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EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.),2020

Use 4 decimal places for numerical answers

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

Table 1.a

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

$\bar{Y}_i = 3.2125$

$\bar{X}_i = 77.625$

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1. (Note: *NIID* = Normally, Identically, and Independently Distributed).

Find $\beta_1 = \dfrac{\sum (Y_i - \bar{Y})(X_i - \bar{X})}{\sum (X_i - \bar{X})^2}$ $\qquad \beta_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$

-0.4 -14.6 0.2 -5.6 -0.2 0.4 0.3 3.4 0.4 9.4 -0.2 -2.6 -0.5 -2.6 0.5 12.4

$$= \frac{(2.8-3.2)(63-77.6)+(3.4-3.2)(72-77.6)+(3-3.2)(78-77.6)+(3.5-3.2)(81-77.6)+(3.6-3.2)(87-77.6)+(3.0-3.2)(75-77.6)+(2.7-3.2)(75-77.6)+(3.7-3.2)(90-77.6)}{(63-77.6)^2+(72-77.6)^2+(78-77.6)^2+(81-77.6)^2+(87-77.6)^2+(75-77.6)^2+(75-77.6)^2+(90-77.6)^2}$$

$\hat{\beta}_1 = \dfrac{17.44}{511.84} = 0.03$ #

$\hat{\beta}_0 = 3.2 - 0.03(77.6)$

$\hat{\beta}_0 = 0.87$ #

$\hat{Y}_i =$?

1.2 For each observation i, find $\widehat{Y_i}$ and $\widehat{u_i}$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$\hat{Y}_1 = (0.87) + (0.03)(63) = 2.76$ $\qquad \hat{u}_1 = 0.44$

$\hat{Y}_2 = (0.87) + (0.03)(72) = 3.03$ $\qquad \hat{u}_2 = 0.17$

$\hat{Y}_3 = (0.87) + (0.03)(78) = 3.21$ $\qquad \hat{u}_3 = -0.01$

$\hat{Y}_4 = (0.87) + (0.03)(81) = 3.3$ $\qquad \hat{u}_4 = -0.1$

$\hat{Y}_5 = (0.87) + (0.03)(87) = 3.48$ $\qquad \hat{u}_5 = -0.28$

$\hat{Y}_6 = (0.87) + (0.03)(75) = 3.12$ $\qquad \hat{u}_6 = 0.08$

$\hat{Y}_7 = (0.87) + (0.03)(75) = 3.12$ $\qquad \hat{u}_7 = 0.08$

$\hat{Y}_8 = (0.87) + (0.03)(90) = 3.57$ $\qquad \hat{u}_8 = -0.37$

So, $\sum_{i=0}^{N} \hat{u}_i = 0.01 \approx 0$ #

$$SSR = \sum (\hat{Y} - \bar{Y})^2 \qquad SST = \sum_i (X_i - \bar{X})^2$$

1.3 Find $var(\widehat{u_i}), var(\widehat{\beta_0}), var(\widehat{\beta_1})$

$$Var(\hat{u}_i) = \hat{\sigma}^2 = \frac{SSR}{n-2} = \frac{1.02875}{8-2}$$

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$\bar{X} = 20$

$\bar{Y} = 9.1$

2. Data is listed in the table

X_i	Y_i	$Y_i - \bar{Y}$	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
10	0	-9.1	-10	100
12	2	-7.1	-8	64
14	5	-4.1	-6	36
16	6	-3.1	-4	16
18	7	-2.1	-2	4
22	10	0.9	2	4
24	10	0.9	4	16
26	15	5.9	6	36
28	16	6.9	8	64
30	20	10.9	10	100

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i, u_i \sim NIID(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

$$\beta_1 = \frac{\sum (Y_i - \bar{Y})(X_i - \bar{X})}{\sum (X_i - \bar{X})^2}$$

$$\beta_1 = \frac{(-9.1)(-10) + (-7.1)(-8) + (-4.1)(-6) + (-3.1)(-4) + (-2.1)(-2) + (0.9)(2) + (0.9)(4) + (5.9)(6) + (6.9)(8) + (10.9)(10)}{100 + 64 + 36 + 16 + 4 + 4 + 16 + 36 + 64 + 100}$$

$$\beta_1 = \frac{394}{440} = 0.89545455$$

$$Y = -8.8 + 0.8955 X_i$$

$$\beta_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$$

$$= 9.1 - 0.8955(20) = -8.81$$

$\hat{Y} = \beta_0 + \beta_1 x_i$

2.2 Find the value of $\widehat{Y}_i$ and $\widehat{u}_i$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$\hat{Y}_1 = -8.81 + 0.8955(10) = 0.155$

$\hat{Y}_2 \quad -8.81 + 0.8955(12) = 1.946$

$\hat{Y}_3 \quad -8.81 + 0.8955(14) = 3.737$

$\hat{Y}_4 \quad -8.81 + 0.8955(16) = 5.528$

$\hat{Y}_5 \quad -8.81 + 0.8955(18) = 7.319$

$\hat{Y}_6 \quad -8.81 + 0.8955(22) = 10.901$

$\hat{Y}_7 \quad -8.81 + 0.8955(24) = 12.692$

$\hat{Y}_8 \quad -8.81 + 0.8955(26) = 14.483$

$\hat{Y}_9 \quad -8.81 + 0.8955(28) = 16.274$

$\hat{Y}_{10} \quad -8.81 + 0.8955(30) = 18.065$

$\hat{u}_1 = -4.945 \quad \hat{u}_6 = 1.801$

$\hat{u}_2 = -7.154 \quad \hat{u}_7 = 3.592$

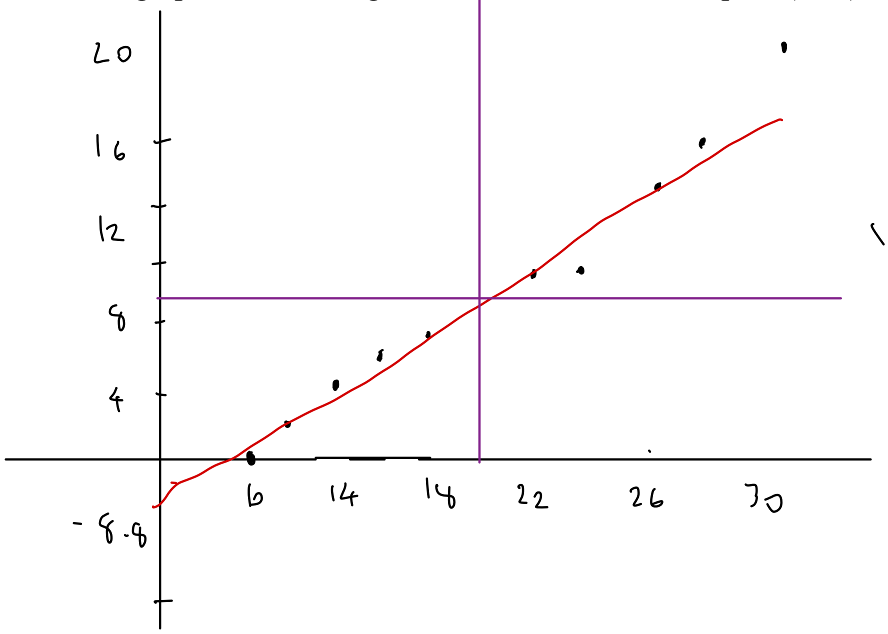
$\hat{u}_3 = -5.763 \quad \hat{u}_8 = 5.343$

$\hat{u}_4 = -3.572 \quad \hat{u}_9 = 7.174$

$\hat{u}_5 = -1.721 \quad \hat{u}_{10} = 4.965$

$\sum \hat{u}_i = 0$ #

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?



$\beta_0 + \beta_1 x_i$

$Y = -8.8 + 0.8556(20) = 9.1$

Pass #

2.4 If X_i =16, what is the predicted Y?

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2.5 Find $var(\widehat{u}_i)$, $var(\widehat{\beta_0})$, $var(\widehat{\beta_1})$

$SSR = \sum (\hat{Y} - \bar{Y})^2 = 366.9$

$SST = \sum (X - \bar{X})^2 = 440$

$Var(\hat{u}_i) = \frac{SSR}{n-2} = \frac{366.9}{10-8} = 45.8625$

$Var(\hat{\beta}_0) = \frac{\sigma^2 n^{-1} \sum_{i=1} x_i^2}{SST} = \frac{(45.8625)(4440)}{10(440)} = 46.2794$

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$Var(\hat{\beta}_1) = \frac{\sigma^2}{SST} = \frac{46.8625}{440} = 0.104233$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i \,,$$

Where $u_i \sim NIID(0, \sigma^2)$. Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$Y_i = \beta_1 X_i + u_i$

$\text{argmin} \; \Sigma_i (Y_i - \tilde{\beta}_1 X_i)^2$

F.O.C w.r.t. $\tilde{\beta}_1$

$$\frac{\partial \Sigma_i (Y_i - \tilde{\beta}_1 X_i)^2}{\partial \tilde{\beta}_1} = 0$$

$$\Sigma_i \, 2(Y_i - \tilde{\beta}_1 X_i)(-X_i) = 0$$

$$\Sigma_i X_i (Y_i - \tilde{\beta}_1 X_i) = 0 \; (-2)$$

$$\Sigma_i X_i Y_i - \Sigma_i \tilde{\beta}_1 X_i = 0$$

$$\Sigma_i X_i Y_i = \Sigma_i \tilde{\beta}_1 \hat{X}_i$$

$$\hat{\beta}_1 = \frac{\Sigma_i X_i Y_i}{\Sigma_i X_i^2}$$

Substitute $Y_i = \beta_1 X_i + u_i$

$$\beta_1 = \frac{\Sigma_i X_i (\beta_1 X_i + u_i)}{\Sigma_i X_i^2}$$

$$\bar{\beta}_1 = \frac{\Sigma_i \beta_1 X_i^2}{\Sigma_i X_i^2} + \frac{\Sigma_i u_i X_i}{\Sigma_i X_i^2}$$

$E(\bar{\beta}_1) = E(\beta_1) + E\left(\frac{\Sigma u_i X_i}{\Sigma_i X_i^2}\right)$ by SLR4 $E(u_2 | X_2) = 0$

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$E(\bar{\beta}_1) = \beta_1$