Discussion handout 3: *EE320 Semester 1/2017*

Keywords: Derivative; Differential; Single-variable Optimization

Question 1: *Taylor's approximation*

Consider a two-year government bond with the following pricing equation,

$$P = \frac{CF}{1+r} + \frac{CF+FV}{(1+r)^2}$$

where $CF = \frac{c}{100} * FV$ denotes the amount of coupon offered by the two-year bond.

Following the bond pricing formula, one knows that if c = r then P would be equal to FV. Now consider the following questions.

1.1) For the given level of CF and FV, how does "r" affect "p"? Use the derivative concept the calculate the rate of change of price with respect to "r".

1.2) Suppose that c = 3% and r = 3%. The two-year bond has its face value (FV) equal to $100. What is the new level of bond price when r has increased from 3% to 4%.

1.3) Use the first-order differential concept and derive the approximate change in price when r has changed from 3% to 4%. Compare your answer with the answer that you obtained in (1.2)

1.4) Redo 1.3 by using the second-order differential concept under the same change in the value of r (3% to 4%).

Question 2: Given firm i, a perfect competitor in a labor maker and in an output market, with a short-run production function $Q_i = -L_i^3 + 6L_i^2 + 2L_i$ where Q_i is the firm's output (in hundreds of units per day) and L_i is the labor employed by the firm (in hundreds of labor-hours per day.)

Assume that the market demand supply schedules of labor are:

$$L^d = 770 - 40w \text{ and } L^S = -100 + 20w$$

where L^d and L^S are the market quantities of labor demanded and supplied (in hundreds of thousands of labor-hours per day) and w is the market wage (dollars per hour).

Assume that the market demand and supply schedules for the product are:

$$Q^d = 50 - 20P \text{ and } Q^S = -20 + 15P$$

where Q^d and Q^S are the market quantities of output demanded and supplied (in hundreds of thousands of labor-hours per day) and w is the market wage (dollars per hour).

1.1) Derive the firm's average product of labor (AP_L) and marginal product of labor (MP_L)
1.2) Find the level of employment where diminishing returns to labor first set in for the firm i.
1.3) Show that $MP_L = AP_L$ at the minimum of (AP_L)
1.4) Determine the level of profit-maximizing employment for firm "i" in the short-run and the level of profit-maximizing output produced by the firm.
1.5) Suppose that the market supply of labor fell to $L^S = -310 + 20w$, redo the question 1.4
1.6) Suppose that the market demand of output increased to $Q^D = 85 - 20P$, redo the question 1.4