding the division of a tax burden between buyers and sellers or producers and consumers.

Tax incidence is related to the price elasticity of supply and demand. When supply is more elastic than demand, the tax burden falls on the buyers. If demand is more elastic than supply, producers will bear the cost of the tax.

Digression

What is elasticity ? How can we derive elasticity?

From linear demand function in (2):

………………………………………………………………………………………

With P on Y-axis and Q on x-axis, the inverse demand function is:

………………………………………………………………………………………

From linear supply function in (3):

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The inverse supply function is:

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$-b$ is

………………………………………………………………………………………

………………………………………………………………………………………

$-d$ is

……………………………………………………………………………………………

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To be able to compare how responsive of change in quantity to change in price across different commodities, the concept of "elasticity" comes in handy.

Elasticity is the measurement of the percentage change of one economic variable in response to one percentage change in another.

(4)

……………………………………………………………………………………………

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The price elasticity of demand is:

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The price elasticity of supply is:

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……………………………………………………………………………………………

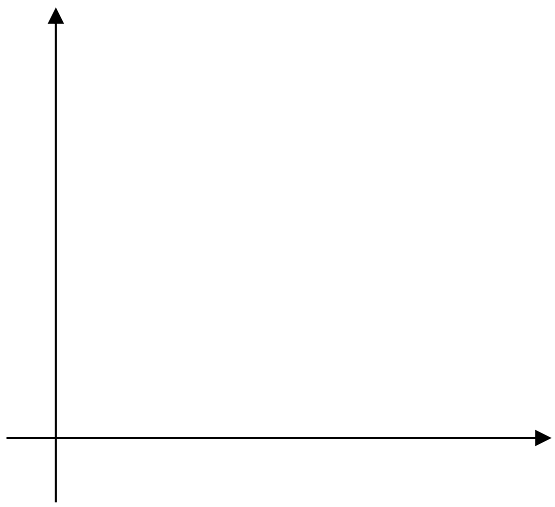
Inelastic vs. Elastic vs. Unit elastic

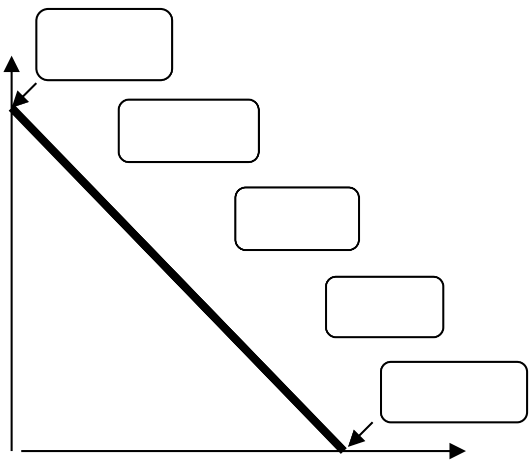
$E_p < 1$

$E_p > 1$

$E_p = 1$

Price Elasticity of Demand at different point on inverse demand function:



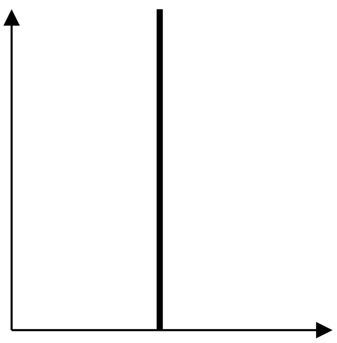


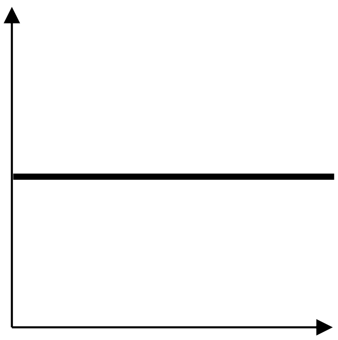
The price elasticity of demand at each point of linear demand function, with negative slope, is not equal to other point. This is because:

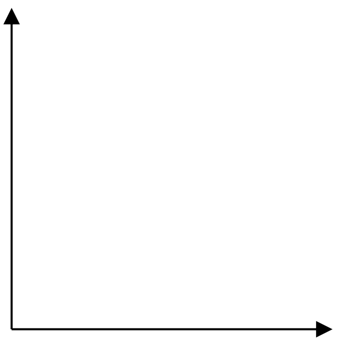
$$\frac{P}{Q}$$

$$\frac{\Delta Q}{\Delta P} = -b$$

In which case are the price elasticities of demand for different points equal?







Tax incidence and Elasticity

Whoever cannot adjust themselves instantaneously to change in price will have to bear a larger portion of the tax burden.