PROSPECT THEORY

EPISODE 2

$\pi(p) \neq p$

EE 434 Behavioral Finance, SEM1/2022

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objective probability p

the probability weight $\pi(p)$

LOTTERY TICKETS & INSURANCE

Lottery tickets and Insurance

- Why do people who buy lottery tickets also purchase insurance?

Lottery tickets and Insurance

Expected value of ticket < price of ticket

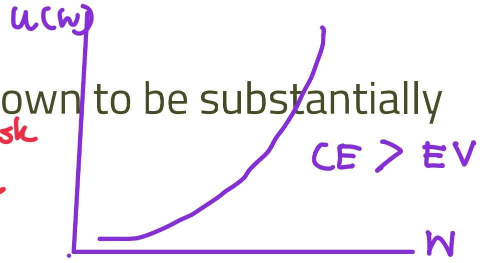
- In the expected utility framework, this is puzzling because:

if we look at utility fn

- With a lottery, a person is seeking risk.
  - The expected payoff from a lottery is well known to be substantially less than the price of a ticket.
  - Very small chance to win a prize

pay to get risk

risk loving

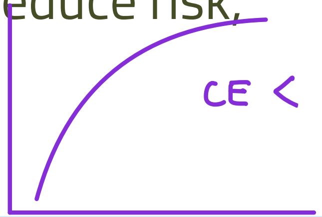


CE > EV

- With insurance, the same person may pay to reduce risk, appearing to be risk averse.

pay to reduce risk

risk averse



CE < EV

Lottery tickets and Insurance

- Prospect theory can well account for the observation that some people buy lottery tickets and insurance at the same time.

  - Overweighting low-probability events

Lottery

gain domain

A B

Choose between prospects ($5,000, 0.001) and ($5, 1).

$A \succ B$

risk-seeking in the gain domain

Lottery

Choose between prospects ($5,000, 0.001) and ($5, 1).

- The expected values of the two prospects are equal $5.
- Many people prefer ($5,000, 0.001) to ($5, 1) consistent with risk seeking.
- This is indicative of risk seeking in the domain of gains.

Insurance

Choose between prospects $\overset{A}{(-\$5{,}000,\ 0.001)}$ and $\overset{B}{(-\$5,\ 1)}$.

$B \succsim A$

risk-averse in loss domain

Insurance

Choose between prospects (–\$5,000, 0.001) and (–\$5, 1).

- In this case we often see people choosing prospect (–\$5, 1) over (–\$5,000, 0.001), consistent with risk aversion.
- This implies risk aversion in the loss domain.

COMMON-RATIO EFFECT

Allais (1953)

Problem 3&4 in Kahneman&Tversky (1979)

if ($3k, 1) ≻ ($4k, 0.8)
⇒ risk aversion in the gain domain

Problem 3:
Choose between ($4,000 ; 0.80) and ($3,000, 1).

0.25 0.25

Problem 4:
Choose between ($4,000 ; 0.20) and ($3,000, 0.25).

if ($4k, 0.2) ≻ ($3k, 0.25)

Problem 3&4 in Kahneman&Tversky (1979)

Problem 3:
Choose between ($4,000 ; 0.80) and ($3,000, 1). risk averse

Problem 4:
Choose between ($4,000 ; 0.20) and ($3,000, 0.25). risk seeking

K&T found that

$$(3000, 1) \succ (4000, 0.80) \text{ and } (4000, 0.20) \succ (3000, 0.25)$$

Problem 3&4 in Kahneman&Tversky (1979)

Problem 3:
Choose between ($4,000 ; 0.80) and ($3,000, 1).

Problem 4:
Choose between ($4,000 ; 0.20) and ($3,000, 0.25).

Problem 4 is identical to Problem 3, except that the probabilities are multiplied by a common ratio of 0.25.

This pattern of choices violates EUT.

To prove this → Write Expected utility for each pattern of choice to find the contradiction

THE COMMON-RATIO EFFECT (Allais (1953))

The common-ratio effect refers to the observation that the more risky of two simple prospects becomes relatively more attractive when the probability of winning is reduced by equal proportion in both prospects.

PROBABILITY WEIGHTING

Think about ($5000, p), p objective prob

(1.) $p = 0\%$ $\xrightarrow{}$ $p = 1\%$ ⇒ "possibity effect"

impossibility $\Delta p = 1\%$

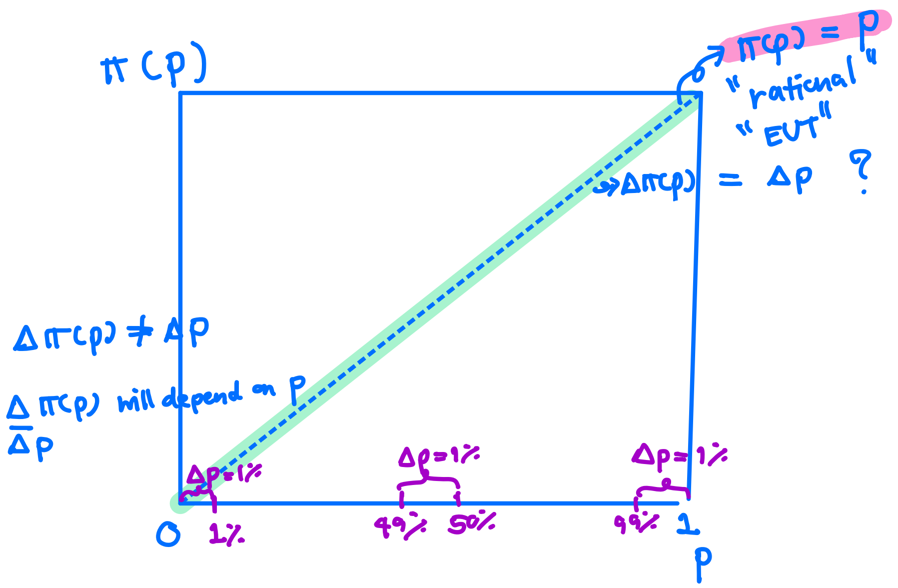
(2.) $p = 49\%$ $\xrightarrow{}$ $p = 50\%$

$\Delta p = 1\%$

(3.) $p = 99\%$ $\xrightarrow{}$ $p = 100\%$ ⇒ " certainty effect"

$\Delta p = 1\%$ "certainty"

$$\pi(p) \neq p$$



Recall: Prospect theory

A prospect can be written as $(x, p; y, q)$ with $p + q \leq 1$.
Note: $p + q < 1$ implies prospect yields 0 with probability $1 - p - q$.

A person evaluates a prospect $(x, p; y, q)$ according to the functional

" Prospect-theory value "

$$V(x, p; y, q) = \pi(p)v(x) + \pi(q)v(y)$$

probability weighting function

Probability p vs. The weighting of probability $\pi(p)$

- The decision weights that people assign to outcomes might NOT be identical to the objective probabilities of these outcomes.

$(\$10{,}000,\ \underbrace{0.0001}_{\text{objective probability } p})$

$\neq$

decision weight $\pi(p)$

Probability p vs The weighting of probability $\pi(p)$

- We have different sensitivity to change in probability depending on the level of probability being considered.

- Psychological effect:
  - Possibility effect
  - Certainty effect

The possibility effect: 0% → 1%

- The move from a 0% chance to a 1% possibility of winning a prize or losing something transform the situation.

- It creates a possibility that did not exist earlier.

The possibility effect: 0% → 1%

- The influence we give to the move from 0% probability to 1% illustrates the possibility effect.

- The weights reflect:
  - The hope of winning
  - The worry of losing

The possibility effect: 0% → 1%

- The possibility effect causes highly unlikely outcomes to be weighted disproportionately more than they "deserve" if we evaluate the change in probability objectively.

- Overweighting of small probabilities increases the attractiveness of both lottery ticket and insurance policies.

The certainty effect: 99% → 100%

- Outcomes that are almost certain are given less weight than their probabilities justifies.

Example

impossibility

sure thing

$\Delta p = 1\%$

Prob(%) of winning a gamble	0	1	2	5	10	20	50	80	90	95	98	99	100
Decision weight	0	5.5	8.1	13.2	18.6	26.1	42.1	60.1	71.2	79.3	87.1	91.2	100

$\Delta p = 1\%$ $\Delta p = 1\%$

$\Delta\pi(p) = 2.6\%$

$\Delta\pi(p) = 5.5\%$

"diminishing sensitivity"

8.8%

$\pi(p) > p$ "overweighting"

$\pi(p) < p$ "underweighting"

- The decision weights are identical to the corresponding probabilities at the extremes: the impossible $\pi(0) = 0$ and the sure thing $\pi(1) = 1$.

- The decision weights ***depart sharply*** from probabilities near the impossible and near the sure thing.

Example

Prob(%) of winning a gamble	0	1	2	5	10	20	50	80	90	95	98	99	100
Decision weight	0	5.5	8.1	13.2	18.6	26.1	42.1	60.1	71.2	79.3	87.1	91.2	100

- At 2%, the rare event is overweighted by a factor of 4.
- At 98%, a 2% risk of not winning the prize reduces the weight by 13%, from 100 to 87.

Example

Prob(%) of winning a gamble	0	1	2	5	10	20	50	80	90	95	98	99	100
Decision weight	0	5.5	8.1	13.2	18.6	26.1	42.1	60.1	71.2	79.3	87.1	91.2	100

- **Inadequate sensitivity** for intermediate probabilities.
- Sometimes the very small probabilities get ignored, instead of getting overweighted.

Probability Weighting in Kahneman & Tversky (1979)

Kahneman & Tversky (1979) suggest that the probability-weighting function $\pi(p)$ has several features such as:

- Overweighting of small probabilities $\pi(p) > p$,
- Underweighting of large probabilities $\pi(p) < p$,
- Subcertainty $\pi(p) + \pi(1-p) < 1$,
- A discontinuity at the endpoints.

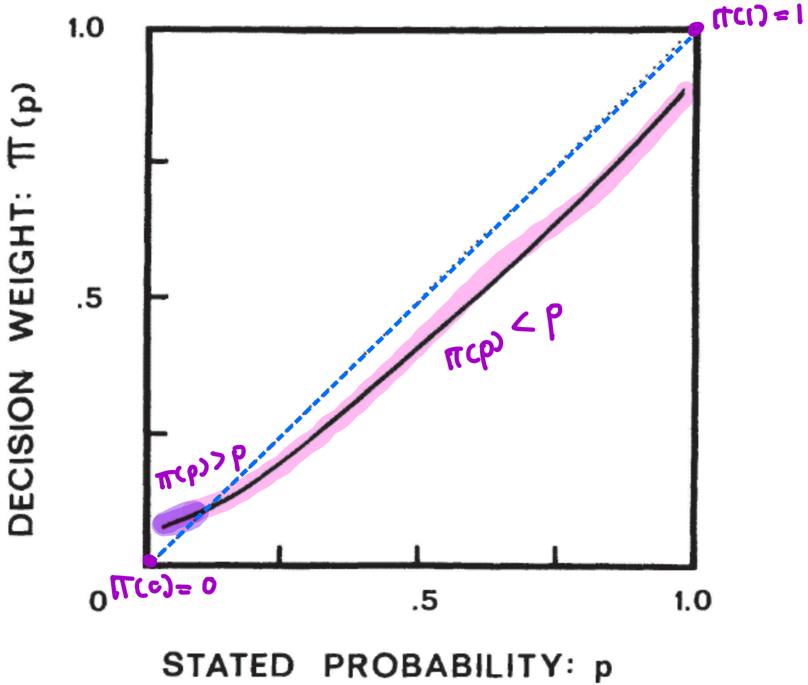


FIGURE 4.—A hypothetical weighting function.

Inverse S-Shaped Probability Weighting Functions

- In the ensuing years, people started to eliminate the discontinuity at the endpoints by assuming an inverse S-shaped probability weighting function.

Inverse S-Shaped Probability Weighting Functions

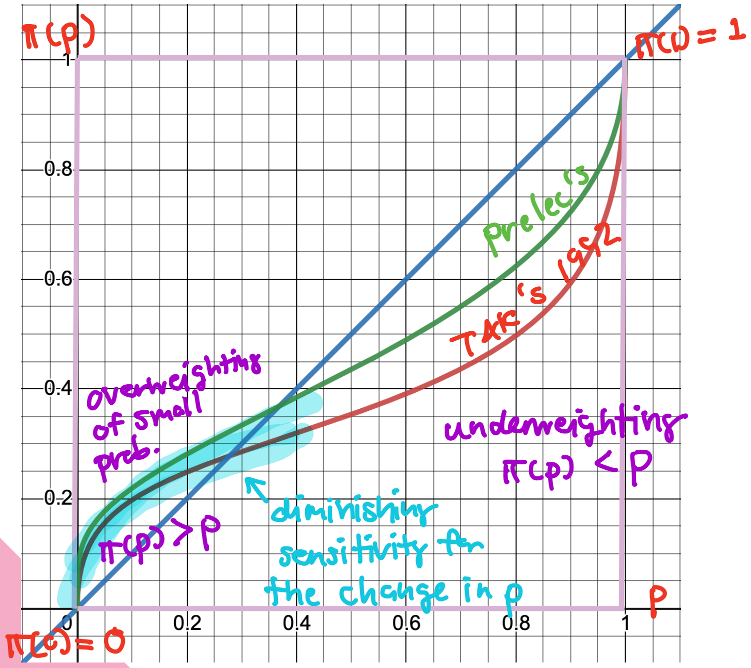
- Tversky & Kahneman (JRU 1992) suggest

$$\pi(p) = \frac{p^\gamma}{(p^\gamma+(1-p)^\gamma)^{1/\gamma}} \qquad for\ some\ \gamma \in (0.279,1)$$

- Prelec (ECTA 1998) suggests

$$\pi(p) = \exp(-(-lnp)^\alpha) \qquad for\ some\ \alpha \in (0,1)$$

Inverse S-Shaped Probability Weighting Functions



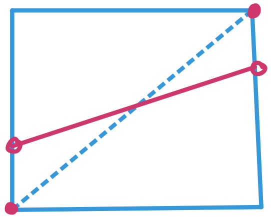
$$\pi(p) = \exp(-(-lnp)^{\underline{0.5}})$$

$$\pi(p) = \frac{p^{0.5}}{(p^{0.5} + (1-p)^{0.5})^{1/0.5}}$$

https://www.desmos.com/

A Simple Functional Form

$$\pi(p) = \begin{cases} 0 & \text{if } p = 0 \\ \alpha + \beta p & \text{if } p \in (0,1) \\ 1 & \text{if } p = 1 \end{cases}$$



If $0 < \alpha < 1$ and $\beta < 1 - 2\alpha$, then this functional form generates overweighting of small probabilities, underweighting of large probabilities, subcertainty, and a discontinuity at the endpoints.

Differential Weighting for Gains vs. Losses

- Tversky & Kahneman (JRU 1992) propose differential probability weighting for gains and losses.

- Weights on losses and weights on gains derived from two separate probability-weighting functions π^- and π^+ .

Example

Let

$$\boxed{\pi(p) = \frac{p^{\gamma}}{(p^{\gamma}+(1-p)^{\gamma})^{1/\gamma}} \text{ and } \gamma = 0.65}$$

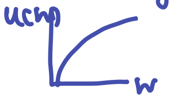
$$v(x) = \begin{cases} x^{0.88} & if\ x > 0 \\ -2.25(-x)^{0.88} & if\ x < 0 \end{cases}$$

Example

Consider the lottery scenario choosing between prospects: ($5,000, 0.001) and ($5, 1.0)

($5,000, 0.001) vs. ($5, 1)

note: if it's risk-averse utility fn,



$V(5{,}000,\ 0.001) =$

$= \pi(0.001)\, v(5{,}000) + \pi(0.999)\, \cancel{v(0)} = 0$

$= \pi(0.001)\, v(5{,}000)$

$= \underbrace{\dfrac{0.001^{0.65}}{\left(0.001^{0.65} + (1-0.001)^{0.65}\right)^{1/0.65}}}_{*\ 0.011\ *} \times (5{,}000)^{0.88}$

$= 0.011 \times 1{,}799$

$= 19.8$

$V(5,\ 1) = \pi(1)\, v(5)$

$= 1 \times 5^{0.88}$

$= 1 \times 4.12$

$= 4.12$

$V(\$5000,\ 0.001) > V(\$5,\ 1) \ \therefore\ (\$5{,}000,\ 0.001) \succ (\$5,\ 1)$

if there is no prob. weighting (if $\pi(p) = p$),

$\Rightarrow V(5{,}000, 0.001) = 0.001 \times 1{,}799$

$= 1.799$ smaller than

when we have $\pi(p) >> p$

Example

➢ The fact that a typical decision-maker likes to buy a lottery ticket is driven by the fact that the decision weight on $p = 0.001$ is about 10 times higher than the objective probability.

Stocks as Lotteries: The Implications of Probability Weighting for Security Prices

Barberis & Huang (AER 2008)

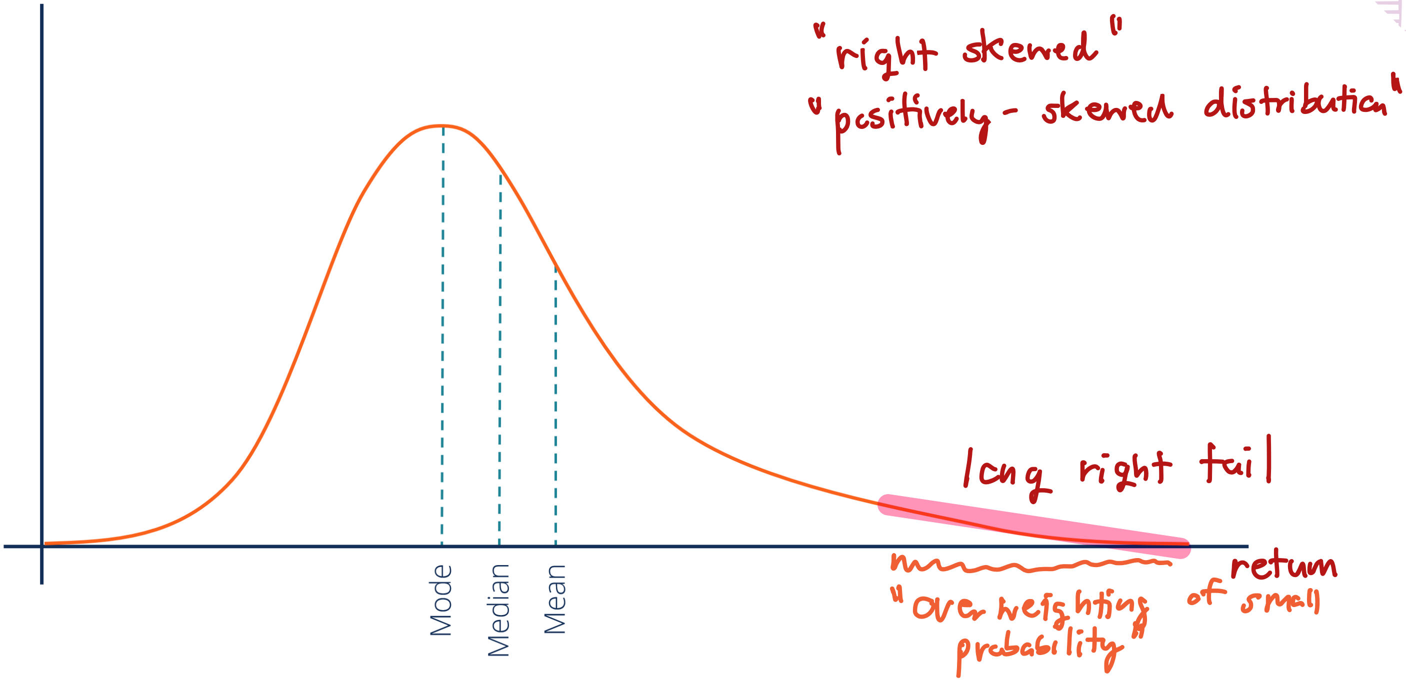
Stocks as Lotteries: The Implications of Probability Weighting for Security Prices

By Nicholas Barberis and Ming Huang*

We study the asset pricing implications of Tversky and Kahneman's (1992) cumulative prospect theory, with a particular focus on its probability weighting component. Our main result, derived from a novel equilibrium with nonunique global optima, is that, in contrast to the prediction of a standard expected utility model, a security's own skewness can be priced: a positively skewed security can be "overpriced" and can earn a negative average excess return. We argue that our analysis offers a unifying way of thinking about a number of seemingly unrelated financial phenomena. (*JEL* D81, G11, G12)

Over the past few decades, economists and psychologists have accumulated a large body of experimental evidence on attitudes to risk. This evidence reveals that, when people make decisions under risk, they often depart from the predictions of expected utility. In an effort to cap-

American Economic Review 2008, 98:5, 2066–2100 http://www.aeaweb.org/articles.php?doi=10.1257/aer.98.5.2066



In statistics, a positively skewed (or right-skewed) distribution is a type of distribution in which most values are clustered around the left tail of the distribution while the right tail of the distribution is longer.

Barberis & Huang (AER 2008)

- Rank-dependent probability weighting yields a preference for assets with positively skewed returns.

- In an economy with cumulative prospect theory investors, the skewed security can become overpriced relative to the prediction of the expected utility model, and can earn a negative average excess return.

Barberis & Huang (AER 2008)

- Some investors take a large, undiversified position in the skewed security, because by doing so, they make the distribution of their overall wealth more lottery-like, which, as people who overweight tails, they find highly desirable.

Barberis & Huang (AER 2008)

- It is not easy for expected utility investors to remove the effect: while expected utility investors can try to exploit the overpricing by taking short positions in skewed securities, there are risks and costs to doing so, and this limits the impact of their trading.

DANKE!

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