EE 320 Introductory Mathematical Economics

Semester 2/2012

Problem Set 6

Derivatives of More-Than-One Independent Variable Function[1]

1. The demand for money, M, in the United States for the period 1929-1952 has been estimated as

$$M = 0.14Y + 76.03(r-2)^{-0.84}, \qquad (r > 2)$$

where Y is the annual national income, and r is the interest rate measured in percent per year. Find $\partial M/\partial Y$ and $\partial M/\partial r$ and discuss their signs.

2. The demand for a product depends on the price p of the product and on the price q charged by a competing producer

$$D(p,q) = a - bpq^{-\alpha}$$

where a, b, and α are positive constants with $\alpha < 1$. Find $D'_p(p,q)$ and $D'_q(p,q)$, and comments on the signs of the partial derivatives.

3. Let x and y be the populations of two cities and d the distance between them. Suppose that the number of travelers T between the cities is given by

$$T = kxy/d^n \qquad \text{(k and n are positive constants.)}$$

Find $\partial T/\partial x$, $\partial T/\partial y$, and $\partial T/\partial d$, and discuss their signs.

4. Let $D(p, m)$ indicate a typical consumer's demand for a particular commodity, as a function of its price p and the consumer's own income m. Show that the proportion pD/m of income spent on the commodity increases with income if $\varepsilon_m > 1$ (in which case the good is a "luxury" whereas it is a "necessity" if $\varepsilon_m < 1$).

5. The annual herring catch is given by the function $Y(K,S) = 0.06157K^{1.356}S^{0.562}$ of the catching effort (K) and the herring stock (S).

   (a) Find $\partial Y/\partial K$ and $\partial Y/\partial S$.

   (b) If K and S are both doubled, what happens to the catch?

6. Suppose that a firm produces $Q = f(L) = \sqrt{L}$ units of commodity using L units of labor. Assume that $f'(L) > 0$ and $f''(L) < 0$, so f is strictly increasing and strictly concave.

[1] All questions in this problem set are from Sydsaeter and Hammond, 2008.

(a) If the firm gets P baht per unit produced and pays w baht for a unit of labor, write down the profit function, and find the first-order condition for profit maximization at $L^* > 0$.

(b) By implicit differentiation of the first-order condition, examine how changes in P and w influence the optimal choice of L* (i.e. find $\partial L^*/\partial P$ and w).

7. The Nerlove-Ringstad production function $y = y(K, L)$ is defined implicitly by

$$y^{1+c\ln y} = AK^{\alpha}L^{\beta}$$

where A, α, and β are positive constants. Find the marginal products of y w.r.t both K (capital) and L (labor) – that is, find $\partial y/\partial K$ and $\partial y/\partial L$. (Hint: Take the logarithm of each side and then differentiate implicitly).

8. Suppose production X depends on the number of workers N according the formula

$$X = Ng(\frac{\varphi(N)}{N})$$

where g and φ are given differentiable functions. Find expressions for dX/dN and d^2X/dN^2.

9. Find the marginal rate of substitution (MRS) between y and x when:

(a) $U(x, y) = 2x^{0.4}y^{0.6}$

(b) $U(x, y) = xy + y$

(c) $U(x, y) = 10(x^{-2} + y^{-2})^{-4}$

10. An equilibrium model of labor demand and output pricing leads to the following system of equations:

$$pF'(L) - w = 0$$
$$pF(L) - wL - B = 0 \qquad (*)$$

Suppose that F is twice differentiable with $F'(L) > 0$ and $F''(L) < 0$, and all the variables are positive. Treat w and B as exogenous, so that p and L are endogenous variables which are functions of w and B.

(a) Find expressions for $\partial p/\partial w$, $\partial p/\partial B$, $\partial L/\partial w$, and $\partial L/\partial B$ by implicit differentiation.
(b) What can be said about the signs of these partial derivatives? Show that $\partial L/\partial w < 0$.