Term Structure of Interest Rates

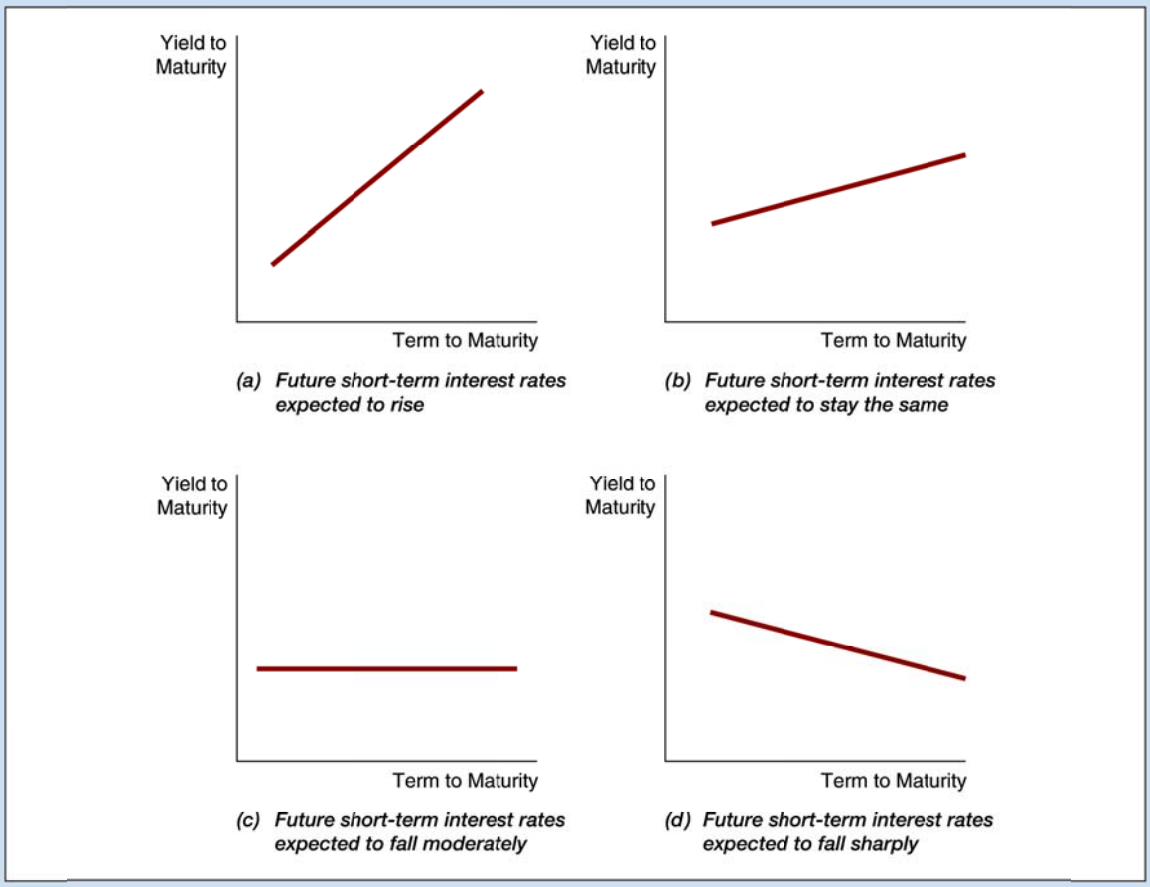

FIGURE 6 **Yield Curves and the Market's Expectations of Future Short-Term Interest Rates According to the Liquidity Premium (Preferred Habitat) Theory**

Term Structure of Interest Rate

- Bonds with identical risk, liquidity, and tax characteristics may have different interest rates because the time remaining to maturity is different
- Yield curve—a plot of the yield on bonds with differing terms to maturity but the same risk, liquidity and tax considerations
  - Upward-sloping ⇒ long-term rates are above short-term rates
  - Flat ⇒ short- and long-term rates are the same
  - Inverted ⇒ long-term rates are below short-term rates

Facts Theory of the Term Structure of Interest Rates Must Explain

1. Interest rates on bonds of different maturities move together over time
2. When short-term interest rates are low, yield curves are more likely to have an upward slope; when short-term rates are high, yield curves are more likely to slope downward and be inverted
3. Yield curves almost always slope upward

Three Theories to Explain the Three Facts

1. Expectations theory explains the first two facts but not the third
2. Segmented markets theory explains fact three but not the first two
3. Liquidity premium theory combines the two theories to explain all three facts

Expectations Theory

- The interest rate on a long-term bond will equal an average of the short-term interest rates that people expect to occur over the life of the long-term bond
- Buyers of bonds do not prefer bonds of one maturity over another; they will not hold any quantity of a bond if its expected return is less than that of another bond with a different maturity
- Bonds like these are said to be perfect substitutes

Example 1

The current interest rate on a one-year bond is 9.50 percent, and you expect the interest rate on the one-year next year to be 11.25 percent. What is the expected return over the two years? What interest rate must a two-ear bond have to equal the two one-year bonds?

Example 2

The one-year interest rates over the next five years are expected to be 5.10 percent, 6.20 percent, 7.70 percent, 8.80 percent and 10.50 percent. Given this information, what are the interest rates on a two-year, three-year, four-year, and five-year bonds? Explain what is happening to the yield curve.

Example 3

Fill in the missing spot rates and construct yield curve from the following information:

Maturity	Year 1	Year 2	Year 3	Year 4	Year 5
Rate	6.00 percent	7.00 percent			10.00 percent

Maturity	Year 6	Year 7	Year 8	Year 9	Year 10
Rate					12.00 percent

Bootstrapping Process

Process to construct default-free theoretical spot rate curve from the observed Treasury yield curve

Example 4

Calculate spot interest rate curve from the observed Treasury yields:

Treasury Securities	Coupon Rate	Yield-to-Maturity	Price
1 year	0.00%	4.00%	
2 years	5.00%	5.00%	
3 years	6.00%	6.00%	
4 years	7.00%	7.00%	

Forward Interest Rate

- From the yield curve we can extrapolate the theoretical spot rates. In addition, we can extrapolate the market's consensus of future interest rates.
- To see the importance of knowing the market's consensus for future interest rates, consider the following two investment alternatives for an investor who has a one-year investment horizon.
- Alternative 1: Buy a one-year instrument.
- Alternative 2: Buy a six-month instrument and when it matures in six months, buy another six-month instrument.
- With alternative 1, the investor will realize the one-year spot rate and that rate is known with certainty but with alternative 2, the investor will realize the 6-month spot rate, but the 6-month rate 6 months from now is unknown.
- Therefore, for alternative 2, the rate that will be earned over one year is not known with certainty.

Example 5

From example 4, compute the following forward rates:

- $F^{1,2}$
- $F^{2,3}$
- $F^{1,3}$
- $F^{2,4}$
- $F^{1,4}$

Arbitrage Free Valuation

- Valuation of a bond based on spot rates, spot rates for a Treasury securities as a rate should be sued to discount a default-free cash flow with the same maturity
- Traditional valuation methodology is to discount every cash flow of a security by the same interest rate (or discount rate), thereby incorrectly viewing each security as the same package of cash flows

Example 6

Using spot rates from example 4, given 3-year corporate bond with 10.00 percent coupon rate, compute price of this bond using arbitrage free valuation with 2.00 percent default spread.

Example 7

Using spot rates from example 4, given 4-year corporate bond with 12.50 percent coupon rate, compute price of this bond using arbitrage free valuation with default spreads of 0.50 percent, 1.25 percent, 1.75 percent and 2.15 percent in year 1 to year 4, respectively.

Conclusions from Expectations Theory

- Explains why the term structure of interest rates changes at different times
- Explains why interest rates on bonds with different maturities move together over time (fact 1)
- Explains why yield curves tend to slope up when short-term rates are low and slope down when short-term rates are high (fact 2)
- Cannot explain why yield curves usually slope upward (fact 3)

Exercise

1. Given the following 6-month forward rates, compute the value of a 3-year 8% coupon bond using these forward rates.

Period	1	2	3	4	5	6
Annual Forward Rate	4.00%	4.40%	5.00%	5.60%	6.00%	6.40%

2. Suppose that the annual yield-to-maturity for the 6-month and 1-year Treasury bills are 4.6% and 5.0%, respectively. These yields represent the 6-month and 1-year spot rates. Also assume the following Treasury yield curve (price 100 each) has been estimated for 6-month periods out to a maturity of 3 years.

Maturity	1.5	2.0	2.5	3.0
YTM	5.4%	5.8%	6.4%	7.0%

Compute the 1.5-year, 2-year, 2.5-year, and 3-year spot rates.

Maturity	1.5	2.0	2.5	3.0
Spot Rate				

Given these spot rates, compute the arbitrage-free value of a 3-year Treasury security with a coupon rate of 8%.

3. Assume the following Treasury spot rates:

Period	**Years to maturity**	**Spot rate**
1	0.5	5.00%
2	1.0	5.40%
3	1.5	5.80%
4	2.0	6.40%
5	2.5	7.00%
6	3.0	7.20%
7	3.5	7.40%
8	4.0	7.80%

Compute the following forward rates:

a. The 6-month forward rate six months from now.
b. The 6-month forward rate one year from now.

c. The 6-month forward rate three years from now.
d. The 2-year forward rate one year from now.
e. The 1-year forward rate two years from now.

Demonstrate that the 6-month forward rate six months from now is the rate that will produce at the end of one year the same future dollar as investing either (1) at the current 1-year spot rate or (2) at the 6-month spot rate and reinvest at the 6-month forward rate from now.

Segmented Markets Theory

- Bonds of different maturities are not substitutes at all
- The interest rate for each bond with a different maturity is determined by the demand for and supply of that bond
- Investors have preferences for bonds of one maturity over another
- If investors have short desired holding periods and generally prefer bonds with shorter maturities that have less interest-rate risk, then this explains why yield curves usually slope upward (fact 3)

Liquidity Premium Theory

- The interest rate on a long-term bond will equal an average of short-term interest rates expected to occur over the life of the long-term bond plus a liquidity premium that responds to supply and demand conditions for that bond
- Bonds of different maturities are substitutes but not perfect substitutes

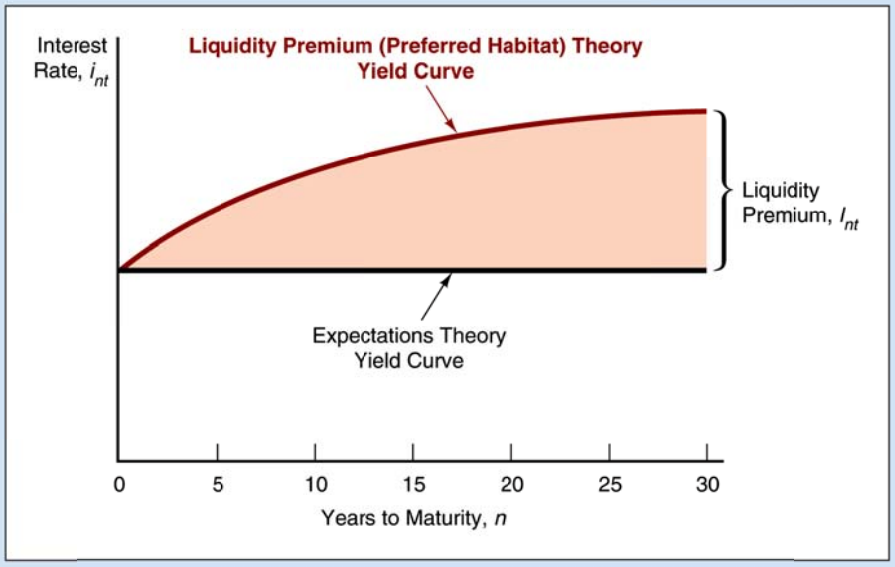

FIGURE 5 **The Relationship Between the Liquidity Premium (Preferred Habitat) and Expectations Theory**

Example 8

Suppose that the one-year interest rates over the next five years are expected to be 5.10 percent, 6.20 percent, 7.70 percent, 8.80 percent and 10.50 percent.
Investors' preferences for holding short-term bonds have the liquidity premiums for one-year to five-year bonds as 0.00 percent, 0.25 percent, 0.50 percent, 0.75 percent, and 1.25 percent, respectively. What are the interest rates on one-year to five-year bonds? Compare this finding with example 2.

Exercise

4. The table below shows current and expected future one-year interest rates, as well as current interest rates on multi-year bonds. Use the table to calculate the liquidity premium for each multi-year bond.

Year	One-year Bond Rate	Multi-year Bond Rate
1	2.20 percent	2.20 percent
2	3.50 percent	3.10 percent
3	3.75 percent	3.50 percent
4	4.00 percent	4.00 percent
5	4.20 percent	5.60 percent

5. A customer asks a bank if it would be willing to commit to making the customer a one-year loan at an interest rate of 8.25 percent one year from now. To compensate for the costs of making the loan, the bank needs to charge 1.30 percent more than the expected interest rate on a Treasury security with the same maturity if it to make a profit. If the bank manager estimates the liquidity premium to be 0.45 percent, and the one-year Treasury security rate is 6.10 percent and the two-year bond rate is 7.30 percent, should the manager be willing to make the commitment?

Preferred Habitat Theory

- Investors have a preference for bonds of one maturity over another
- They will be willing to buy bonds of different maturities only if they earn a somewhat higher expected return
- Investors are likely to prefer short-term bonds over longer-term bonds

Liquidity Premium and Preferred Habitat Theories, Explanation of the Facts

- Interest rates on different maturity bonds move together over time; explained by the first term in the equation
- Yield curves tend to slope upward when short-term rates are low and to be inverted when short-term rates are high; explained by the liquidity premium term in the first case and by a low expected average in the second case
- Yield curves typically slope upward; explained by a larger liquidity premium as the term to maturity lengthens

The Yield Curve as a Forecasting Tool for Inflation and the Business Cycle

Because the yield curve contains information about future expected interest rates, it should also have the capacity to help forecast inflation and real output fluctuations. The rising interest rates are associated with economic booms and falling interest rates with recessions. When the yield curve is either flat or downward-sloping, it suggests that future short-term interest rates are expected to fall and, therefore, that the economy is more likely to enter recession. Indeed, the yield curve is found to be an accurate predictor of the business cycle.

Nominal interest rate is composed of a real interest rate and expected inflation, implying that the yield curve contains information about both the future path of nominal interest rates and future inflation. A steep yield curve predicts a future increase in inflation, while a flat or downward-sloping yield curve forecasts a future decline in inflation.

The ability of the yield curve to forecast business cycles and inflation is one reason why the slope of the yield curve is part of the tool kit of many economic forecasters and is often viewed as a useful indicator of the stance of monetary policy, with a steep yield curve indicating loose policy and a flat or downward-sloping yield curve indicating tight policy.