Statistics for Business and Economics (13e)

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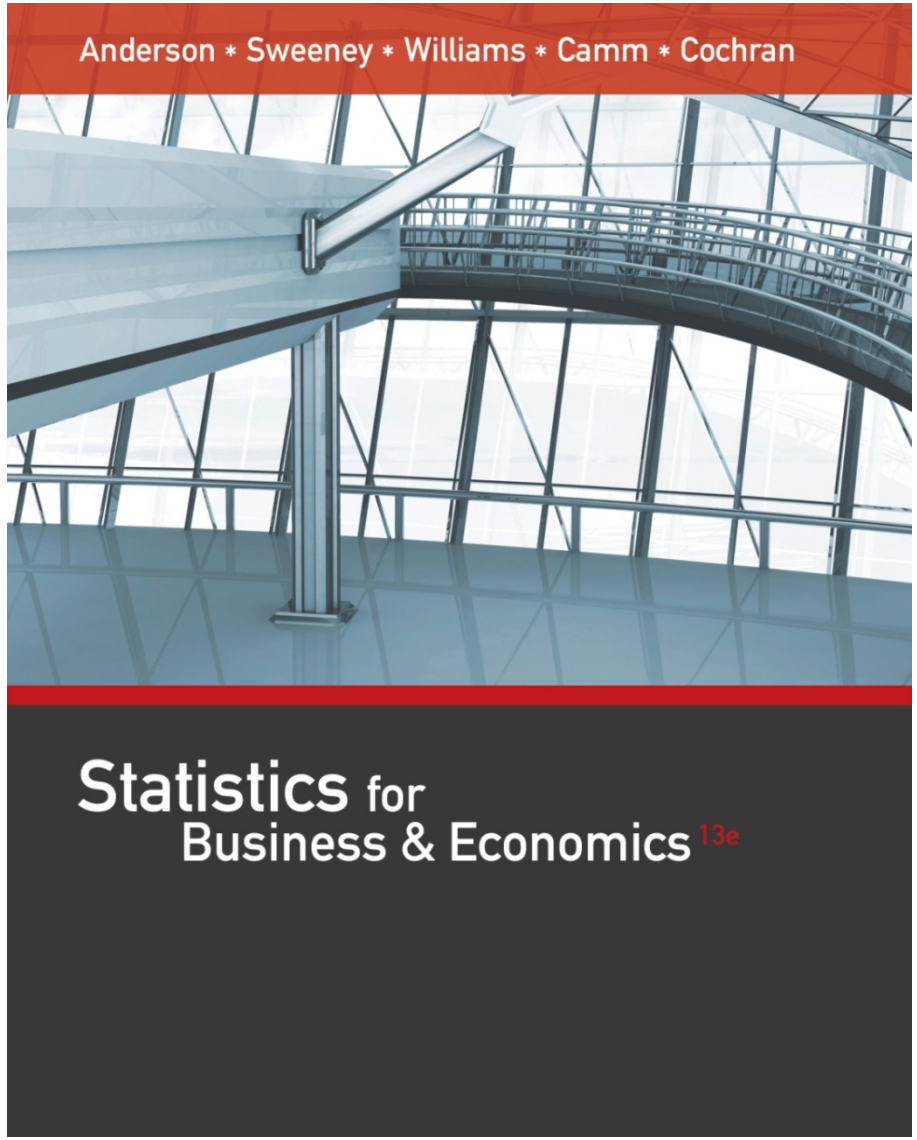


Chapter 20
Index Numbers

- Price Relatives
- Aggregate Price Indexes
- Computing an Aggregate Price Index from Price Relatives
- Some Important Price Indexes
- Deflating a Series by Price Indexes
- Price Indexes: Other Considerations
- Quantity Indexes

Price Relatives

- Price relatives are helpful in understanding and interpreting changing economic and business conditions over time.
- A price relative shows how the current price per unit for a given item compares to a base period price per unit for the same item.
- A price relative expresses the unit price in each period as a percentage of the unit price in the base period.
- A base period is a given starting point in time.

$$\text{Price relative in period } t = \frac{\text{Price in period } t}{\text{Base period price}}(100)$$

Price Relatives

- Example: Besco Products

  The prices Besco paid for newspaper and television ads in 2004 and 2014 are shown below. Using 2004 as the base year, compute a 2014 price index for newspaper and television ad prices.

	2004	2014
Newspaper	$14,794	$29,412
Television	11,469	23,904

Price Relatives

Newspaper

$$I_{2014} = \frac{29{,}412}{14{,}794}(100) = 199$$

Television

$$I_{2014} = \frac{23{,}904}{11{,}469}(100) = 208$$

Television advertising cost increased at a greater rate.

Aggregate Price Indexes

- An aggregate price index is developed for the specific purpose of measuring the combined change of a group of items.
- An unweighted aggregate price index in period t, denoted by I_t, is given by

$$I_t = \frac{\sum P_{it}}{\sum P_{i0}} (100)$$

where

P_{it} = unit price for item i in period t

P_{i0} = unit price for item i in the base period

Aggregate Price Indexes

- With a weighted aggregate index each item in the group is weighted according to its importance, which typically is the quantity of usage.
- Letting Q_i = quantity for item i, the weighted aggregate price index in period t is given by

$$I_t = \frac{\sum P_{it} Q_i}{\sum P_{i0} Q_i} (100)$$

where the sums are over all items in the group

Aggregate Price Indexes

- When the fixed quantity weights are determined from the base-year usage, the index is called a Laspeyres index.
- When the weights are based on period *t* usage, the index is a Paasche index.

Aggregate Price Indexes

- Example: City of Rockdale

  Data on energy consumption and expenditures by sector for the city of Rockdale are given on the next slide. Construct an aggregate price index for energy expenditures in 2014 using 1993 as the base year.

Aggregate Price Indexes

- Example: City of Rockdale

	Quantity (BTU)		Unit Price ($/BTU)	
Sector	1993	2014	1993	2014
Residential	9,473	8,804	$2.12	$10.92
Commercial	5,416	6,015	1.97	11.32
Industrial	21,287	17,832	.79	5.13
Transport.	15,293	20,262	2.32	6.16

Aggregate Price Indexes

- Example: City of Rockdale

Unweighted Aggregate Price Index

$$I_{2014} = \frac{10.92 + 11.32 + 5.13 + 6.16}{2.12 + 1.97 + .79 + 2.32}(100) = 466$$

Aggregate Price Indexes

- Example: City of Rockdale

Weighted Aggregate Index (Laspeyres Method)

$$I_{2014} = \frac{10.92(9{,}473) + \cdots + 6.16(15{,}293)}{2.12(9{,}473 + \cdots + 2.32(15{,}293)} (100) = 443$$

Weighted Aggregate Index (Paasche Method)

$$I_{2014} = \frac{10.92(8{,}804) + \cdots + 6.16(20{,}262)}{2.12(8{,}804) + \cdots + 2.32(20{,}262)} (100) = 415$$

The Paasche value being less than the Laspeyres indicates usage has increased faster in the lower-priced sectors.

Aggregate Price Indexes

- Example: Annual Cost of Lawn Care

  Dina Evers is pleased with her lovely lawn, but she is concern about the increasing cost of maintaining it. The cost includes mowing, fertilizing, watering, and more.

  Dina wants an index that measures the change in the overall cost of her lawn care. Price and quantity data for her annual lawn expenses are listed on the next slide.

Aggregate Price Indexes

- Example: Annual Cost of Lawn Care

Item	Quantity (Units)	Unit Price ($) 2010	2014
Mowing	32	57.00	79.00
Leaf Removal	3	56.00	71.00
Watering (1000s gal.)	40	1.83	2.78
Fertilizing	2	56.00	67.00
Sprinkler Repair	1	109.00	128.00

Aggregate Price Indexes

Unweighted

$$I_i = \frac{\sum P_{it}}{\sum P_{i0}}(100)$$

where:

P_{it} = unit price for item *i* in period *t*

P_{i0} = unit price for item *i* in the base period

$$I_{2014} = \frac{79.00 + 71.00 + 2.78 + 67.00 + 128.00}{57.00 + 56.00 + 1.83 + 56.00 + 109.00}(100) = 124$$

Annual lawn care expenses increased 24% from 2010 to 2014.

Aggregate Price Indexes

Weighted (Fixed Quantity)

$$I_t = \frac{\sum P_{it} Q_i}{\sum P_{i0} Q_i} (100)$$

where: Q_i = quantity for item i

$$I_{2014} = \frac{79.00(32)+71.00(3)+2.78(40)+67.00(2)+128.00(1)}{57.00(32)+56.00(3)+1.83(40)+56.00(2)+109.00(1)} (100) = 136$$

Annual lawn care expenses increased 36% from 2010 to 2014.

Aggregate Price Indexes

Weighted (Base-Period Quantity) Laspeyres Index

Special case of the fixed quantity index

$$I_t = \frac{\sum P_{it} Q_{i0}}{\sum P_{i0} Q_{i0}} (100)$$

More widely used than the Paasche index

Weighted (Period t Quantity) Paasche Index

A variable-quantity index

$$I_t = \frac{\sum P_{it} Q_{it}}{\sum P_{i0} Q_{it}} (100)$$

Pro: Reflects current usage;
Con: Weights require continual updating

Aggregate Price Indexes

Item i	Unit Price ($) 2010 P_{i0}	2014 P_{it}	Price Relative $(P_{it}/P_{i0})100$
Mowing	57.00	79.00	138.6
Leaf Removal	56.00	71.00	126.8
Water (1000s gal.)	1.83	2.78	151.9
Fertilizing	56.00	67.00	119.6
Sprinkler Repair	109.00	128.00	117.4

The 5-year increases in unit price ranged from a low of 17.4% for sprinkler repair to a high of 51.9% for water.

Aggregate Price Indexes

Item	Price Relative $(P_{it}/P_{i0})100$	Base Price ($) P_{i0}	Quantity Q_i	Weight $w_i = P_{i0}Q_i$	Weighted Price Relative $(P_{it}/P_{i0})(100)w_i$
Mowing	138.6	57.00	32	1,824.0	252,806.40
Leaves	126.8	56.00	3	168.0	21,302.40
Water	151.9	1.83	40	73.2	11,119.08
Fertilize	119.6	56.00	2	112.0	13,395.20
Sprinkler	117.4	109.00	1	109.0	12,796.60
			Total	2,286.2	311,419.68

$$I_t = \frac{\sum \frac{P_{it}}{P_{i0}}(100)w_i}{\sum w_i} = \frac{311{,}419.68}{2{,}286.2} = 136$$

This value is the same as the one identified by the weighted aggregate index computation.

Some Important Price Indexes

- Consumer Price Index (CPI)
  - Primary measure of the cost of living in U.S.
  - Based on 400 items including food, housing, clothing, transportation, and medical items.
  - Weighted aggregate price index with fixed weights derived from a usage survey.
  - Published monthly by the U.S. Bureau of Labor Statistics.
  - Its base period is 1982-1984 with an index of 100.

Some Important Price Indexes

Consumer Price Index (CPI)

Base 1982-1984 = 100.0

Year	CPI	Year	CPI	Year	CPI	Year	CPI
1980	82.4	1989	124.0	1998	163.0	2007	207.3
1981	90.9	1990	130.7	1999	166.6	2008	215.3
1982	96.5	1991	136.2	2000	172.2	2009	214.5
1983	99.6	1992	140.3	2001	177.1	2010	218.1
1984	103.9	1993	144.5	2002	179.9	2011	224.9
1985	107.6	1994	148.2	2003	184.0	2012	229.6
1986	109.6	1995	152.4	2004	188.9	2013	233.0
1987	113.6	1996	156.9	2005	195.3	2014	236.7
1988	118.3	1997	160.5	2006	201.6	2015	???

Note: For 1982 – 1984, (96.5 + 99.6 + 103.9)/3 = 100.0

Also note: CPI for 2009 was lower than CPI for 2008.

Some Important Price Indexes

- Producer Price Index (PPI)
  - Measures the monthly changes in prices in primary markets in the U.S.
  - Used as a leading indicator of the future trend of consumer prices and the cost of living.
  - Covers raw, manufactured, and processed goods at each level of processing.
  - Includes the output of manufacturing, agriculture, forestry, fishing, mining, gas and electricity, and public utilities.
  - Is a weighted average of price relatives using the Laspeyres method.

Some Important Price Indexes

- Dow Jones Averages
  - Indexes designed to show price trends and movements on the New York Stock Exchange.
  - The Dow Jones Industrial Average (DJIA) is based on common stock prices of 30 industrial firms.
  - The DJIA is not expressed as a percentage of base-year prices.
  - Another average is computed for 20 transportation stocks, and another for 15 utility stocks.

Some Important Price Indexes

Dow Jones Industrial Average (DJIA)

30 Companies (as of 03/2015)

3M	General Electric	Nike
American Express	Goldman Sachs	Pfizer
Apple	The Home Depot	Procter & Gamble
Boeing	IBM	Travelers
Caterpillar	Intel	UnitedHealth Group
Chevron Corp.	Johnson & Johnson	United Technologies
Cisco Systems	JPMorgan Chase	Verizon
Coca-Cola	McDonald's	Visa
DuPont	Merck	Wal-Mart
ExxonMobil	Microsoft	Walt Disney

Deflating a Series by Price Indexes

- In order to correctly interpret business activity over time when it is expressed in dollar amounts, we should adjust the data for the price-increase effect.
- Removing the price-increase effect from a time series is called deflating the series.
- Deflating actual hourly wages results in real wages or the purchasing power of wages.

Deflating a Series by Price Indexes

- Example: McNeer Cleaners

  McNeer Cleaners, with 46 branch locations, has had the total sales revenues shown on the next slide for the last five years. Deflate the sales revenue figures on the basis of 1982-1984 constant dollars. Is the increase in sales due entirely to the price-increase effect?

Deflating a Series by Price Indexes

- Example: McNeer Cleaners

Year	Total Sales ($1000)	CPI
2010	8,446	218.1
2011	9,062	224.9
2012	9,830	229.6
2013	10,724	233.0
2014	11,690	236.7

Deflating a Series by Price Indexes

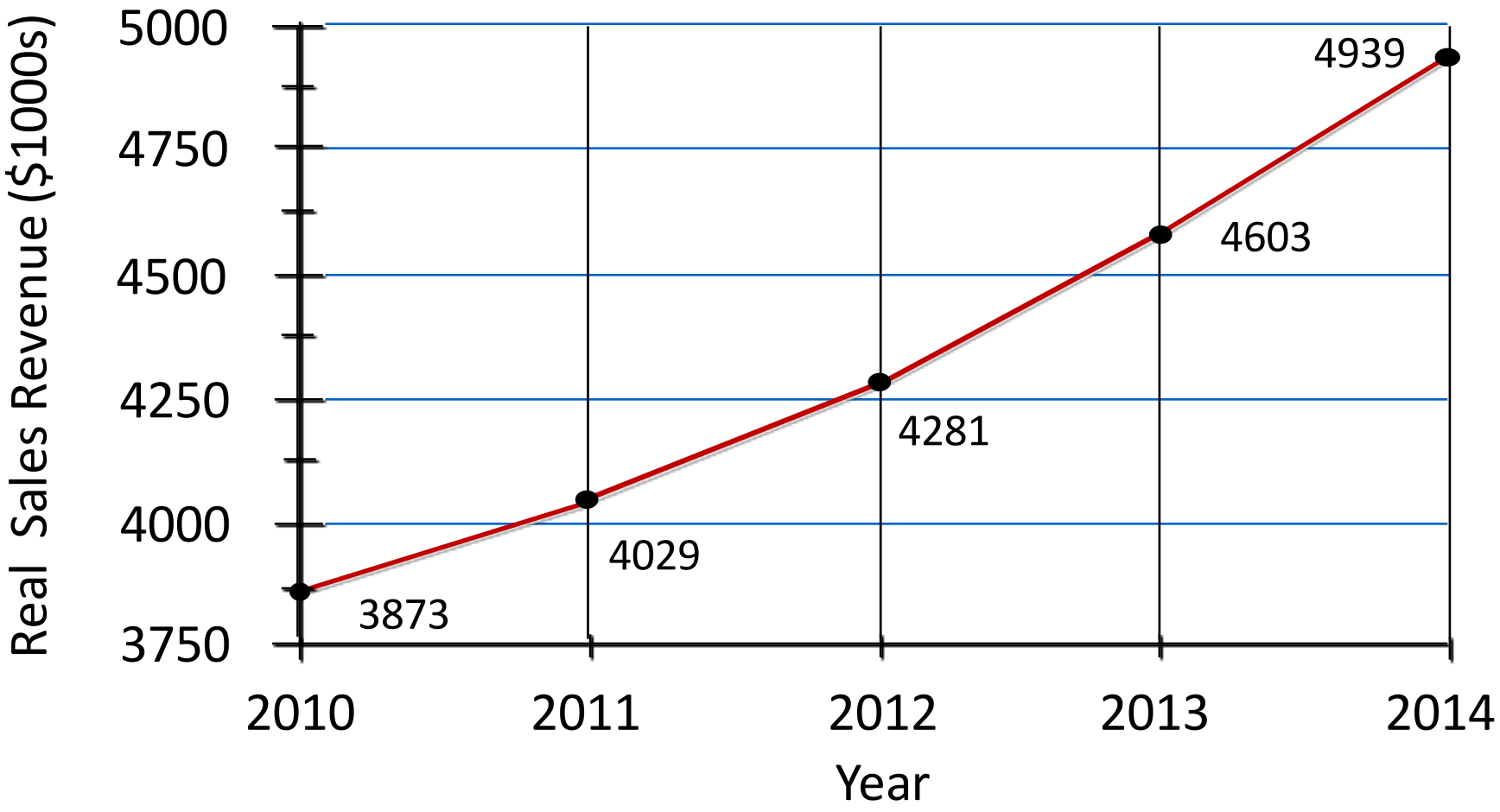
- Adjusting Revenue For the Price-Increase Effect

Year	Deflated Sales ($1000)	Annual Change (%)
2010	(8,446/218.1)(100) = 3,873	
2011	(9,062/224.9)(100) = 4,029	+4.0
2012	(9,830/229.6)(100) = 4,281	+6.3
2013	(10,724/233.0)(100) = 4,603	+7.5
2014	(11,690/236.7)(100) = 4,939	+7.3

After adjusting, revenue is still increasing at an average rate of 6.3% per year.

Deflating a Series by Price Indexes

A real sales increase of 27.5% from 2010 to 2014



Price Indexes: Other Considerations

- Selection of Items
  - When the class of items is very large, a representative group (usually not a random sample) must be used.
  - The group of items in the aggregate index must be periodically reviewed and revised if it is not representative of the class of items in mind.

Price Indexes: Other Considerations

- Selection of a Base Period
  - As a rule, the base period should not be too far from the current period.
  - For example, a CPI with a 1945 base period would be difficult for most individuals to understand, due to unfamiliarity with conditions in 1945.
  - The base period for most indexes is adjusted periodically to a more recent period of time.
  - The CPI's base period was changed to 1982-84 in 1988.

Price Indexes: Other Considerations

- Quality Changes
  - A basic assumption of a price index is that prices over time are identified for the same item.
  - Is a product that has undergone a major quality change the same product it was ?
  - An increase in an item's quality may or may not result in a price increase . . . and a decrease in quality may or may not result in a price decrease.

Quantity Indexes

- An index used to measure changes in quantity levels over time is called a quantity index.
- A quantity relative shows how the current quantity level for a single item compares to a base period quantity level for the same item.
- A weighted aggregate quantity index is computed in much the same way as a weighted aggregate price index.

$$I_t = \frac{\sum Q_{it} w_i}{\sum Q_{i0} w_i} (100)$$

Quantity Indexes

- Example: Appliance Mart

  Appliance Mart reports the 2005 and 2014 sales for three major kitchen appliances as shown below. Compute quantity relatives and use them to develop a weighted aggregate quantity index for 2014.

Kitchen Appliance	Sales Price (2005)	Sales (Units) 2005	Sales (Units) 2014
Dishwasher	$325	720	950
Range	$450	540	610
Refrigerator	$710	980	1110

Quantity Indexes

$$\text{Quantity relative in period } t = \frac{\text{Quantity in period t}}{\text{Base period quantity}}(100)$$

Kitchen Appliance	Sales (Units) 2005	2014	Quantity Relative
Dishwasher	720	950	(950/720)100 = 131.9
Range	540	610	(610/540)100 = 113.0
Refrigerator	980	1110	(1110/980)100 = 113.3

Quantity Indexes

Unweighted

$$I_t = \frac{\sum Q_{it}}{\sum Q_{i0}}(100)$$

where: Q_{it} = quantity for item *i* in period *t*

Q_{i0} = quantity for item *i* in the base period

$$I_{2014} = \frac{950+610+1110}{720+540+980}(100) = 119$$

Quantity Indexes

Weighted (Base-Period Price)

$$I_t = \frac{\sum Q_{it} P_{i0}}{\sum Q_{i0} P_{i0}} (100)$$

where: P_{i0} = price for item *i* in the base period

$$I_{2014} = \frac{950(325)+610(450)+1110(710)}{720(325)+540(450)+980(710)} (100) = 117$$

Quantity Indexes

Weighted (Fixed Price)

$$I_t = \frac{\sum Q_{it} P_i}{\sum Q_{i0} P_i}(100)$$

Weighted (Period t Price)

$$I_t = \frac{\sum Q_{it} P_{it}}{\sum Q_{i0} P_{it}}(100)$$

End of Chapter 20