


Production and Costs in the Long run
EE311

Chayun Tantivasadakarn
Faculty of Economics, Thammasat University


Chayun Tanti



Topics to be Discussed

- Isoquants
- Production with Two Variable Inputs
- Returns to Scale
- Cost in the Long Run
- Production with Two Outputs--Economies of Scope
- Learning Curve

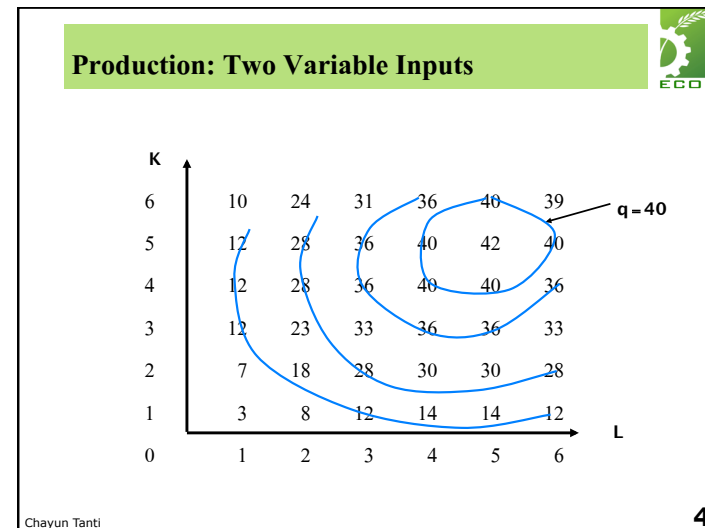
Chayun Tanti 2



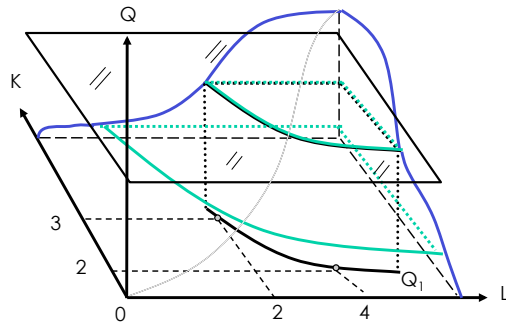
Production: Two Variable Inputs

- Firm can produce output by combining different amounts of labor and capital
- In the long-run, capital and labor are both variable.
- We can look at the output we can achieve with different combinations of capital and labor

Chayun Tanti 3



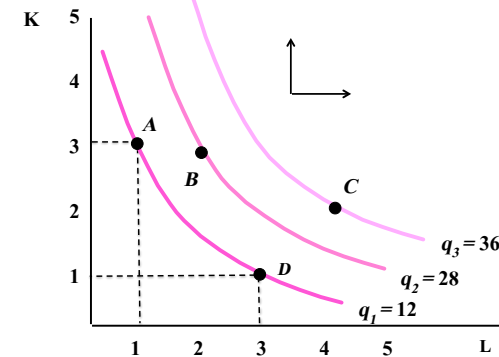
Isoquants are the contour on the production hill



Chayun Tanti

5

Isoquant Map



Chayun Tanti

6

Derivation of an Isoquant

- From a production function $Q = K^{0.5}L^{0.5}$
- At a fixed Q , $K = Q^2/L$
- Suppose $Q = 100$, the function for this isoquant is: $K = 10000/L$
- Any combination of K & L according to the above function will always give $Q = 100$ and the slope is declining as L increases

Chayun Tanti

7

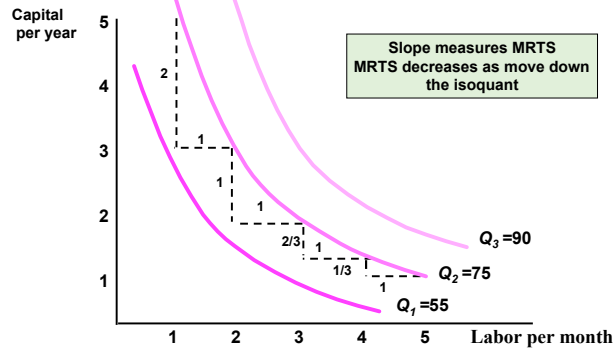
Production: Two Variable Inputs

- Substituting Among Inputs
 - Slope of the isoquant shows how one input can be substituted for the other and keep the level of output the same.
 - Negative slope is the **marginal rate of technical substitution (MRTS) = dK/dL**
 - Amount by which the quantity of one input can be reduced when one extra unit of another input is used, so that output remains constant.

Chayun Tanti

8

Marginal Rate of Technical Substitution



Chayun Tanti

9

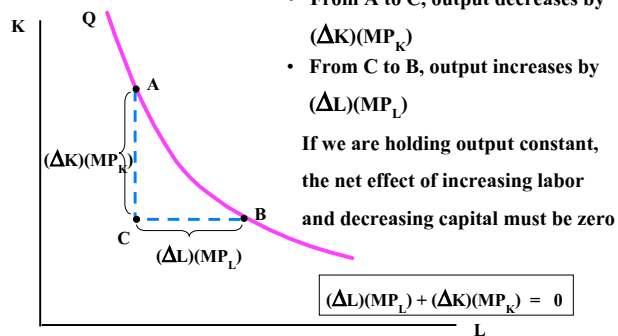
MRTS and Isoquants

- We assume there is diminishing MRTS
 - Increasing labor in one unit increments from 1 to 5 results in a decreasing MRTS from 1 to 1/2.
 - Productivity of any one input is limited
- Diminishing MRTS occurs because of diminishing returns and implies isoquants are convex.
- There is a relationship between MRTS and marginal products of inputs

Chayun Tanti

10

MRTS and Marginal Products



Chayun Tanti

11

MRTS and Marginal Products

- Rearranging equation, we can see the relationship between MRTS and MPs

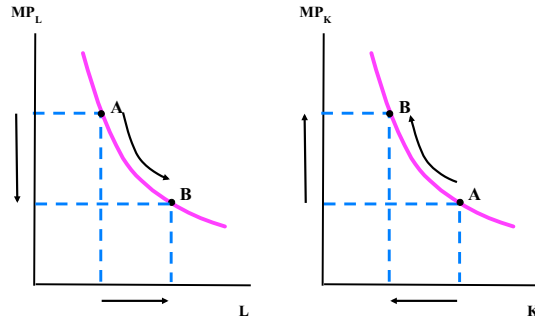
$$\begin{aligned}
 (MP_L)(\Delta L) + (MP_K)(\Delta K) &= 0 \\
 (MP_L)(\Delta L) &= -(MP_K)(\Delta K) \\
 \frac{(MP_L)}{(MP_K)} &= -\frac{\Delta K}{\Delta L} = \text{MRTS}
 \end{aligned}$$

Chayun Tanti

12

MRTS and Marginal Products

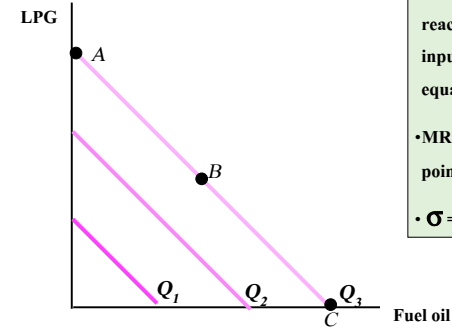
As we move down the isoquant, MP_L decreases while MP_K increases. Hence $MRTS = MP_L / MP_K$ decreases



Chayun Tanti

13

Perfect Substitutes

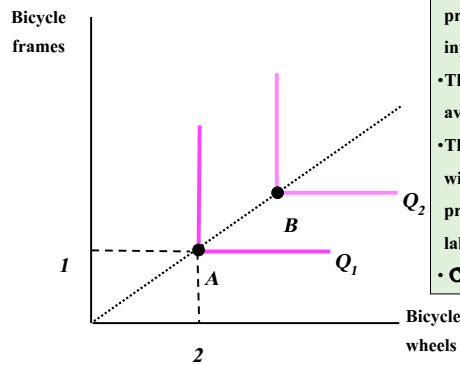


- Same output can be reached with mostly one input (A or C) or with equal amount of both (B)
- MRTS is constant at all points on isoquant
- $\sigma = \text{infinity}$

Chayun Tanti

14

Fixed-Proportions Production Function



- Same output can only be produced with one set of inputs.
- There is no substitution available between inputs.
- The output can be made with only a specific proportion of capital and labor
- $\sigma = 0$

Chayun Tanti

15

Returns to Scale

- How does a firm decide, in the long run, the best way to increase output
 - Can change the scale of production by increasing all inputs in proportion
- Rate at which output increases as inputs are increased proportionately
 - Constant returns to scale
 - Increasing returns to scale
 - Decreasing returns to scale

Chayun Tanti

16

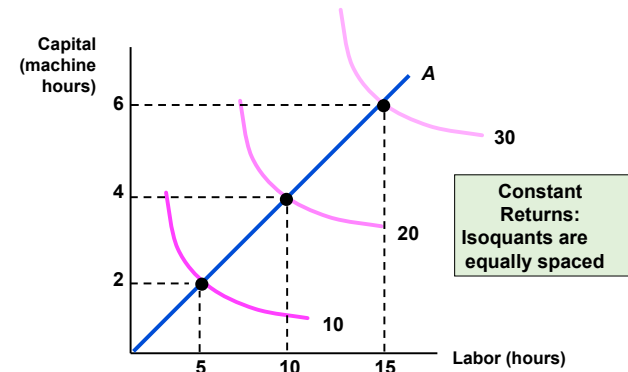
Returns to Scale

- **Constant returns to scale:** output doubles when all inputs are doubled
 - Size does not affect productivity
 - May have a large number of producers
 - Isoquants are equidistant apart

Chayun Tanti

17

Returns to Scale



Chayun Tanti

18

Returns to Scale

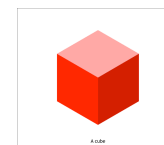
- **Increasing returns to scale:** output more than doubles when all inputs are doubled
 - Larger output associated with lower cost (cars)
 - One firm is more efficient than many (utilities)
 - The isoquants get closer together
- Reasons
 - Specialization and division of labor
 - Technical increasing returns to scale
- It leads to a declining cost per unit



Chayun Tanti

19

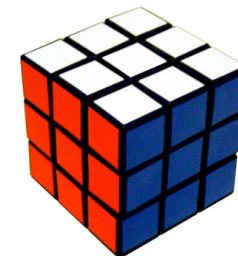
Technical increasing returns to scale



$1 \times 1 \times 1 = 1$
cubic



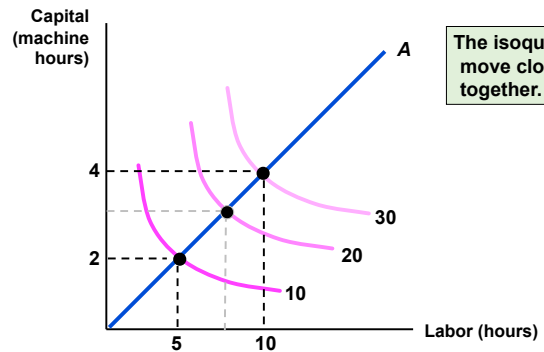
$2 \times 2 \times 2 = 8$
cubics



$3 \times 3 \times 3 = 27$
cubics

Chayun Tanti

Increasing Returns to Scale



Chayun Tanti

21

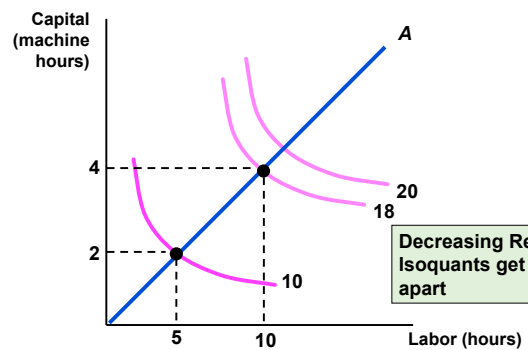
Returns to Scale

- **Decreasing returns to scale:** output less than doubles when all inputs are doubled
 - Decreasing efficiency with large size
 - Reduction of entrepreneurial abilities
 - Isoquants become farther apart

Chayun Tanti

22

Returns to Scale



Chayun Tanti

23

Returns to Scale: Example

$$\begin{aligned}
 Q_0 &= AL_0^\alpha K_0^\beta \\
 Q_1 &= A(mL)_0^\alpha (mK)_0^\beta, \quad m > 1 \\
 &= Am^\alpha L_0^\alpha m^\beta K_0^\beta \\
 &= m^{\alpha+\beta} AL_0^\alpha K_0^\beta = m^{\alpha+\beta} Q_0
 \end{aligned}$$

$a + b > 1$, Increasing returns to scale

$a + b = 1$, Constant returns to scale

$a + b < 1$, Decreasing returns to scale

Chayun Tanti

24

Homogeneous Production Function

- A function is homogeneous of degree k if an increase in all variables by m times causes the value of the function to increase by m^k times or

$$F(mL, mK) = m^k F(L, K)$$

- Example: $Q = L^a K^b$ is homogeneous degree 1 if $a + b = 1$
- When $k = 1$ it is also referred to as linearly homogeneous function
 - All APs, MPs, MRTSs are constant for a given K/L
 - Euler's Theorem holds: $L MP_L + K MP_K = Q$

Homogeneous Production function

- ตัวอย่าง: Cobb-Douglas function $Q(L, K) = K^\alpha L^{(1-\alpha)}$
- $AP_L = K^\alpha L^{(1-\alpha)}/L = K^\alpha L^{-\alpha} = (K/L)^\alpha$
- $AP_K = K^\alpha L^{(1-\alpha)}/K = K^{\alpha-1} L^{(1-\alpha)} = (L/K)^{(1-\alpha)}$
- $MP_L = (1-\alpha) K^\alpha L^{(1-\alpha)-1} = (1-\alpha) (K/L)^\alpha$
- $MP_K = \alpha K^{\alpha-1} L^{(1-\alpha)} = \alpha (L/K)^{(1-\alpha)}$
- $MRTS = -MP_L/MP_K = -(1-\alpha) (K/L)^\alpha / \alpha (L/K)^{(1-\alpha)}$
 $= -[(1-\alpha)/\alpha](K/L)$
- $L MP_L + K MP_K = L(1-\alpha) (K/L)^\alpha + K\alpha (L/K)^{(1-\alpha)}$
 $(1-\alpha) K^\alpha L^{(1-\alpha)} + \alpha K^\alpha L^{(1-\alpha)} = K^\alpha L^{(1-\alpha)} = Q$

Cost in the Long Run

- In the long run a firm can change all of its inputs
- Assumptions
 - Two Inputs: Labor (L) & capital (K)
 - Price of input: wage rate (w), rental rate (r)
- The Isocost Line
 - A line showing all combinations of L & K that can be purchased for the same cost
 - Total cost of production is sum of firm's labor cost, wL and its capital cost rK

$$TC = wL + rK$$

Cost in the Long Run

- Rewriting TC as an equation for a straight line:
 - $K = TC/r - (w/r)L$
 - Slope of the isocost: $\Delta K / \Delta L = -(w/r)$
- w/r is the ratio of the wage rate to rental cost of capital.
- This shows the rate at which capital can be substituted for labor with no change in cost.

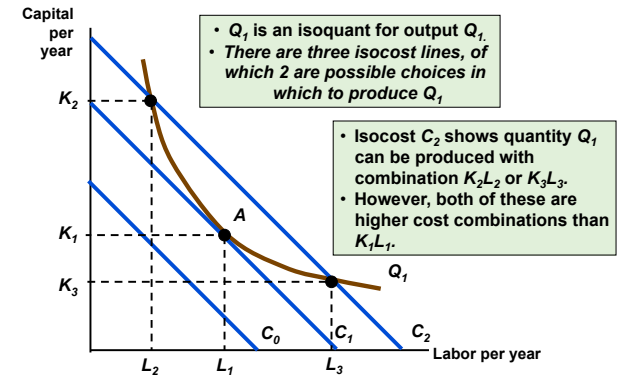
Least Cost Combination of Inputs

- We will address how to minimize cost for a given level of output by combining isocosts with isoquants
- We choose the output we wish to produce and then determine how to do that at minimum cost
 - Isoquant is the quantity we wish to produce
 - Isocost is the combination of K and L that gives a set cost

Chayun Tanti

29

Least Cost Combination of Inputs



Chayun Tanti

30

Cost in the Long Run

$$MRTS = -\frac{\Delta K}{\Delta L} = \frac{MP_L}{MP_K}$$

$$\text{Slope of isocost line} = \frac{\Delta K}{\Delta L} = -\frac{w}{r}$$

$$\frac{MP_L}{MP_K} = \frac{w}{r} \text{ when firm minimizes cost}$$

- The minimum cost combination can then be written as:

$$\frac{MP_L}{w} = \frac{MP_K}{r}$$

- Minimum cost for a given output will occur when each dollar of input added to the production process will add an equivalent amount of output.

Chayun Tanti

31

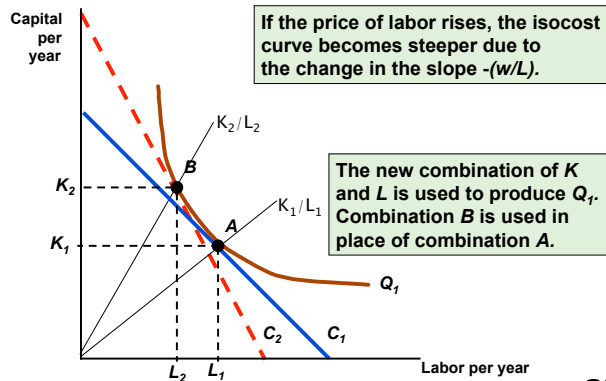
Input Substitution When an Input Price Change

- If the price of labor changes, then the slope of the isocost line change, w/r
- It now takes a new quantity of labor and capital to produce the output
- If price of labor increases relative to price of capital, and capital is substituted for labor

Chayun Tanti

32

Input Substitution When an Input Price Change



Chayun Tanti

33

Exercise

- When Ratchadaphisek Road was first built, there were so many large restaurants popped up along the road. Later they are replaced by offices, condominiums, hotels, massage parlors. Large restaurants are now congregated on the newly built Kaset-Navamin Road. Why?



Chayun Tanti

34

Elasticity of substitution

- $\% \Delta$ in capital-labor ratio divided by the $\% \Delta$ in the slope of the isoquant

$$\sigma = \frac{\% \Delta (K/L)}{\% \Delta \text{MRTS}} = \frac{d(K/L)}{d(\text{MRTS})} \frac{\text{MRTS}}{K/L} = \frac{\partial \ln(K/L)}{\partial \ln(\text{MRTS})}$$

$\sigma = 0$ means fixed proportion

$\sigma = \infty$ means perfect substitutes

Chayun Tanti

35

Example

From Cobb-Douglas production function: $Q = K^\alpha L^\beta$,

$$\text{MRTS} = \frac{MP_L}{MP_K} = \frac{\beta K^\alpha L^{\beta-1}}{\alpha K^{\alpha-1} L^\beta} = \frac{\beta K}{\alpha L}$$

$$\frac{K}{L} = \frac{\alpha}{\beta} \text{MRTS}$$

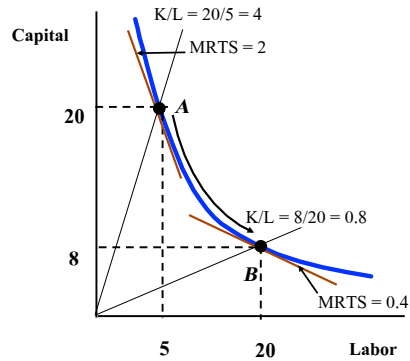
$$\ln \frac{K}{L} = \ln \frac{\alpha}{\beta} + \ln \text{MRTS}$$

$$\sigma = 1$$

Chayun Tanti

36

Elasticity of substitution



- Point A is more capital-intensive, and B is more labor-intensive.
- Moving from A to B, K/L decreases by 80% and $MRTS$ also decreases by 80% $\rightarrow \sigma = 1$

Chayun Tanti

37

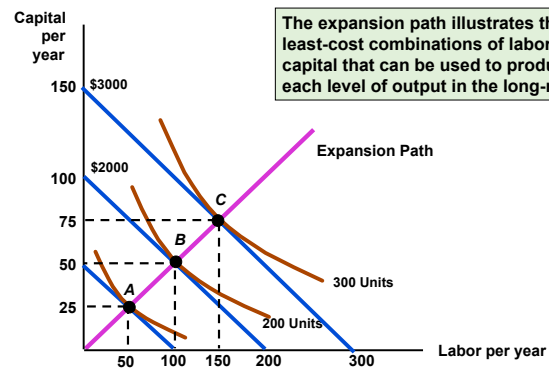
Cost in the Long Run

- Cost minimization with Varying Output Levels
 - For each level of output, there is an isocost curve showing minimum cost for that output level
 - A firm's **expansion path** shows the minimum cost combinations of labor and capital at each level of output.

Chayun Tanti

38

A Firm's Expansion Path



The expansion path illustrates the least-cost combinations of labor and capital that can be used to produce each level of output in the long-run.

Chayun Tanti

39

Expansion Path & Long-run Costs

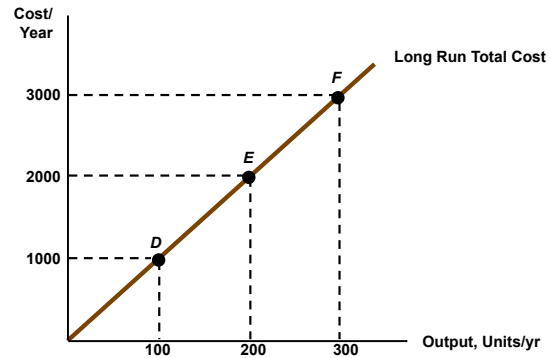
- Firms expansion path has same information as long-run total cost curve
- To move from expansion path to LR cost curve
 - Find tangency with isoquant and isocost
 - Determine min cost of producing the output level selected
 - Graph output-cost combination

Chayun Tanti

40

A Firm's Long-Run Total Cost Curve: CRTS

case



Chayun Tanti

41

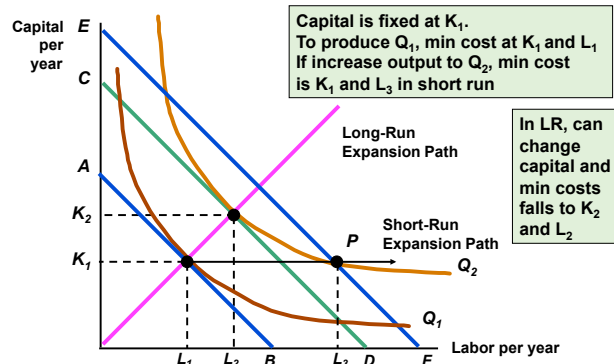
Long-Run Versus Short-Run Cost Curves

- In the short run some costs are fixed
- In the long run firm can change anything including plant size
 - Can produce at a lower average cost in long run than in short run
 - Capital and labor are both flexible
- We can show this by holding capital fixed in the short run and flexible in long run

Chayun Tanti

42

The Inflexibility of Short-Run Production



Chayun Tanti

43

Long-Run Versus Short-Run Cost Curves

- Long-Run Average Cost (LAC)
 - Most important determinant of the shape of the LR AC and MC curves is relationship between scale of the firm's operation and inputs required to min cost
- Constant Returns to Scale
 - If input is doubled, output will double
 - AC cost is constant at all levels of output.

Chayun Tanti

44

Long-Run Versus Short-Run Cost Curves



- Increasing Returns to Scale
 - If input is doubled, output will more than double
 - AC decreases at all levels of output.
- Decreasing Returns to Scale
 - If input is doubled, output will less than double
 - AC increases at all levels of output

Chayun Tanti

45

Long-Run Versus Short-Run Cost Curves

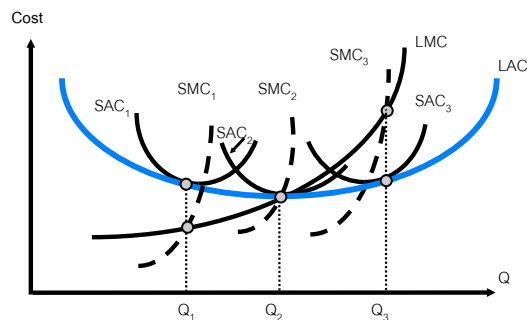


- In the long-run:
 - Firms experience increasing and decreasing returns to scale and therefore long-run average cost is “U” shaped.
 - Source of U-shape is due to returns to scale instead of diminishing marginal returns like the short run curve
 - Long-run marginal cost curve measures the change in long-run total costs as output is increased by 1 unit

Chayun Tanti

46

Short and Long-run Average Cost Curves



Chayun Tanti

47

Long-Run Versus Short-Run Cost Curves



- Long-run marginal cost leads long-run average cost:
 - If $LMC < LAC$, LAC will fall
 - If $LMC > LAC$, LAC will rise
 - Therefore, $LMC = LAC$ at the minimum of LAC
- In special case where LAC is constant, LAC and LMC are equal

Chayun Tanti

48

Long Run Costs

- **Economies of Scale:** a situation when LAC declines with a larger output due to
 - increasing returns to scale
 - On a larger scale, workers can better specialize
 - Firm can use more efficient machine
 - Lumpiness in investment
 - Scale can provide flexibility – managers can organize production more effectively
 - Firm may be able to get inputs at lower cost if can get quantity discounts. Lower prices might lead to different input mix

Chayun Tanti

49

Long Run Costs

- **Diseconomies of scale:** a situation when LAC increases with a larger output due to
 - Factory space and machinery may make it more difficult for workers to do their job efficiently
 - Managing a larger firm may become more complex and inefficient as the number of tasks increase
 - Bulk discounts can no longer be utilized. Limited availability of inputs may cause price to rise

Chayun Tanti

50

Long Run Costs

- Economies of scale are measured in terms of cost-output elasticity, E_C
- E_C is the percentage change in the cost of production resulting from a 1-percent increase in output

$$E_C = \frac{\Delta LTC / LTC}{\Delta Q / Q} = \frac{\Delta LTC / \Delta Q}{LTC / Q} = \frac{LMC}{LAC}$$

Chayun Tanti

51

Long Run Costs

- E_C is equal to 1, $LMC = LAC$
 - Costs increase proportionately with output
 - Neither economies nor diseconomies of scale
- $E_C < 1$ when $LMC < LAC$
 - Economies of scale
 - LAC are declining
- $E_C > 1$ when $LMC > LAC$
 - Diseconomies of scale
 - Both LMC and LAC are rising

Chayun Tanti

52

Production with Two Outputs: Economies of Scope

- Many firms produce more than one product and those product are closely linked
- Examples:
 - Chicken farm--poultry and eggs
 - Automobile company--cars and trucks
 - University--Teaching and research
 - Nation Group
 - Choke Chai Farm

Production with Two Outputs: Economies of Scope

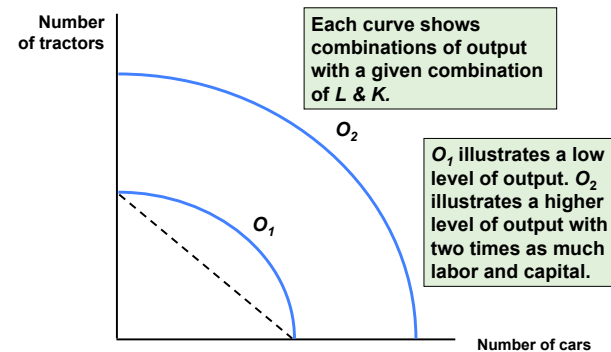
Advantages

- Both use capital and labor.
- The firms share management resources.
- Both use the same labor skills and type of machinery.

Production with Two Outputs: Economies of Scope

- Firms must choose how much of each to produce.
- The alternative quantities can be illustrated using **product transformation curves**
 - Curves showing the various combinations of two different outputs (products) that can be produced with a given set of inputs

Product Transformation Curve



Product Transformation Curve



- Product transformation curves are negatively slope
 - To get more of one output, must give up some of the other output
- Constant returns exist in this example
 - Second curve lies twice as far from origin as the first curve
- Curve is concave
 - Joint production has its advantages

Production with Two Outputs: Economies of Scope



- There is no direct relationship between economies of scope and economies of scale.
 - May experience economies of scope and diseconomies of scale
 - May have economies of scale and not have economies of scope

Dynamic Changes in Costs: The Learning Curve



- Firms may lower their costs not only due to economies of scope, but also due to managers and workers become more experienced at their jobs
- As management and labor gain experience with production, the firm's marginal and average costs may fall

Dynamic Changes in Costs: The Learning Curve



- Reasons
 - Speed of work increases with experience
 - Managers learn to schedule production processes more efficiently
 - More flexibility is allowed with experience. May include more specialized tools and plant organization
 - Suppliers become more efficient passing savings to company

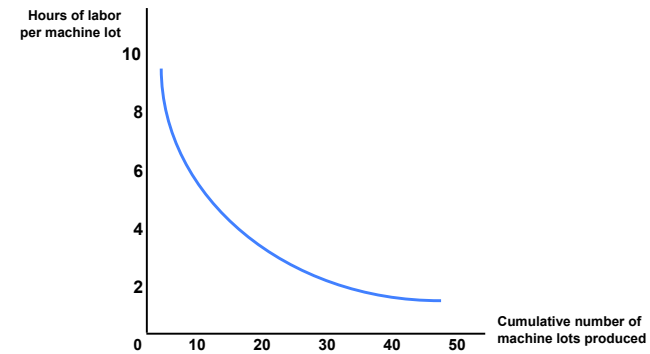
Dynamic Changes in Costs: The Learning Curve

- The **learning curve** measures the impact of worker's experience on the costs of production.
- It describes the relationship between a firm's cumulative output and amount of inputs needed to produce a unit of output.

Chayun Tanti

61

The Learning Curve



Chayun Tanti

62

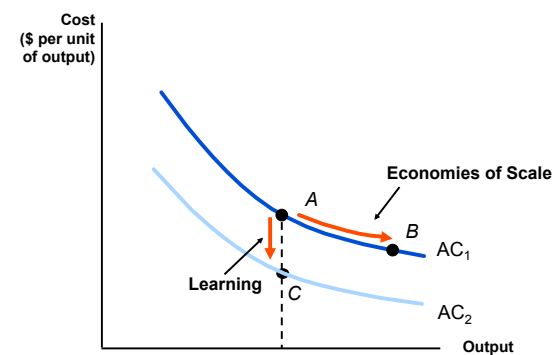
Dynamic Changes in Costs: The Learning Curve

- Observations
 - New firms may experience a learning curve, not economies of scale.
 - Should increase production of many lots regardless of individual lot size
 - Older firms have relatively small gains from learning.
 - Should produce its machines in very large lots to take advantage of lower costs associated with size

Chayun Tanti

63

Economies of Scale Versus Learning



Chayun Tanti

64