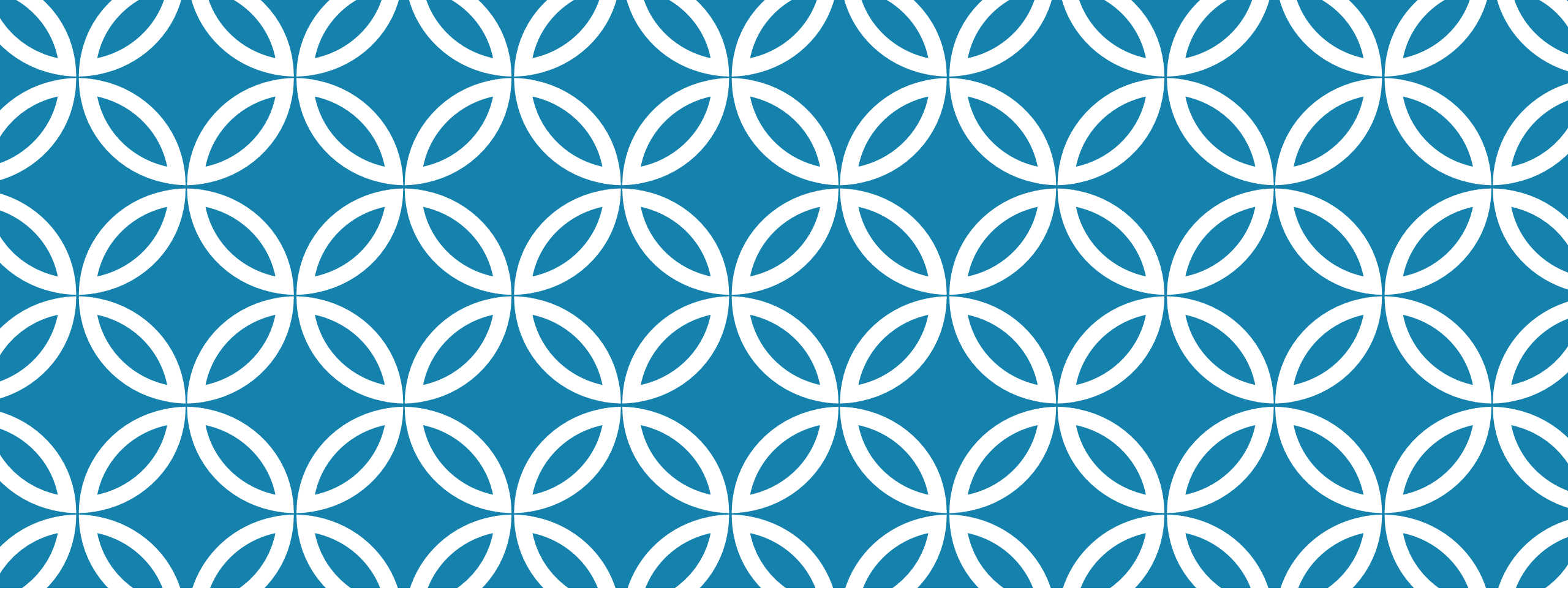


RISK AND TERM-STRUCTURE OF INTEREST RATE

EE431 Semester 1/2017
Kittichai Saelee

TOPICS

- ~~Bond pricing and yield to maturity~~
- ~~Bond returns and Interest rate risk.~~
- ~~Bond price and interest rate determination~~
- ~~Risk and~~ Term structure of interest rates

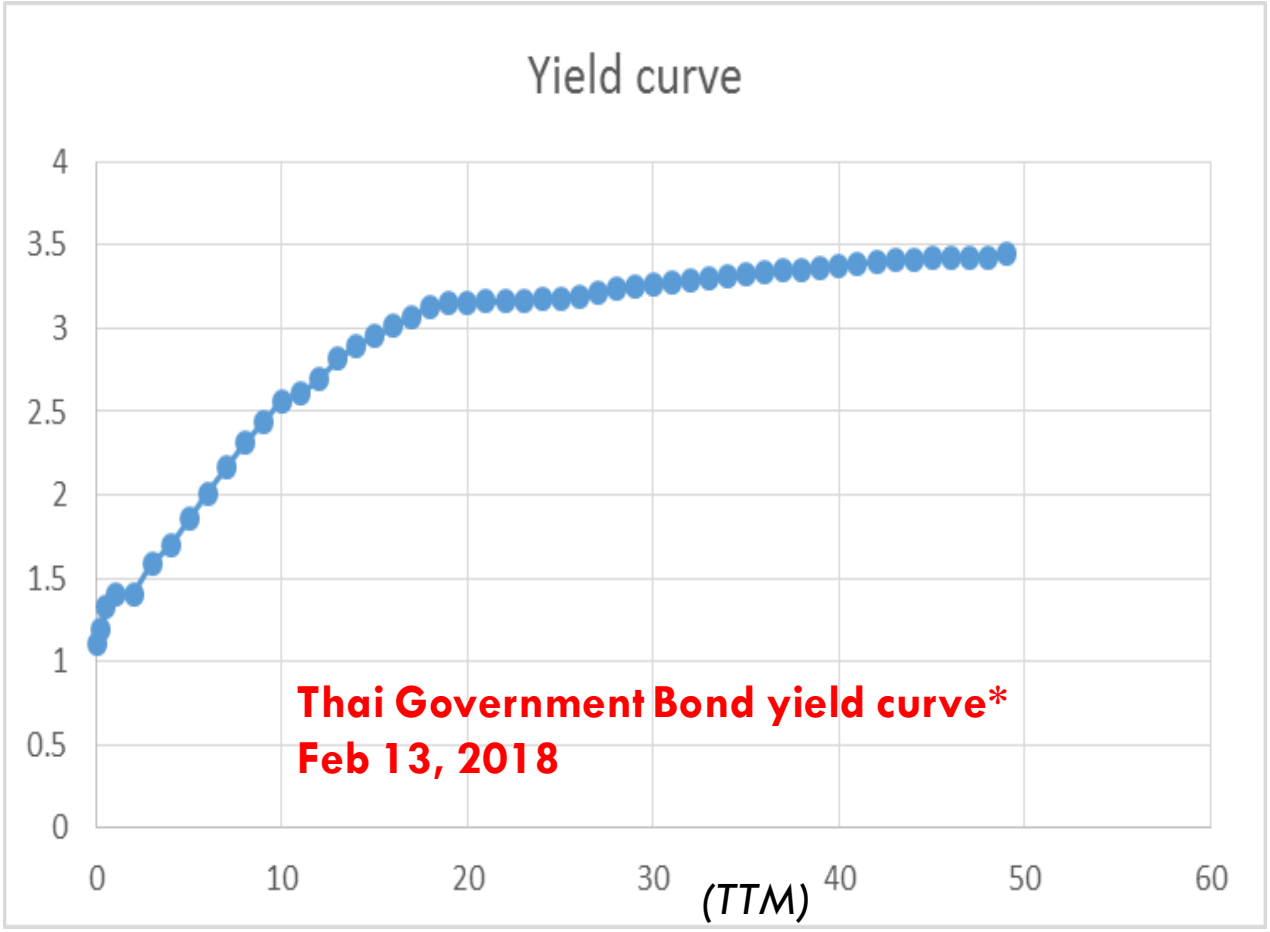
Chapter 6 Mishkin

Also note/extra reading lists uploaded on the BE Moodle.

TERM STRUCTURE OF INTEREST RATE

- Interest rates vary with *the remaining periods to maturity date (TTM).*

- Explaining about the *dynamic of yield curve.*



THREE STYLIZED-FACTS FOR YIELD CURVE

- **Fact 1:** Yields at different TTM co-move.

- **Fact 2:** If short-term yield is (exceptionally) low (high), yield curve tends to be *steep* upward (downward) sloping.
  - Upward sloping ➔ SR yield < LR yield
  - Downward sloping ➔ SR yield > LR yield

- **Fact 3:** *Normally,* yield curve is upward sloping.

HOW TO RATIONALIZE STYLIZED FACTS?

- *Expectations Theory*
  - 1&2, but not 3

- *Segmented Markets Theory*
  - *3, but not the first two.*

- *Liquidity Premium (Preferred Habitat) Theory:*
  - *1&2&3 can be explained (combined together the first two theories.)*

EXPECTATION HYPOTHESIS: ASSUMPTION

- **Assumptions**
  1. Investors consider investing money in bond with different maturities.
  2. Each type of bonds can be perfectly substituted; no biased preference on one over the others.
  3. Investors are risk neutral.

Risk neutral investors look at the expected return on the investment; they don't care about the risk.

PREDICTION UNDER THE SIMPLEST VERSION

TTM = 1 year

$$i_{t=0,2} = \frac{i_{t=0,1} + i^{e}_{t=1,1}}{2}$$

Yield of 2-year bond = average of yield of 1-year bond over 2 years

TTM 2 year

Expected yield of 1-year bond at time "t", (in the next year)

EXPECTATION HYPOTHESIS: DERIVATION/INTUITION

- Suppose a representative risk-neutral investor consider an investment decision for 2 years.

- As the investor is risk-neutral, he/she cares for the *highest expected return* from the investment.

- The investor can compare the outcome under two possible strategies, namely roll-over and once-and-for-all.

THE TWO POSSIBLE INVESTMENT STRATEGIES: ROLL-OVER STRATEGY

- **Option 1:** Roll-over strategy with one-year bond in each year. (short-term strategy)

- Suppose that yield of 1-year bond is $i_{t=0,1}$.

- In the next year, yield of 1-year bond is expected to be $i^{e}_{t=1,1}$

EXPECTED RETURN UNDER OPTION 1

Expected return from Option 1 =

$$\left(1 + i_{t=0,1}\right)\left(1 + i^{e}_{t=1,1}\right) - 1$$

On its approximation

$$i_{t=0,1} + i^{e}_{t=1,1}$$

THE TWO POSSIBLE INVESTMENT STRATEGIES: ONCE-AND-FOR-ALL STRATEGY

- **Option 2:** *Long-term investment* with 2-year bond.
  - Suppose that yield of the 2-year bond is $i_{t=0,2}$

Expected return from Option 2

$$\left(1 + i_{t=0,2}\right)^2 - 1$$

On its approximation

$$2 \times i_{t=0,2}$$

EQUILIBRIUM: NO-ARBITRAGE CONDITION

- Equilibrium occurs when the return on two alternative choices of investment strategies are equal. (why?)

$$2i_{t=0,2} = i_{t=0,1} + i^{e}_{t=1,1}$$

$$i_{t=0,2} = \frac{1}{2}\left(i_{t=0,1} + i^{e}_{t=1,1}\right)$$

- The expectation hypothesis to term structure of interest rate captures the **No-arbitrage condition**; investors will move around until returns get equalized.

EXPECTATIONS HYPOTHESIS: *GENERALIZATION*

- Long-term T-period investment v.s. Roll-over short-term investment for T periods.

$$i_{t=0,T} = \frac{i_{t=0,1} + \sum_{j=1}^{T-1} i^{e}_{t=j,1}}{T}$$

Long-term yield should be equal to the average of expected short-term yield over the duration of the remaining times/periods to maturity of the long-term bond.

EXPECTATION HYPOTHESIS AND FACT 1

- Why does short-term yield and long-term yield co-move?

  - Suppose that current short-term yield ($i_{t=0,1}$) increases, while given that $i^{e}_{t=1,1}$, $i^{e}_{t=2,1}$, …. are fixed.

$$i_{t=0,T} = \frac{i_{t=0,1} + \sum_{j=1}^{T-1} i^{e}_{t=j,1}}{T}$$

  - Mathematically implied by EH, the average of interest rate will increase, and hence $i_{t=0,T}$ ↑.

HOW TIGHT DO THEY COMMOVE?: EVIDENCE FROM 2005M1-2017M9.

	R1M	R3M	R6M	R1YR
R1M	1	0.997995	0.990083	0.968111
R3M	0.997995	1	0.996559	0.979335
R6M	0.990083	0.996559	1	0.991306
R1YR	0.968111	0.979335	0.991306	1
R2YR	0.902792	0.919897	0.942647	0.97503
R3YR	0.832165	0.851757	0.879886	0.926238
R4YR	0.747176	0.767354	0.79788	0.853303
R5YR	0.706364	0.724983	0.754556	0.812287
R6YR	0.663297	0.681325	0.711142	0.772238
R7YR	0.629757	0.646797	0.676096	0.738146
R8YR	0.602851	0.618559	0.64687	0.709318
R9YR	0.580742	0.595844	0.623935	0.686988
R10YR	0.546415	0.561243	0.5894	0.653821
R11YR	0.527782	0.541557	0.568603	0.632919
R12YR	0.515876	0.529029	0.555341	0.619423
R13YR	0.510568	0.523043	0.548554	0.612347
R14YR	0.493402	0.504989	0.529659	0.59297
R15YR	0.490684	0.50125	0.524726	0.586896
R16YR	0.493704	0.503572	0.52582	0.586933
R17YR	0.492329	0.501395	0.522362	0.582328
R18YR	0.495001	0.503508	0.523495	0.582194
R19YR	0.500082	0.508064	0.527442	0.585437
R20YR	0.504188	0.511843	0.531026	0.588574

- In practice, **correlation is not 1**.
  - $i_{t=0,1} \uparrow \rightarrow i^{e}_{t=1,1}$, $i^{e}_{t=2,1}$,…. may be also adjusting in the way that offsets the directional change of the current short-tem yield, and hence causing an imperfect correlation.
  - This point is to be explained later when we discuss about the dynamic yield curve.

EXPECTATION HYPOTHESIS AND FACT 2

- Yield curve is "downward sloping" if short-term rate is exceptionally high.
  - First, note that downward sloping = STR > LTR!
  - $(i_{t=0,1})$ is exceptionally high → $i^{e}_{t=1,1}$, $i^{e}_{t=2,1}$ is expected to drop because current short-term rate is "too" high.
  - Average would be lower than $(i_{t=0,1})$.
  - Following the Expected hypothesis, long-term interest rate will be lower than the current short-term rate.

EXPECTATION HYPOTHESIS AND FACT 3: TROUBLE

Fact 3: Normally, yield curve is (mostly) upward sloping.

- To explain fact#3, it requires that average of expected future short-term interest rate is higher than today.

$$i_{t=0,T} = \frac{i_{t=0,1} + \sum_{j=1}^{T-1} i^{e}_{t=j,1}}{T}$$

- **Reasonable?** No, because it implies that we always need a rising trend of market interest rate to account for the regularity of upward sloping yield curve.

SEGMENTED MARKET MODEL (SMM)

- Shortcomings of the EH → an assumption is relaxed
- Bonds with different TTM are **NOT** substitutable.
- Investors and Borrowers have made choice of investment decision and fund raising with *biasedness*.
- SMM assumes a strong version of *biased preference*; no substitution at all, and hence implies that markets are *completely segmented.*
- Implication: The determination of yield at each TTM will be separately determined in each market.

SEGMENTED MARKET MODEL: BOND HOLDER

- Biased preference implies that investors prefer some types of bods to others.

- Biasedness over TTM of bonds: relatively high demand of short-term bond to long-term bond.

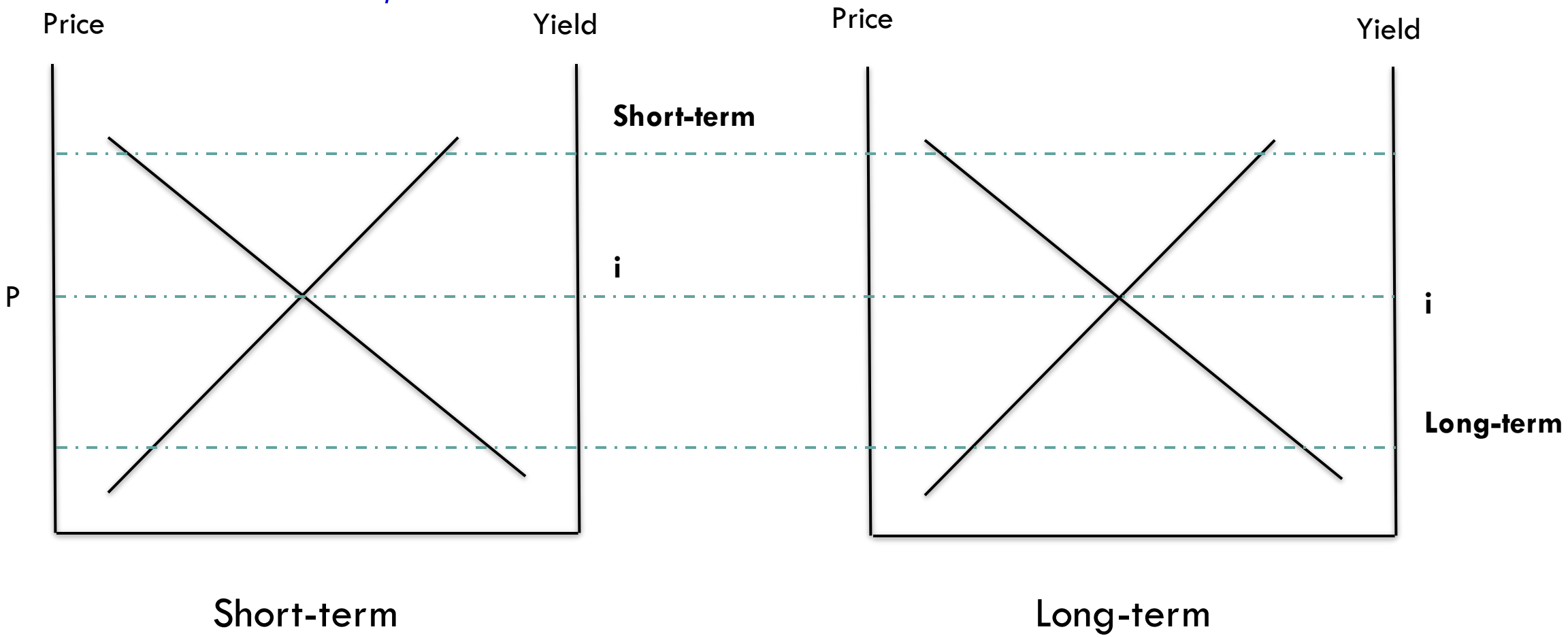
- Why? Uncertainty resolved sooner; the better.
  - Event Risk is lower.
  - Inflation Risk is lower.

SEGMENTED MARKET MODEL: BOND ISSUER

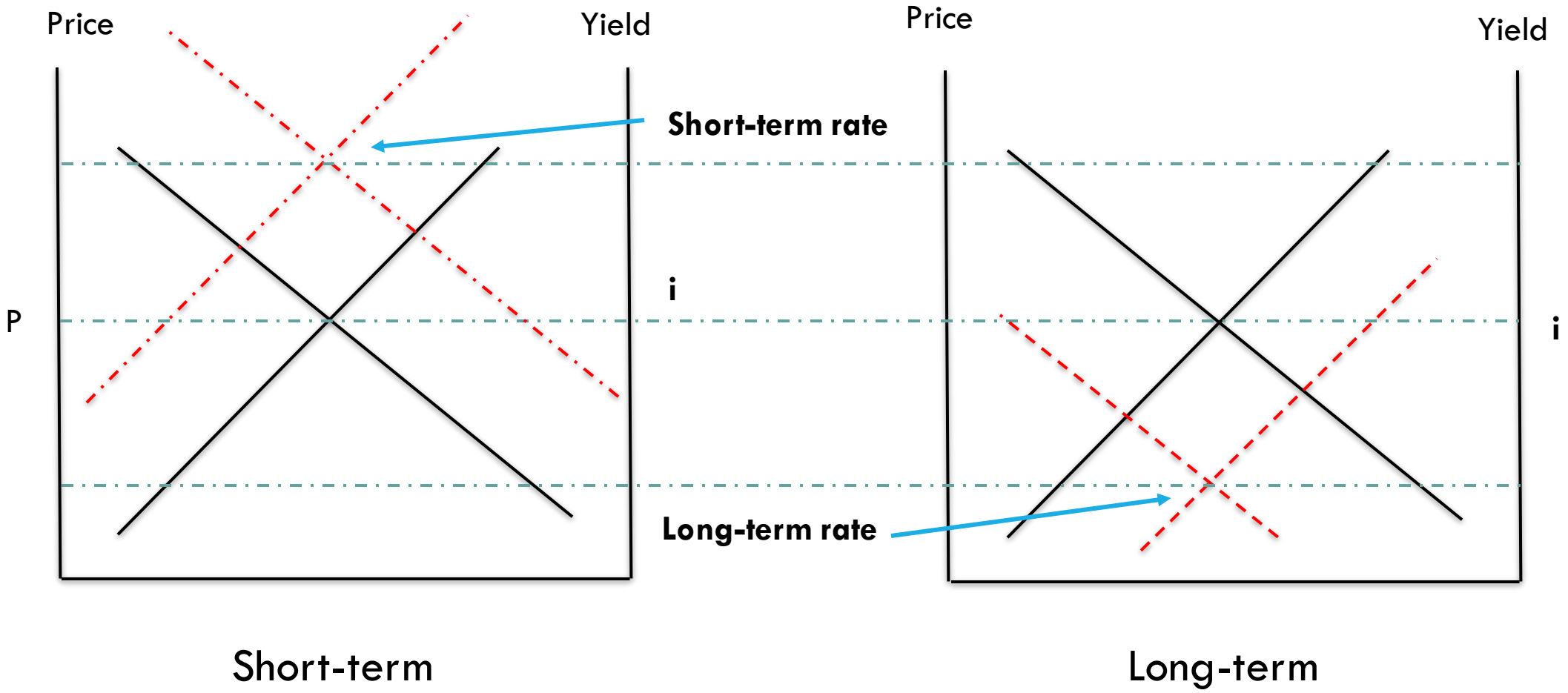
- Similarly, bond issuers usually prefer securing long-term funds, rather than short-term funds.

- There is a risk associated with the failure to refinance, i.e. roll-over risk.

RELATIVE DEMAND AND SUPPLY FOR SHORT-TERM AND LONG-TERM BOND: NO BIASED / IDENTICAL BONDS



RELATIVE DEMAND AND SUPPLY FOR SHORT-TERM AND LONG-TERM BOND: WITH BIASED PREFERENCE TAKEN INTO ACCOUNT.



A SHORTFALL OF THE SMM MODEL

- No linkage between Short-term and long-term market.

- Changes in short-term market will not affect long-term market as we assume that each of them is separate on their own.

- So, our assumption is **too strong**.

LIQUIDITY PREMIUM OR PREFERRED HABITAT

- Risk aversion investor!
  - Not only expected return still matters; but now risk also matters as well.
- What risk?
  - Event risk / Inflation risk/ Interest rate risk/ Liquidity risk
- All these risks vary with respect to **time horizon** of investment.
  - The longer, the bigger.

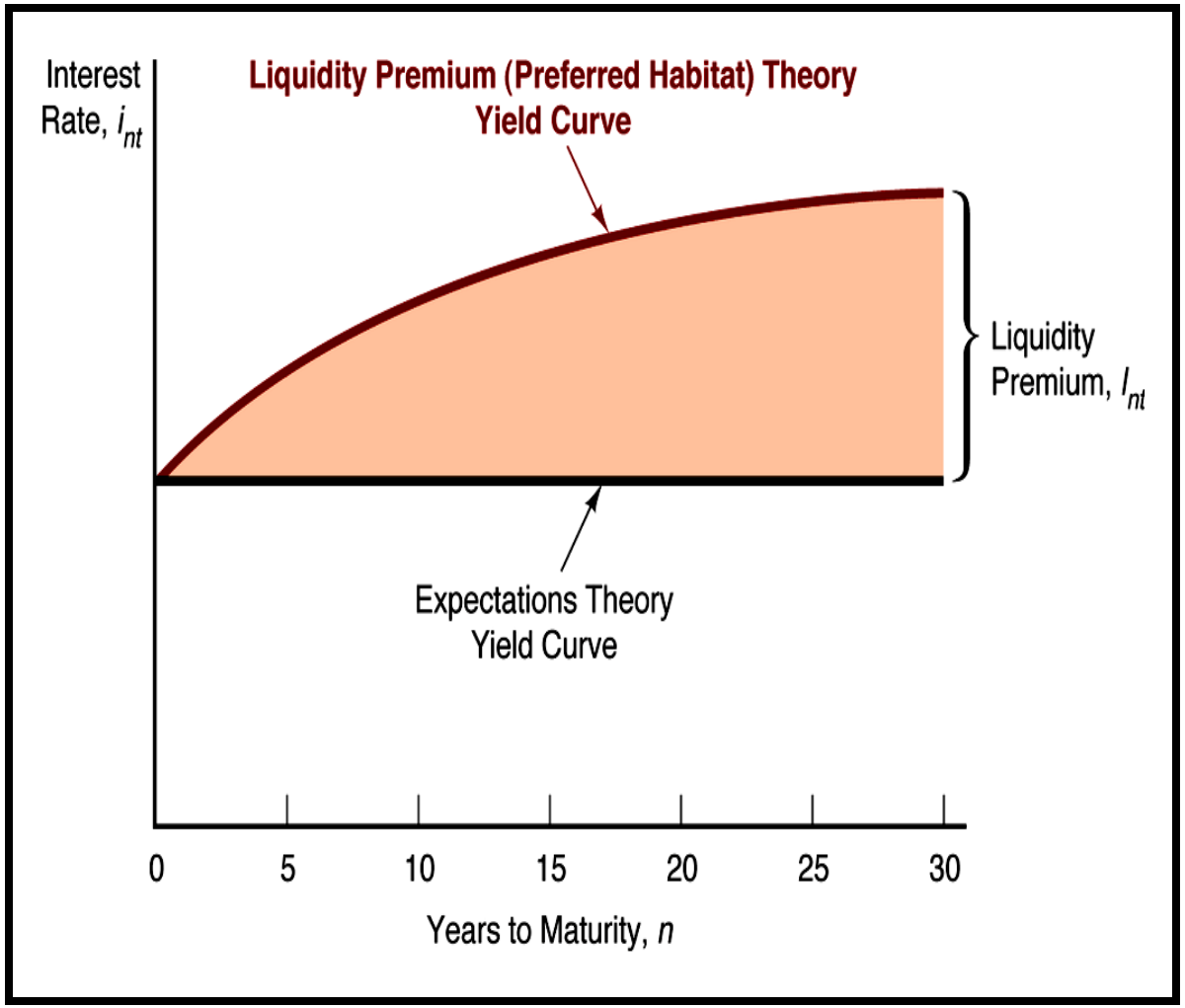
LIQUIDITY PREMIUM OR PREFERRED HABITAT

- Long-term bond is more riskier than short-term bond; you need to get some extra return on the top, i.e. the required **term premium**.
- Term premium $i(T)$ varies over TTM.
  - The longer TTM, the higher required premium.

$$i_{t=0,T} = \frac{i_{t=0,1} + \sum_{j=1}^{T-1} i^{e}_{t=j,1}}{T} + i(T)$$

$i(T)$: **an increasing function with respect to "time to maturity"**

LIQUIDITY PREMIUM OR PREFERRED HABITAT



Fact 3 can now be explained under more general (reasonable) assumptions.

Even if investors expected a constant path of future short-term rate, yield curve can still be upward sloping.

APPLICATIONS

- Dynamic yield curve
  - Term structure theorem provides the foundation for explaining the movement of yield curve.
  - To be discussed next time.

- Financial applications:
  - We use EH to compute the implied forward interest rate.

CALCULATING FORWARD RATE

- Suppose we observe market interest rates at different TTM.

$$i_{t=0,2} = \frac{1}{2}\left(i_{t=0,1} + i^{e}_{t=1,1}\right)$$

→ Solving for an unknown, $i^{e}_{t=1,1}$ can be backed out.

- In finance, the rate is called "forward interest rate at a specific TTM".

- The implied figure represents a fair quote of interest rate that a forward contractual loan can be committing to.

EXAMPLE..

- Below summarizes data of yield to maturity of bonds with TTMs.
- Calculate the implied one-year forward rate of one-year bond and two-year bond.

TTM	Yield
1 year	3.5%
2 year	4.25%
3 year	5%