1. Find cournot equilibrium when there are 3 firms in the market?

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$P = a - bQ$; $Q = q_1 + q_2 + q_3$

$C_1 = C_2 = C_3 = C$

what is equilibrium price?

what are firm profit?

$$\pi_i = (P-C)q_i = (a - b(q_1+q_2+q_3))\, q_1 - c)$$

$$\text{Max } \pi_i : \frac{\partial \pi_i}{\partial q_i} = 0 \longrightarrow \frac{\partial \pi_i}{\partial q_1} = (a - b(q_1+q_2+q_3)) = 0$$

$$q_1 = a - bq_1 - bq_2 - bq_3 - bq_1$$

$$0 = a - bq_2 - bq_3 - 2bq_1$$

$$2bq_1 = a - bq_2 - bq_3$$

$$q_1^* = \frac{a - bq_2 - bq_3}{2b} \quad \text{—— (1)}$$

$$q_2^* = \frac{a - bq_1 - bq_3}{2b} \quad \text{—— (2)}$$

$$q_3^* = \frac{a - bq_1 - bq_2}{2b} \quad \text{—— (3)}$$

sub (2) in (1) ; $q_1^* = \dfrac{a - b\left(\frac{a - bq_1 - bq_3}{2b}\right) - bq_3}{2b}$

$$q_1 \times 2b = a - \frac{1}{2}a + \frac{1}{2}c + \frac{bq_1}{2} + \frac{bq_3}{2} - bq_3 \quad ① \times 2$$

$$4b \times q_1 = 2a - a + bq_1 + bq_3 - 2bq_3$$

$$4bq_1 = a + bq_1 - bq_3$$

$$3bq_1 = a - bq_3$$

$$q_1^* = \frac{a - bq_3}{3b}$$

sub (1) in (2) ; $q_2^* = \dfrac{a - b\left(\frac{a - bq_1 - bq_3}{2b}\right) - bq_3}{2b}$

$$q_2^* = \frac{a - bq_3}{3b}$$

sub q_1^* and q_3^* in (3) ; $q_3^* = \dfrac{a - b\left(\frac{a - bq_3}{3b}\right) - b\left(\frac{a - bq_3}{3b}\right)}{2b}$

$$2b\, q_3^* = a - \frac{a}{3} + \frac{bq_3}{3} - \frac{a}{3} + \frac{bq_3}{3} \quad ③ \times 3$$

$$6bq_3^* = 3a - a + bq_3 - a + bq_3$$

$$6bq_3^* = a + 2bq_3$$

$$4bq_3^* = a$$

$$q_3^* = \frac{a}{4b} \quad \#$$

sub q_3^* ; $q_1^* = \dfrac{a - b\left(\frac{a}{4b}\right)}{3b}$

$$3bq_1^* = a - \frac{a}{4}$$

$$12bq_1^* = 4a - a$$

$$q_1^* = \frac{3a}{12b}$$

$$q_1^* = \frac{a}{4b} \quad \#$$

$$q_2^* = \frac{a}{4b} \quad \#$$

$$Q = q_1 + q_2 + q_3 = \frac{a}{4b} + \frac{a}{4b} + \frac{a}{4b} = \frac{3a}{4b}$$

$$P = a - b\,\frac{3a}{4b}$$

$$P = a - \frac{3a}{4}$$

$$P = a - 0.75a$$

$$P = 0.25a \quad \#$$

$$\pi_i = (P-C)q_i = (0.25a - C)\,\frac{a}{4b}$$

$$\pi_1 = (0.25a - C)\,\frac{a}{4b} = \frac{a^2}{16b} - C$$

$$\pi_1 = \pi_2 = \pi_3 = \frac{a^2}{16b} - C \quad \#$$

2 If there are N firms

$q_i^* = f(N), P = f(N), \pi_i = f(N)$

assume $q_1 + q_2 + \ldots + q_n = A$

$P = a - b(q_1 + q_2 + \ldots + q_n)$

$P = a - bq_1 - bq_2 - \ldots - bq_n)$

$\pi_i = (P-C)q_i \quad \pi_1 = (a - bq_1 - bq_2 - \ldots - bq_n)q_1 - C_1$

$$\frac{\partial \pi_1}{\partial q_1} = 0 \Longrightarrow q_1 = \frac{a}{2b} - 0.5(q_2 + q_3 + \ldots + q_n)$$

$$\vdots$$

$$q_n = \frac{a}{2b} - 0.5(q_2 + q_3 + \ldots + q_{n-1})$$

$$\boxed{\therefore q_1^* = q_2^* = \ldots = q_n^* = \frac{a}{b} - A} \quad (1)$$

$A = q_1 + q_2 + \ldots + q_n$

$A = n(\frac{a}{b} - A)$

$A = \frac{na}{b} - nA$

$A + nA = n(\frac{a}{b})$

$A(n+1) = n(\frac{a}{b}) \longrightarrow A = \frac{na}{(n+1)b}$

sub A in to (1) ; $q_1 = \frac{a}{(n+1)b}$

$\therefore q_i = \frac{a}{(n+1)b}$

$P = a - bA$

$= a - (\frac{n}{n+1})a$

$= \frac{a-(n+1)-na}{n+1}$

$P = \frac{a}{n+1}$

$\pi_i = P \cdot q_i + C_i$

$= \frac{a}{n+1} \cdot \frac{a}{(n+1)b} - C_i$

$\therefore \pi_i = \frac{a^2}{(n+1)^2 b} - C_i$

3. From question 2, what happen if $N \to \infty$

⊢——//——⊣ $N = 1$

In general

- If $n \to \infty$, it mean the output has driven to a competitive level and the price (p^*) converges to marginal cost. The market is perfectly competitive market.
- In cournot model, there must be 2 or more firms competing in the market. So, if $n = 1$, firm will become monopolist

if $n \to \alpha$;

- $q_i = \frac{a}{(n+1)b}$ $\longrightarrow$ nearly 0 and each firm will sell at Q nearly 0 unit
- $A = \frac{na}{(n+1)b}$ $\longrightarrow$ nearly 0. Q of every firms combinded will be nearly $\to \alpha$ unit
- $P = \frac{a}{n+1}$ $\longrightarrow$ make $P = \frac{a}{n+1}$ nearly 0. when supply increase, P will decrease, nearly 0
- $\pi_i = \frac{a^2}{(n+1)^2 b} - c_i$ $\longrightarrow$ each firms will lose their profit

if $n = 1$

- $q_i = \frac{a}{(n+1)b} = \frac{a}{2b}$ $\longrightarrow$ since $q = \frac{a}{2b} < q = \frac{na}{(n-1)b}$, monopoly will sell less
- $A = \frac{na}{(n+1)b} = q$ $\longrightarrow$ it mean than firm will become a monopolist
- $P = \frac{a}{n+1} = \frac{a}{2}$ $\longrightarrow$ since $P_m = \frac{a}{2} > P_c = \frac{a}{n+1}$, monopoly will set higher price.
- $\pi_i = \frac{a^2}{(n+1)^2 b} - c_i = \frac{a^2}{4b} - c_i$ $\longrightarrow$ since $\pi_m > \pi_c$, monopolist will earn higher profit.