Approximation of change of y as a result of change of x

at $B = (2, 14)$

- When $x_1 = 2, y_1 = 14$. If $\Delta x = 0.1$, we can approximate

differential ← speed / slope keep changing

$$\Delta y \approx f'(x_1).\Delta x$$
$$= f'(2).0.1$$
$$= \underbrace{2(2)}_{\text{slope}} \cdot 0.1 = 0.4 \text{ km.}$$

- What is the real Δy?

$x_1 = 2 + 0.1 = 2.1$

$$y_2 = f(2.1) = 10 + (2.1)^2 = 14.41$$
$$\Delta y = y_2 - y_1 = 14.41 - 14 = 0.41 \rightarrow \text{actual change in } y$$

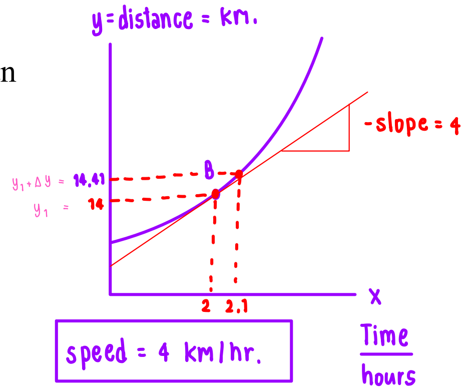
- We underestimate the real change of y.
- What if $\Delta x = -0.2$? Approximate the change of y.

$$\Delta y \approx f'(x) \cdot \Delta x$$
$$= (-0.2) \cdot (4)$$
$$= -0.8$$

Example $y = 10 + x^2$

Point	x	y	slope
	0	10	0
A	1	11	2
B	2	14	4
C	3	19	6

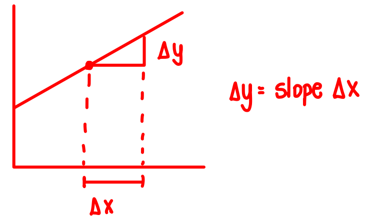
$f'(x) = 2x$



$\Delta x = 0.1$ hr. ⇒ 6 minutes

\# No exact approximation

\# constant speed / slope → exact approximation



$\Delta y = \text{slope } \Delta x$

a) $f(x) = y$

$f(x) = 10 + x^{\frac{1}{2}}$

$f'(x) = 0 + \frac{1}{2} x^{-\frac{1}{2}}$

$f'(x) = \frac{1}{2\sqrt{x}}$ #

HW Given $y = 10 + \sqrt{x}$,

a) Find the derivative $f'(x)$.

b) Fill in the table

$f'(x) = \frac{1}{2\sqrt{x}}$

Point	X	Y	$f'(x)$
	0	10	DNE
A	1	11	$\frac{1}{2} = 0.5$
B	2	1.414 14	$\frac{1}{2\sqrt{2}} = 0.35$
C	3	1.732 19	$\frac{1}{2\sqrt{3}} = 0.29$

c) Does the slope increase as x increase? No, the slope decreases as x increases.

d) Approximate the change in Y when $\Delta x = 0.2$ at $x_1 = 3$. Is the approximation under- or over-estimate?

$$\frac{\Delta y}{\Delta x} \approx f'(x)$$
$$\Delta y \approx f'(3) \cdot \Delta x$$
$$\Delta y \approx \frac{1}{2\sqrt{3}} \cdot 0.2 \approx 0.0577 \approx 0.058$$

Approximation = 11.79

Actual Change

$f(3.2) = 10 + \sqrt{3.2}$

$= 11.789$

Actual Change = 11.789

∴ The approximation is overestimate.

Note: If the function $f(x)$ is linear, the approximation is exact.