CHAPTER 9

Interest Rate Risk II

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Overview

- This chapter discusses a market value-based model for managing interest rate risk, the duration gap model
  - Duration
  - Computation of duration
  - Economic interpretation
  - Immunization using duration
  - Problems in applying duration

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Price Sensitivity and Maturity

- In general, the longer the term to maturity, the greater the sensitivity to interest rate changes
- The longer maturity bond has the greater drop in price because the payment is discounted a greater number of times

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Duration

- Duration
  - Weighted average time to maturity using the relative present values of the cash flows as weights
  - More complete measure of interest rate sensitivity than is maturity
  - The units of duration are years
  - To measure and hedge interest rate risk, FI should manage duration gap rather than maturity gap

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Macaulay's Duration

$$D = \frac{\sum_{t=1}^{N} CF_t \times DF_t \times t}{\sum_{t=1}^{N} CF_t \times DF_t} = \frac{\sum_{t=1}^{N} PV_t \times t}{\sum_{t=1}^{N} PV_t}$$

where

D = Duration measured in years

CF_t = Cash flow received at end of period t

N= Last period in which cash flow is received

DF_t = Discount factor = $1/(1+R)^t$

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Duration

- Since the price (P) of the bond equals the sum of the present values of all its cash flows, we can state the duration formula another way:

$$D = \frac{\sum_{t=1}^{N} PV_t \times t}{P}$$

- Notice the weights correspond to the *relative* present values of the cash flows

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Semiannual Cash Flows

- For semiannual cash flows, Macaulay's duration, D, is equal to:

$$D = \frac{\displaystyle\sum_{t=1/2}^{N} \frac{CF_t \times t}{(1 + R/2)^{2t}}}{\displaystyle\sum_{t=1/2}^{N} \frac{CF_t}{(1 + R/2)^{2t}}}$$

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Duration of Zero-Coupon Bond

- **Zero-coupon bonds**: sell at a discount from face value on issue, pay the face value upon maturity, and have no intervening cash flows between issue and maturity
- Duration equals the bond's maturity since there are no intervening cash flows between issue and maturity
- For all other bonds, duration < maturity because here are intervening cash flows between issue and maturity

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Duration of Consol Bonds

- A bond that pays a fixed coupon each year indefinitely
- Have yet to be issued in the U.S.
- Maturity of a consol (perpetuity):

  $M_c = \infty$
- Duration of a consol (perpetuity):

  $D_c = 1 + 1/R$

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Features of Duration

- Duration and maturity
  - Duration increases with maturity of a fixed-income asset/liability, but at a decreasing rate
- Duration and yield
  - Duration decreases as yield increases
- Duration and coupon interest
  - Duration decreases as coupon increases

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Economic Interpretation

- Duration is a direct measure of interest rate sensitivity, or elasticity, of an asset or liability:

$$[\Delta P/P] \div [\Delta R/(1+R)] = -D$$

- Or equivalently,

$$\Delta P/P = -D[\Delta R/(1+R)] = -MD\,dR$$

where MD is modified duration

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Economic Interpretation Continued

- To estimate the change in price, we can rewrite this as:

$$\Delta P = -D[\Delta R/(1+R)]P = -(MD) \times (\Delta R) \times (P)$$

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Dollar Duration

- Dollar value change in the price of a security to a 1 percent change in the return on the security

$$\text{Dollar duration} = MD \times \text{Price}$$

- Using dollar duration, we can compute the change in price as

$$\Delta P = -\text{Dollar duration} \times \Delta R$$

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Semi-annual Coupon Bonds

- With semi-annual coupon payments, the percentage change in price is calculated as:

$$\Delta P/P = -D[\Delta R/(1+(R/2)]$$

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Immunization

- Matching the *maturity* of an asset with a future payout responsibility does not necessarily eliminate interest rate risk
- Matching the *duration* of a fixed-interest rate instrument (i.e., loan, mortgage, etc.) to the FI's target or investment horizon will immunize the FI against shocks to interest rates

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Balance Sheet Immunization

- Duration gap is a measure of the interest rate risk exposure for an FI
- If the durations of liabilities and assets are not matched, then there is a risk that adverse changes in the interest rate will increase the present value of the liabilities more than the present value of assets is increased

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Immunizing the Balance Sheet of an FI

- Duration Gap:
  - From the balance sheet, A = L+E, which means E = A-L. Therefore, $\Delta E = \Delta A - \Delta L$.
  - In the same manner used to determine the change in bond prices, we can find the change in value of equity using duration.

$$\Delta E = -[D_A - D_L k]A(\Delta R/(1+R))$$

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Duration and Immunizing

- The formula, ΔE, shows 3 effects:
  - Leverage adjusted duration gap
  - The size of the FI
  - The size of the interest rate shock

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Example 9-9

- Suppose D_A = 5 years, D_L = 3 years and rates are expected to rise from 10% to 11%. (Thus, rates change by 1%). Also, A = 100, L = 90, and E = 10. Find ΔE.

$$\Delta E = -[D_A - D_L k]A(\Delta R/(1+R))$$
$$= -[5 - 3(90/100)]100[.01/1.1] = -\$2.09.$$

- Methods of immunizing balance sheet.
  - Adjust D_A, D_L or k.

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Immunization and Regulatory Considerations

- Regulators set target ratios for an FI's capital (net worth) to assets in an effort to monitor solvency and capital positions:
  - Capital (Net worth) ratio = E/A
- If target is to set $\Delta(E/A) = 0$:
  - $D_A = D_L$
- But, to set $\Delta E = 0$:
  - $D_A = kD_L$

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Difficulties in Applying Duration Model

- Duration matching can be costly
  - Growth of purchased funds, asset securitization, and loan sales markets have lowered costs of balance sheet restructurings
- Immunization is a dynamic problem
  - Trade-off exists between being perfect immunization and transaction costs
- Large interest rate changes and convexity

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Convexity

- The degree of curvature of the price-yield curve around some interest rate level
- Convexity is desirable, but greater convexity causes larger errors in the duration-based estimate of price changes

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Basics of Bond Valuation

- Formula to calculate present value of bond:

$$P = \frac{C/m}{(1+i_d/m)^1} + \frac{C/m}{(1+i_d/m)^2} + \cdots + \frac{C/m}{(1+i_d/m)^{Nm}} + \frac{F}{(1+i_d/m)^{Nm}}$$

$$= \frac{C}{m}\sum_{t=1}^{Nm}\left(\frac{1}{1+i_d/m}\right)^t + \frac{F}{(1+i_d/m)^{Nm}}$$

$$= \frac{C}{m}(PVA_{i_d/m,Nm}) + F(PV_{i_d/m,Nm})$$

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Impact of Maturity on Security Values

- Price sensitivity is the percentage change in a bond's present value for a given change in interest rates
- Relationship between bond price sensitivity and maturity is not linear
- As time remaining to maturity on bond increases, price sensitivity increases at decreasing rate

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Incorporating Convexity into the Duration Model

- Three characteristics of convexity:
  - Convexity is desirable
  - Convexity and duration
  - All fixed-income securities are convex

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Modified Duration & Convexity

$$\Delta P/P = -D[\Delta R/(1+R)] + (1/2)\ CX\ (\Delta R)^2, \text{ or}$$

$$\Delta P/P = -MD\ \Delta R + (1/2)\ CX\ (\Delta R)^2$$

– Where MD implies modified duration and CX is a measure of the curvature effect

– CX = Scaling factor × [capital loss from 1bp rise in yield + capital gain from 1bp fall in yield]

– Commonly used scaling factor is 10^8

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Calculation of CX

- Example: convexity of 8% coupon, 8% yield, six-year maturity Eurobond priced at $1,000

$$CX = 10^8[(\Delta P^-/P) + (\Delta P^+/P)]$$

$$= 10^8[(999.53785\text{-}1{,}000)/1{,}000 + (1{,}000.46243\text{-}1{,}000)/1{,}000)]$$

$$= 28$$

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Contingent Claims

- Interest rate changes also affect value of (off-balance sheet) derivative instruments
  - Duration gap hedging strategy must include the effects on off-balance sheet items, such as futures, options, swaps, and caps, as well as other contingent claims

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Pertinent Websites

Bank for International Settlements	www.bis.org
Securities Exchange Commission	www.sec.gov
The Wall Street Journal	www.wsj.com

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