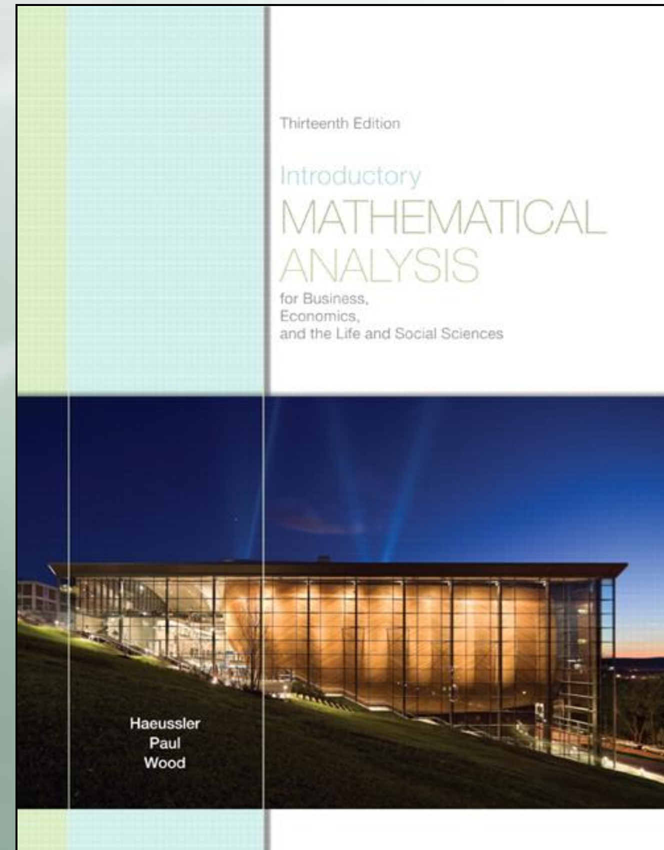
INTRODUCTORY MATHEMATICAL ANALYSIS

For Business, Economics, and the Life and Social Sciences

Chapter 1
Applications and More Algebra



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Chapter Objectives

- To model situations described by linear or quadratic equations.
- To solve linear inequalities in one variable and to introduce interval notation.
- To model real-life situations in terms of inequalities.
- To solve equations and inequalities involving absolute values.
- To write sums in summation notation and evaluate such sums.

Chapter Outline

1.1) Applications of Equations

1.2) Linear Inequalities

1.3) Applications of Inequalities

1.4) Absolute Value

1.5) Summation Notation

1.1 Applications of Equations

- **Modeling:** Translating relationships in the problems to mathematical symbols.

Example 1 - Mixture

A chemist must prepare 350 ml *of a chemical solution made up of two parts alcohol and three parts acid. How much of each should be used?*

Example 1 - Mixture

Solution:

In the next example, we refer to some business terms relative to a manufacturing firm.

- **Fixed cost** is the sum of all costs that are independent of the level of production.
- **Variable cost** is the sum of all costs that are dependent on the level of output.
- **Total cost** = variable cost + fixed cost
- **Total revenue** = (price per unit) x (number of units sold)
- **Profit** = total revenue − total cost

Example 3 – Profit

The Anderson Company produces a product for which the variable cost per unit is \$6 *and the fixed cost is* \$80,000. *Each unit has a selling price of* \$10. *Determine the number of units that must be sold for the company to earn a profit of* \$60,000.

Solution:

Example 5 – Investment

A total of \$10,000 *was invested in two business ventures, A and B. At the end of the first year, A and B yielded returns of* 6%*and* 5.75 %, *respectively, on the original investments. How was the original amount allocated if the total amount earned was* \$588.75?

Example 5 – Investment

Solution:

Example 7 – Apartment Rent

A real-estate firm owns the Parklane Garden Apartments, which consist of 96 apartments. At $550 *per month, every apartment can be rented. However, for each* $25 *per month increase, there will be three vacancies with no possibility of filling them. The firm wants to receive* $54,600 *per month from rent. What rent should be charged for each apartment?*

Example 7 – Apartment Rent

Solution 1:

Let r = rent ($) to be charged per apartment.

Total rent = (rent per apartment) x
(number of apartments rented)

Example 7 – Apartment Rent

Solution 2:

Let n = number of $25 increases.

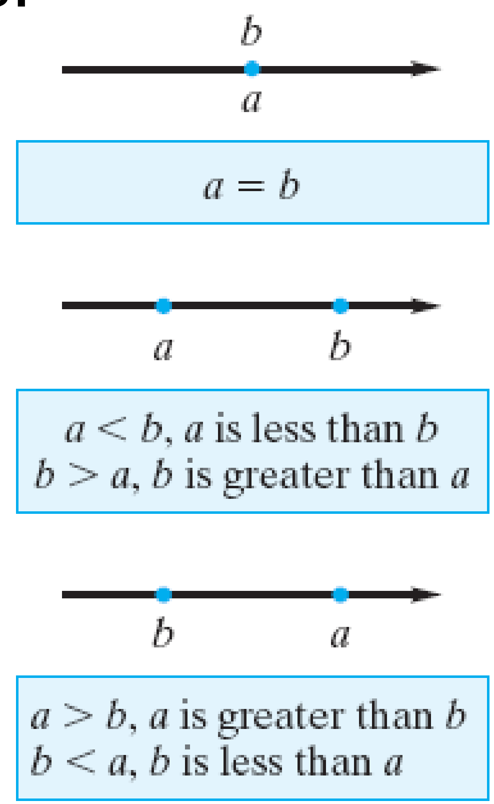
Total rent = (rent per apartment) x
(number of apartments rented)

1.1 Applications of Equations

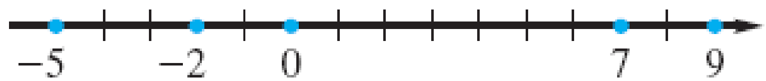
Example 7 – Apartment Rent

1.2 Linear Inequalities

- Supposing *a* and *b* are two points on the real-number line, the relative positions of two points are as follows:

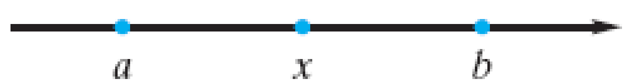


- We use dots to indicate points on a number line.



- Suppose that $a < b$ and x is between a and b.

$$\boxed{a < x < b}$$



- **Inequality** is a statement that one number is less than another number.

- Rules for Inequalities:

1. If $a < b$, then $a + c < b + c$ and $a - c < b - c$.
2. If $a < b$ and $c > 0$, then $ac < bc$ and $a/c < b/c$.
3. If $a < b$ and $c < 0$, then $a(c) > b(c)$ and $a/c > b/c$.
4. If $a < b$ and $a = c$, then $c < b$.
5. If $0 < a < b$ or $a < b < 0$, then $1/a > 1/b$.
6. If $0 < a < b$ and $n > 0$, then $a^n < b^n$.

   If $0 < a < b$, then $\sqrt[n]{a} < \sqrt[n]{b}$.

- **Linear inequality** can be written in the form

$$ax + b < 0$$

where a and b are constants and $a \neq 0$

- To **solve** an inequality involving a variable is to find all values of the variable for which the inequality is true.

Example 1 – Solving a Linear Inequality

Solve $2(x - 3) < 4$.

Solution:

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Example 3 – Solving a Linear Inequality

Solve $(3/2)(s - 2) + 1 > -2(s - 4)$.

Solution:

1.3 Applications of Inequalities

- Solving word problems may involve inequalities.

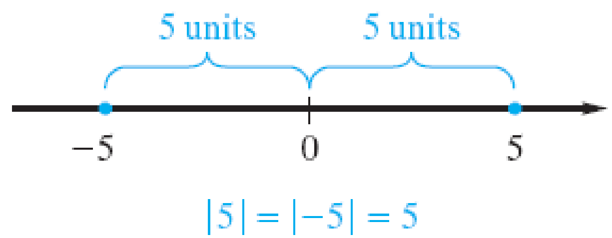
Example 1 - Profit

For a company that manufactures aquarium heaters, the combined cost for labor and material is \$21 *per heater. Fixed costs (costs incurred in a given period, regardless of output) are* \$70,000. *If the selling price of a heater is* \$35, *how many must be sold for the company to earn a profit?*

Solution:

1.4 Absolute Value

- On real-number line, the distance of *x* from 0 is called the **absolute value** of *x*, denoted as $|x|$.



DEFINITION

The **absolute value** of a real number *x*, written $|x|$, is defined by

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

Example 1 – Solving Absolute-Value Equations

a. *Solve* $|x - 3| = 2$

b. *Solve* $|7 - 3x| = 5$

c. *Solve* $|x - 4| = -3$

Example 1 – Solving Absolute-Value Equations

Solution:

Absolute-Value Inequalities

- Summary of the solutions to absolute-value inequalities is given.

Inequality ($d > 0$)	Solution
$\lvert x \rvert < d$	$-d < x < d$
$\lvert x \rvert \le d$	$-d \le x \le d$
$\lvert x \rvert > d$	$x < -d$ or $x > d$
$\lvert x \rvert \ge d$	$x \le -d$ or $x \ge d$

Example 3 – Solving Absolute-Value Equations

a. *Solve* $|x + 5| \geq 7$

b. *Solve* $|3x - 4| > 1$

Solution:

Example 4 : Absolute-Value Notation

Using absolute-value notation, express the following statements:

a. *X* isless than 3 units from 5.

b. X differ from 6 by at least 7.

c. X<3 and X> -3 simultaneously.

d. X is more than 1 unit from -2

e. X is less than σ units from μ.

Properties of the Absolute Value

- 5 basic properties of the absolute value:

1. $|ab| = |a| \cdot |b|$
2. $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}$
3. $|a-b| = |b-a|$
4. $-|a| \le a \le |a|$
5. $|a+b| \le |a| + |b|$

- Property 5 is known as *the triangle inequality*.

1.5 Summation Notation

DEFINITION

The sum of the numbers a_i, with i successively taking on the values m through n is denoted as

$$\sum_{i=m}^{n} a_i = a_m + a_{m+1} + a_{m+2} + \ldots + a_n$$

Example 1 – Evaluating Sums

Evaluate the given sums.

a. $\sum_{n=3}^{7}(5n-2)$

b. $\sum_{j=1}^{6}(j^2+1)$

Solution:

a.

b.

- To sum up consecutive numbers, we have

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

where n = the last number.

Example 3 – Applying the Properties of Summation Notation

Evaluate the given sums.

a. $\sum_{j=30}^{100} 4$ b. $\sum_{k=1}^{100} (5k+3)$ c. $\sum_{k=1}^{200} 9k^2$

Solution: