Lecture 2 Introduction to Mathematical Economics

Read chapter 1

Optimization (calculus with one variable, example of economic optimization)

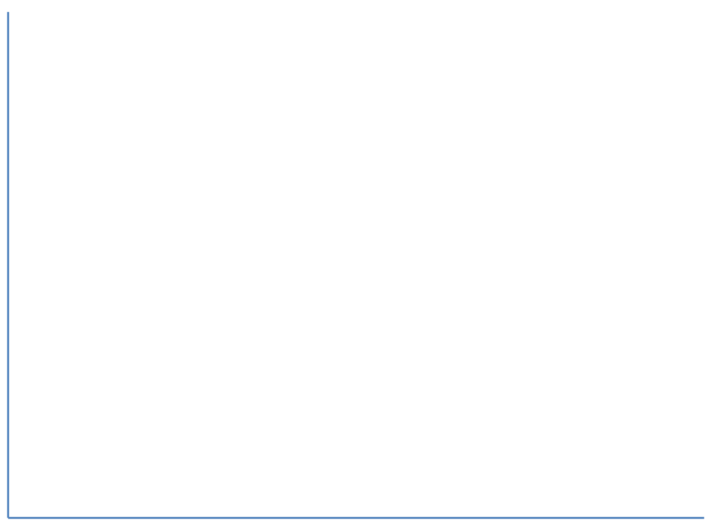
- Second and higher-order derivatives
- Partial derivatives: the effect of a change in x when other variables are held constant.
- Use different notation for derivatives
- Subscripts instead of a prime symbol

Applications of differential calculus: concavity and convexity

- Examples: Production possibility frontier is assumed to be concave; Indifference curves IC are to be convex.
- Why do we need to impose this assumption?
- Function of one variable, $y=f(x)$
- Definition: Strictly globally Concave: the tangent line passing through (x, y) is always on the top or above the function, or $f'' < 0$ for all x's.

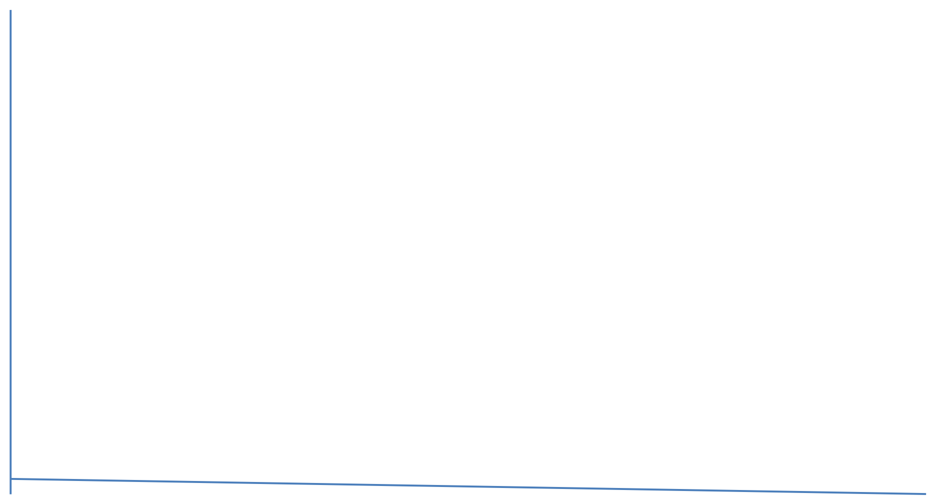
Applications of differential calculus: concavity

- Check PPF: $f(x) = a - bx^2$, $a > 0$, $b > 0$;
- $f' =$
- $f'' = \ldots. < 0$



Applications of differential calculus: convexity

- IC: $y= f(x) = a/ x, a>0;$
- $f' = \qquad < 0$
- $f'' = \ldots. \quad > 0$



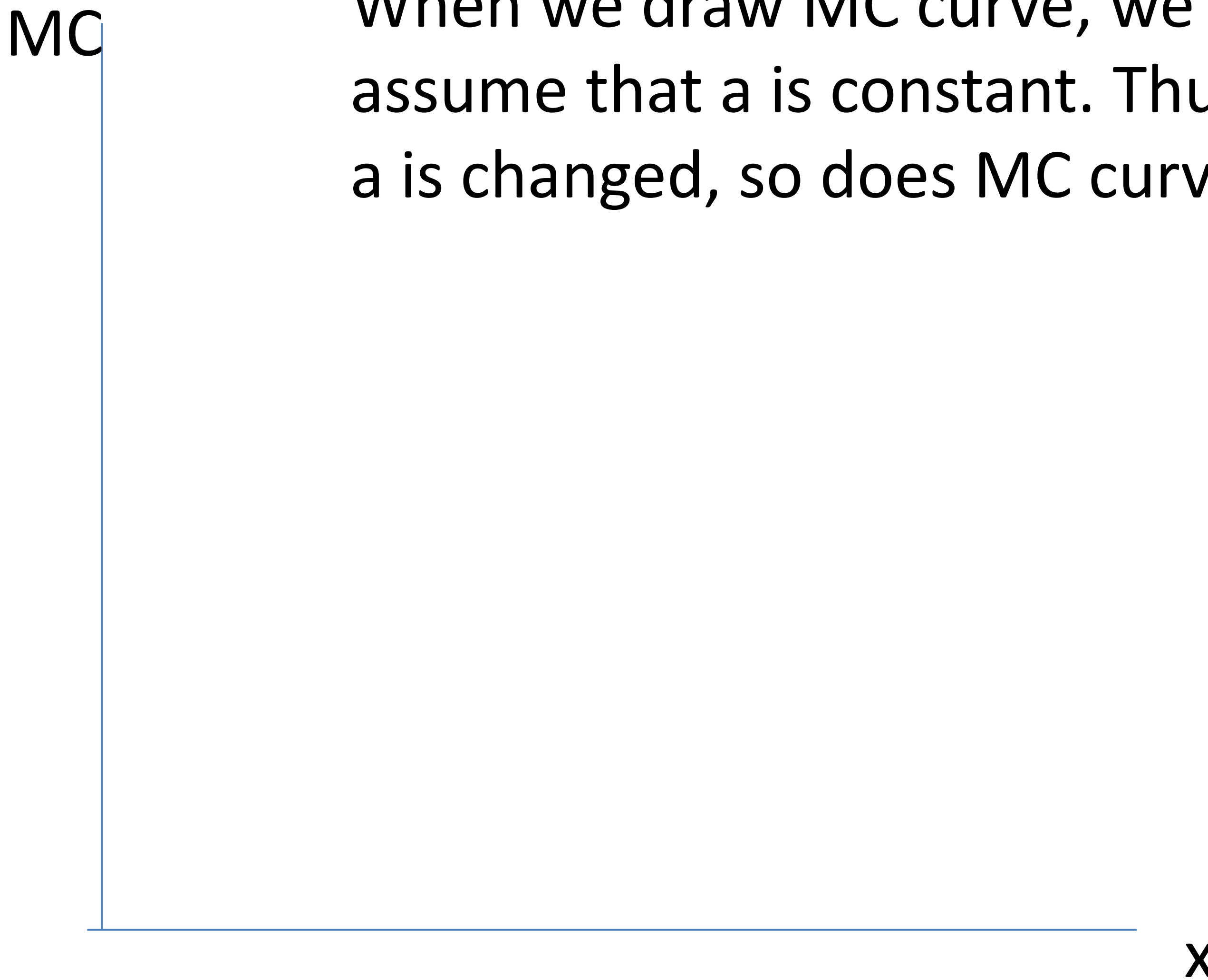
Application: Marginal analysis

- We heard the term marginal cost, marginal benefit, marginal rate of…
- Firm's total cost, y = f(x)=ax^2 +bx + c where x is an output, and a, b, c > 0 (these are parameters)
- Marginal cost or MC: ……….
- *We also want to know how MC changes when output changes*
- $\frac{\partial MC}{\partial x} = 2a; slope\ of\ \ MC\ curve$
- We may also want to know how MC changes when parameter changes (ie. a or b or c)
- $\frac{\partial MC}{\partial a} = 2x;\ MC\ curve\ will\ rotate\ by\ 2x$

Application: Marginal analysis

- *Beware of differences*
- $\frac{\partial MC}{\partial x} = 2a; slope\ of\ MC\ curve$
- while
- $\frac{\partial MC}{\partial a} = 2x;\ MC\ curve\ will\ rotate\ by\ 2x$
- Normally we do not draw a picture of the latter, since we focus on (y, x) pair. In other words, we cannot draw y in terms of all parameters

When we draw MC curve, we assume that a is constant. Thus if a is changed, so does MC curve.



Application: Elasticity

- Suppose we want to know the effect of a price change on demand for computer.
- Consider demand Q(P) where Q' < 0
- If you find derivatives of Q, the value depends on unit of quantity (tons or pounds) and price (Baht or USD)
- To avoid units of measurement and to make them comparable across different types of goods, we use elasticity
- For $y = f(x)$.
- Elasticity is defined as percentage change in y divided by percentage in x. It tell you that if x increase by 1 %, y will change by e%.

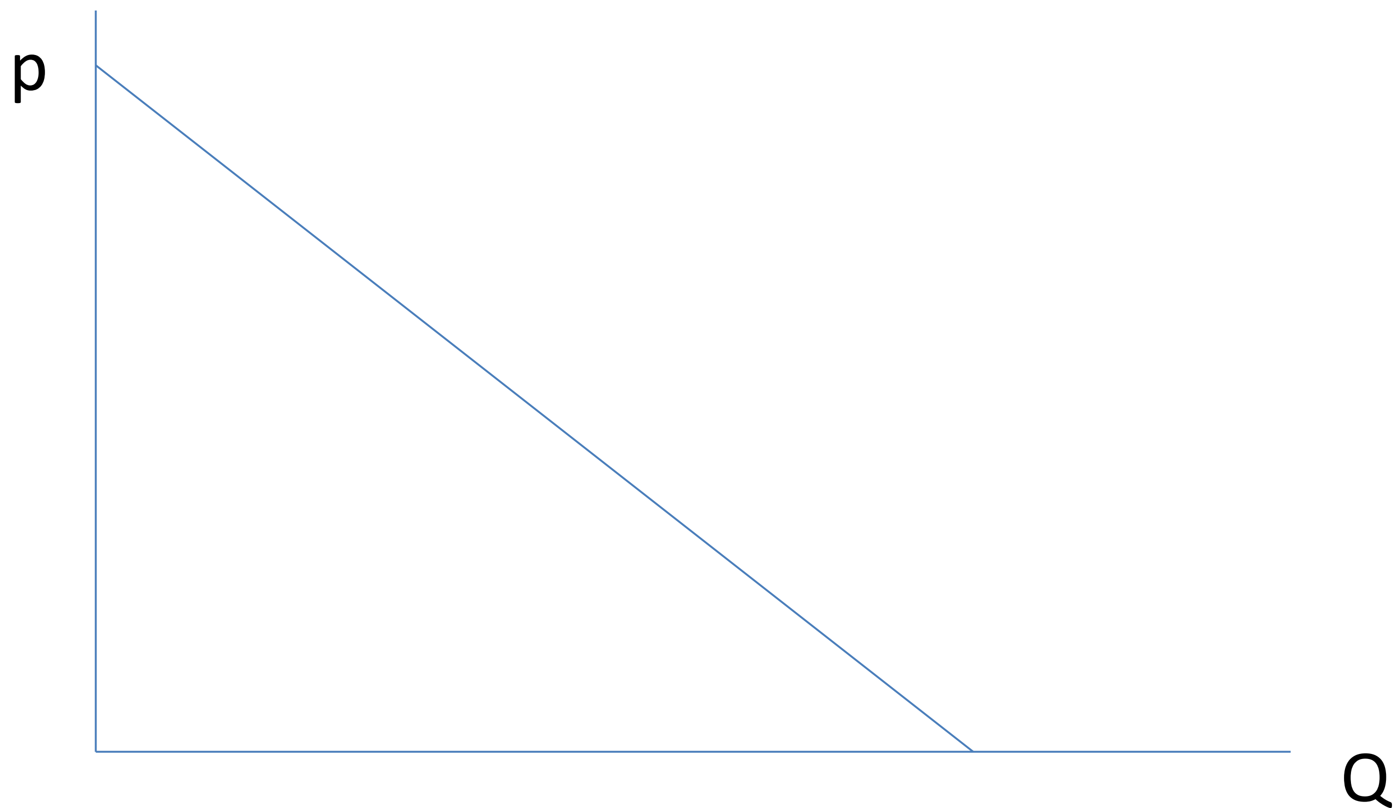
Application: Elasticity

- Consider demand Q(P) where Q' < 0
- Price elasticity of demand, $\varepsilon = \frac{dQ/Q}{dP/P} = Q' \frac{P}{Q}$
- Relation to marginal revenue; R(Q) = P(Q)Q;
- MR(Q) = $\frac{dP}{dQ}Q + P$

$$=\mathrm{P}\left(\frac{dP}{dQ}\frac{Q}{P} + 1\right) = P(\frac{1}{\varepsilon} + 1) = P\left(1 - \frac{1}{|\varepsilon|}\right)$$

Application: Elasticity

- try with $P = a - bQ$, show that $MR = a - 2bQ$



Application: Elasticity

- Relationship in log-log form: $\ln y = a \ln x$, $\varepsilon = a$

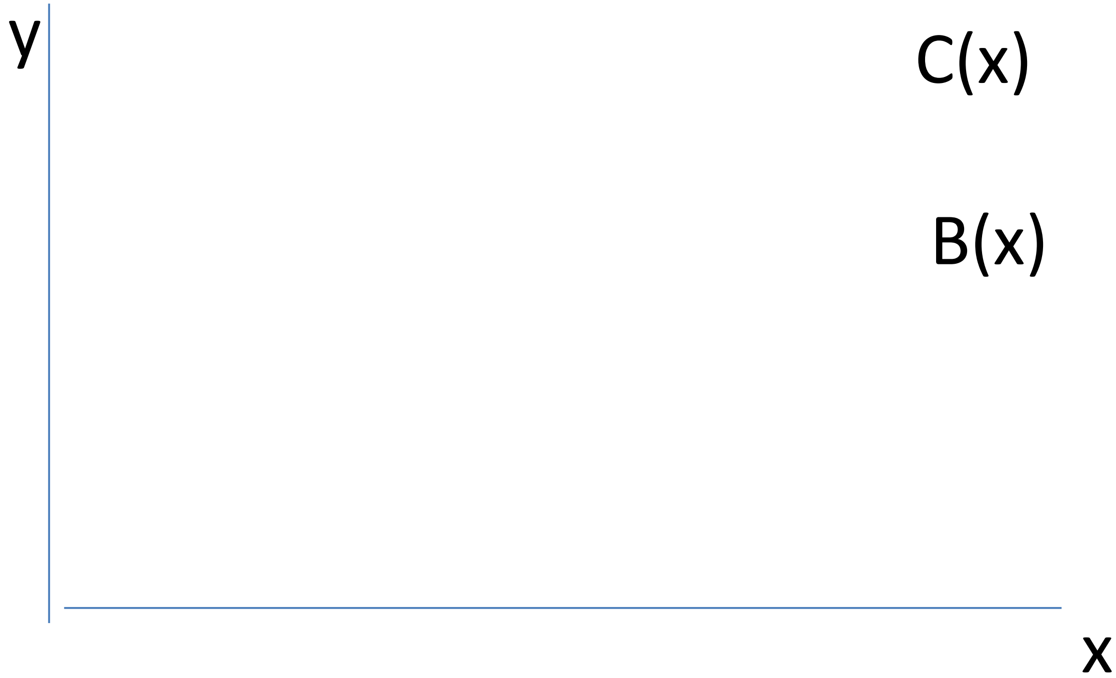
- Use this to transform difficult function. suppose $Q = 7P^3$ we need to find elasticity.
- by taking ln, show that elasticity $\varepsilon = 3$, show that $\varepsilon = d\ln Q \,/\, d\ln P$

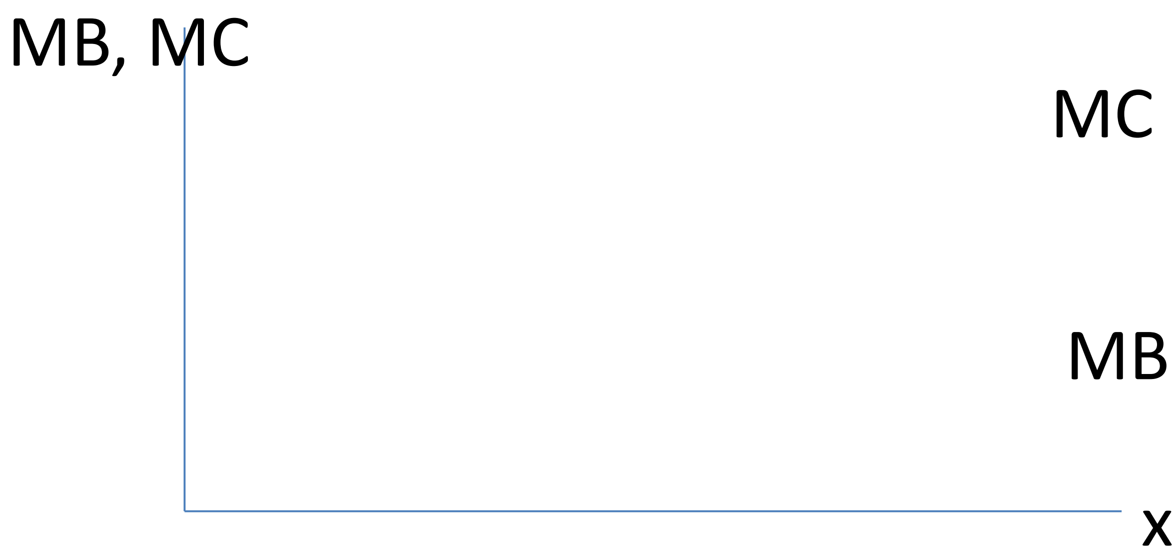
Generic example of economic maximization

- Core idea of microeconomics: agents maximize the net benefits of his action by setting marginal benefits equal to marginal cost.
- Suppose x is the level of economic activity
- Ie. Output of a firm
- Ie. Good consumption level of a consumer
- Assuming B(x) benefits of activity x
- And C(x) cost from activity x
- N(x)= net benefit of x

Generic example of economic maximization

- Consider y= N(x) = B(x) – C(x)
- FONC: B'(x) – C'(x) = 0 or MB = MC
- SOSC: B"(x) – C"(x) < 0 or B"(x) < C"(x)
- Slope of MB < slope of MC
- If MB > MC, shall we stop expanding next unit?


y
C(x)
B(x)
x


MB, MC
MC
MB
x

Generic example of economic maximization

- Suppose $B(x) = ax - bx^2$; $C(x) = cx^2 + f$ and a, b, c, d > 0.
- Show that $x^* = a / 2(b+c)$, and
- SOSC = $-2(b+c) < 0$

Generic example of economic maximization

- Next, find y*= $N(x^{*}) = \frac{a^2}{4(b+c)} - f$.
- Thus, we find our solutions: x* and y*

- We can look at comparative statics of the solutions x* and y*.
- There are four possible checks, depending on a, b, c and f.
- If we check how our solutions change when f or c changes, we get
- $\frac{\partial y*}{\partial f} = -1$; $\frac{\partial x*}{\partial f} = 0$; $\frac{\partial y*}{\partial c} < 0$; $\frac{\partial x*}{\partial c} < 0$;
- Explain meaning in economics