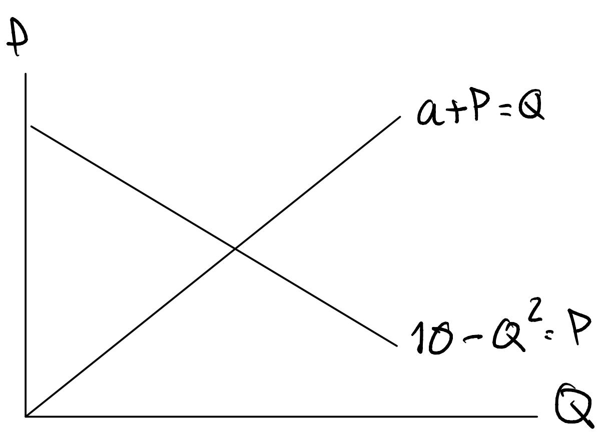
EE320 Placement test

1. Attempt all.
2. Submit your work (in .pdf) on the Moodle. The required format of your filename is studentID_PT
3. You will get **TWO** bonus points if you submit this placement test by the deadline.
4. **This placement test is due on Friday 14th, at 11 AM. Late submission will not be accepted.**

1. Suppose that market demand is given by $P = 10 - Q^2$ and the market supply is given by $Q = a + P$, where P is the unit price, Q is the quantity of output, and a is the coefficient in the supply equation.
   1.1) Graph the market demand and market supply curve in a P-Q diagram. Set the value of a equal to – 14.
   1.2) Solve for the market equilibrium quantity (Q*) and price (P*) when a = -14. Show your work.
   1.3) If "a" increases to -12, what would happen to the market equilibrium quantity and price? State the qualitative predictions without redoing the algebra.

(1.1)



(1.2) solve equilibrium where P = Q

$a + P = 10 - Q^2$ $\quad a = -14$

$\boxed{-14 + P = 10 - Q^2}$

$P = -Q^2 + 24$

$Q^2 = -P + 24$

(1.3) a is sum with P so with a increase will cause quantity to increase

increasing in a make Q to be increase so $-Q^2$ have negative sign increasing in Q make price to be lower

2. Suppose that the revenue function is given by $R(Q) = \ln(Q^2 + 1) + 3\left(\frac{Q}{Q+1}\right), Q \geq 0$. Use the derivative technique and calculate the marginal revenue function. Is the revenue function an increasing or decreasing function?

Use derirative to find marginal revenue

$$\frac{dR(Q)}{dQ} = \frac{1}{Q^2+1} \cdot \frac{dQ^2+1}{dQ} + 3\left[\frac{(Q+1)\left(\frac{dQ}{dQ}\right) - Q\left(\frac{dQ+1}{dQ}\right)}{(Q+1)^2}\right]$$

$$= \left[\frac{1}{Q^2+1} \cdot 2Q\right] + 3\left[\frac{Q+1-Q}{(Q+1)^2}\right]$$

marginal revenue $= \dfrac{2Q}{Q^2+1} + \dfrac{3}{(Q+1)^2}$

3. Suppose that the profit function is given by $\pi(Q) = -\frac{1}{3}Q^3 - Q^2 + 8Q - 1$ where Q is the level of output. Use the calculus and solve for the level of profit-maximizing output. Confirm your answer with the second derivative.

First derirative

$$\frac{d\pi(Q)}{dQ} = -\frac{1}{3}3Q^2 - 2Q + 8$$
$$= -Q^2 - 2Q + 8$$

Second derirative

$$\frac{d\pi(Q)'}{dQ} = -2Q - 2$$

4. Suppose that $A = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, calculate the following object. Show your work.

4.1 A + B

not possible

4.2 A*B

$$\begin{bmatrix} 8(1)+9(4) & 2(8)+5(9) & 8(3)+9(6) \\ (10)(1)+11(4) & 10(2)+11(5) & 10(3)+11(6) \end{bmatrix} = \begin{bmatrix} 44 & 61 & 78 \\ 54 & 75 & 96 \end{bmatrix}$$

4.3 det(A)

$$8(11) - 9(10) = -2$$

4.4 det(B)

not possible

4.5 det(C)

$$\left[\begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 2 \\ 4 & 5 & 6 & 4 & 5 \\ 7 & 8 & 9 & 7 & 8 \end{array}\right]$$

$$\det(C) = 1(5)(9)+2(6)(7)+3(4)(8) - (7)5(3) - (8)(6)1 - 9(4)(2)$$

$$= 0$$

5. Suppose that $U(x, y) = x^a y^b + \ln\left(\frac{x}{x+y}\right)$. Use the partial derivative technique, calculate $\frac{\partial U}{\partial x}$ and $\frac{\partial U}{\partial y}$.

$$\frac{\partial U}{\partial x} = a x^{a-1} y^b + \frac{1}{\frac{x}{x+y}} \cdot \frac{\partial\left(\frac{x}{x+y}\right)}{\partial x}$$

$$= a x^{a-1} y^b + \frac{x+y}{x} \cdot \frac{(x+y) \cdot \frac{\partial x}{\partial x} - x \cdot \frac{\partial x+y}{\partial x}}{(x+y)^2}$$

$$= a x^{a-1} y^b + \frac{x+y}{x} \times \frac{x+y-x}{(x+y)^2}$$

$$\frac{\partial U}{\partial y} = x^a b y^{b-1} + \frac{1}{\frac{x}{x+y}} \cdot \frac{\partial\left(\frac{x}{x+y}\right)}{\partial y}$$

$$= x^a b y^{b-1} + \frac{x+y}{x} \cdot \frac{x+y \cdot \frac{\partial x}{\partial y} - x \cdot \frac{\partial x+y}{\partial y}}{(x+y)^2}$$

$$= x^a b y^{b-1} + \frac{x+y}{x} \cdot \frac{0 - x(1)}{(x+y)^2}$$

$$= x^a b y^{b-1} + \frac{x+y}{x} \cdot \frac{-x}{(x+y)^2}$$