

# Time Series

## EE426 Semester 2/2014

Part 1: OLS with time series data

Chayanee Chawanote

# Overview

- ▶ Stochastic Processes
  - ▶ Stationary processes
  - ▶ Non-stationary processes
- ▶ Concept of Basic Time Series
  - ▶ Unit Root test
  - ▶ Spurious regression
  - ▶ Cointegration
  - ▶ Forecasting (if time allows)

## Example of time series model

- ▶ A static model: look at relationship between contemporaneous variables (same time period)

$$y_t = \beta_0 + \beta_1 x_t + u_t$$

- ▶ A finite distributed lag (FDL) model: an independent variable in other time periods also determine the dependent variable

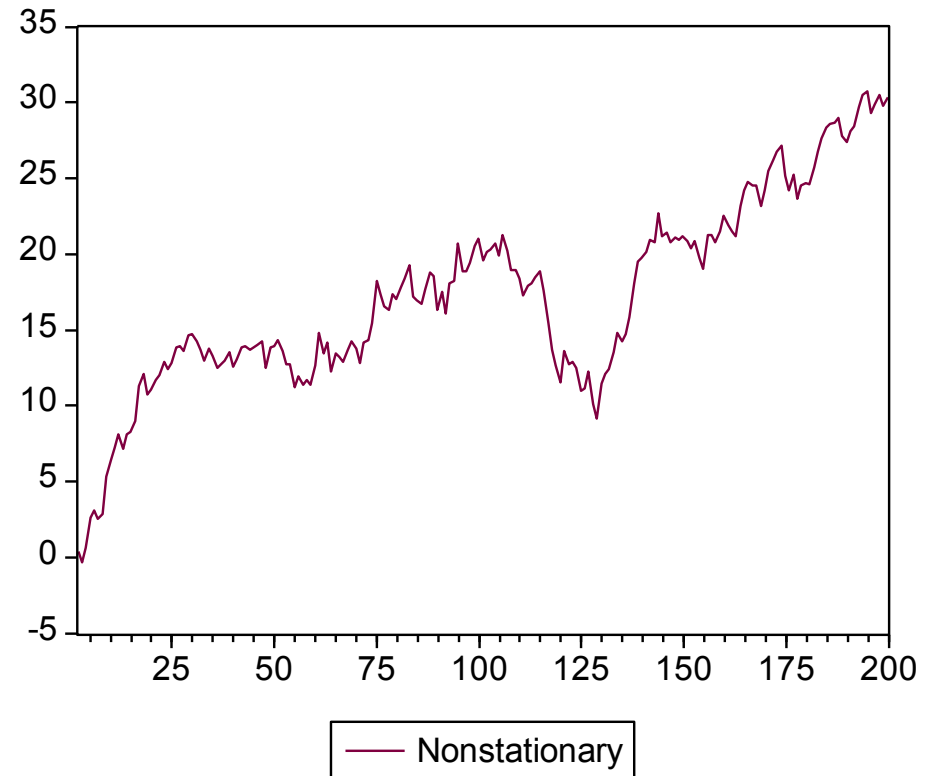
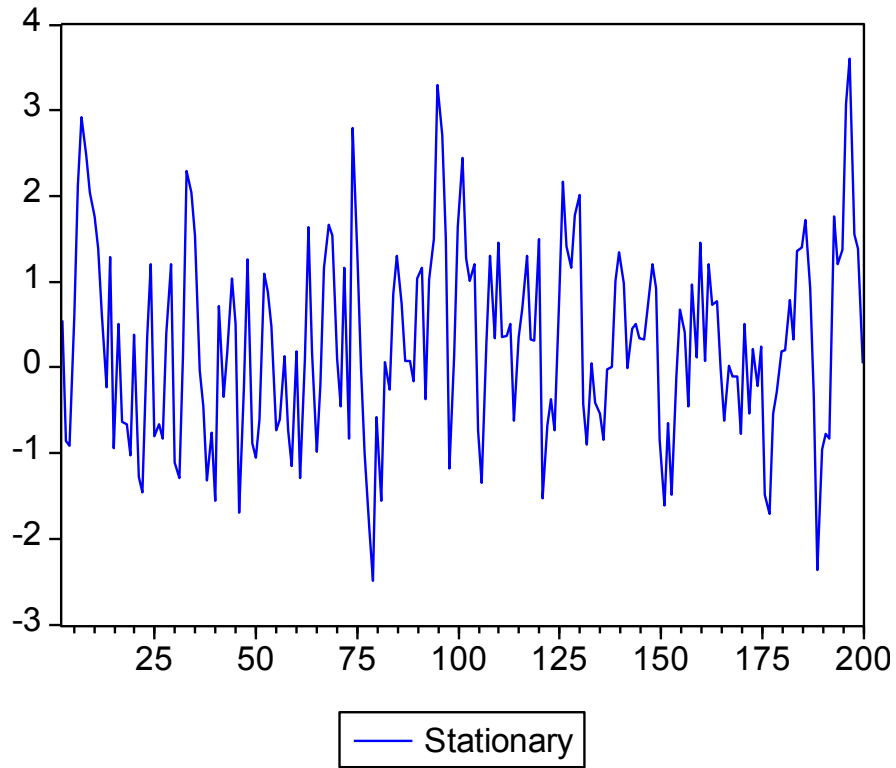
$$y_t = \alpha_0 + \delta_0 x_t + \delta_1 x_{t-1} + \delta_2 x_{t-2} + u_t$$

- ▶  $\delta_0$  - impact propensity, reflecting the immediate change in  $y$
- ▶  $\delta_1$  - effect from  $x$  in the last period that determine  $y$  in the current period
- ▶  $\delta_0 + \delta_1 + \dots + \delta_q$  is long-run propensity (LRP) measures the long-run change in the expected value of  $y$  given a one-unit, permanent increase in  $x$ .

# Stochastic processes

- ▶ OLS assumes that all explanatory variables are nonstochastic process variables
- ▶ Stochastic process: a collection of random variables ordered in time
- ▶ The probability structure of a sequence of random variables is determined by the joint distribution of a stochastic process
  - ▶ Stationary stochastic process: probability distribution is stable over time
  - ▶ Non-stationary stochastic process: otherwise - unstable

# Graphical Analysis



# Stationary processes

- ▶ Any set of values will have the same joint distribution as any other set of values measured at a different point in time.
  - ▶ The stationary process is identically distributed, with the following properties
    - ▶ Mean of series must be stationary:  $E(y_t) = \mu$
    - ▶ Variance of series must be stationary:  $Var(y_t) = \sigma^2$
    - ▶ Covariance of series must be stationary:  
 $Cov(y_t, y_{t+s}) = E(y_t - \mu)(y_{t+s} - \mu) = \gamma$
- (weakly dependent:  $Corr(y_t, y_{t+s}) \rightarrow 0$  and  $s \rightarrow \infty$ )

## Moving average process (MA)

- ▶ MA is an example of a weakly dependent series.
- ▶ Form for MA(1):  $y_t = u_t + \alpha_1 u_{t-1}$ ,  $t = 1, 2, \dots$
- ▶  $u_t \sim iid(0, \sigma_u^2)$ ,  $cov(u_t, u_{t+s}) = 0$
- ▶  $y$  is a weighted average of  $u_t$  and  $u_{t-1}$
- ▶  $Cov(y_t, y_{t-1}) = \alpha_1 \sigma_u^2$ ,  $Corr(y_t, y_{t-1}) = \frac{\alpha_1}{1 + \alpha_1^2}$
- ▶ MA(1) has a finite memory of one period: the last period matters, but observations prior to that have no effect on the current value of the process.

## Autoregressive process (AR)

- ▶ Form for AR(1):  $y_t = \rho_1 y_{t-1} + u_t, t = 1, 2, \dots$
- ▶  $u_t \sim iid(0, \sigma_u^2), cov(u_t, u_{t+s}) = 0$
- ▶  $y$  will be a stable stochastic process if  $|\rho_1| < 1$ .
- ▶ AR(1) process has infinite memory: every past value of the process affects the current value. As the effects of past values are weighted by powers of fraction  $\rho_1$ , those effects damp to zero.
- ▶ Under covariance stationarity and  $\rho_1^2 < 1, \sigma_y^2 = \frac{\sigma_u^2}{1-\rho_1^2}$
- ▶  $Cov(y_t, y_{t+s}) = \rho_1^s \sigma_y^2, Corr(y_t, y_{t+s}) = \rho_1^s$

# OLS regression with time series

- ▶ TS1: Linearity and weak dependence
  - ▶ Assume that a set of variables  $[\mathbf{x}, y]$  is stationary and weakly dependent
- ▶ TS2: No perfect collinearity
- ▶ TS3: Zero conditional mean
  - ▶  $\mathbf{x}_t = (x_{1t}, x_{2t}, \dots, x_{kt})$  are contemporaneously exogenous,  $E[u_t | \mathbf{x}_t] = 0$
  - ▶ This conditional mean does not rule out correlation between, say,  $u_{t-1}$  and  $x_{1t}$
- ▶ Under TS1-3, OLS estimators are consistent, but not necessarily unbiased.

# OLS regression with time series

- ▶ To use standard inference procedures, we need the following assumptions:
- ▶ TS4: Homoskedasticity
  - ▶ The errors are contemporaneously homoskedastic,  
$$\text{Var}(u_t | \mathbf{x}_t) = \sigma^2$$
- ▶ TS5: No serial correlation
  - ▶ For all  $t \neq s$ ,  $E(u_t, u_s | \mathbf{x}_t, \mathbf{x}_s) = 0$
- ▶ It's quite difficult to guarantee that unobservables  $u_t$  are uncorrelated over time.
- ▶ Under TS1-5, OLS estimators are asymptotically normally distributed. The usual OLS standard errors, t and F stats are asymptotically valid.

# Highly persistent time series

- ▶ This means 'strongly dependent': time-varying mean or time-varying variance  $\rightarrow$  nonstationary processes
- ▶ For example, AR(1) with  $\rho_1 = 1$ : Random walk model
  - ▶ Random walk without drift:  $y_t = y_{t-1} + u_t$   
 $E(y_t) = E(y_0 + \sum u_t) = y_0$ ,  $Var(y_t) = t\sigma^2$
  - ▶ Random walk with drift:  $y_t = \delta + y_{t-1} + u_t$   
 $E(y_t) = y_0 + t\delta$ ,  $Var(y_t) = t\sigma^2$

# Highly persistent time series

- ▶ Highly persistent: value of  $y$  today is important for determining the value of  $y$  in the very distant future
  - ▶  $E(y_{t+s}|y_t) = y_t, \forall s \geq 1$
- ▶ A random walk is a special case of what's known as a unit root process (a root of the polynomial)
- ▶ Trending and persistence are different things - a series can be trending but weakly dependent, or a series can be highly persistent without any trend.
- ▶ A random walk with drift is an example of a highly persistent series that is trending.