6004820012
Hongvichny Hor

Seat No................................
 ID.No...6004820012............

Instruction:

Exam time: 30 minutes.

You may use a calculator, turn off cell phones. Phone communication are strictly prohibited during the exam.

For each question, write your answer in the blank space provided.

Manage your time carefully and answer as many questions as you can.

Seat No...............................
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Question1 (40 points)

Your score.....................

Suppose the daily log return r_t of Stock A follows the model:

$$r_t = 0.002 + a_t$$

$$a_t = \sigma_t \epsilon_t$$

where ϵ_t is an independent and identically distributed (iid) sequence of standardized Student-t distribution with 5 degrees of freedom. In addition,

$$\sigma_t^2 = 0.01 + 0.1 a_{t-1}^2$$

.

Question1.1 (10 points)

Your score.....................

From the above model, Find out the unconditional Expectation of a_t :$E(a_t)$ and the unconditional expectation of r_t :$E(r_t)$

, Find out the unconditional Expectation of a_t :$E(a_t)$ / $a_t = \sigma_t \epsilon_t$

$\rightarrow E(a_t) = E(\sigma_t \epsilon_t)$

$= \sigma_t^2$

: unconditional expectation

$r_t = 0.002 + a_t$

$r_t = 0.002 + a_t$ / $E(r_t) = E(\overset{\Rightarrow 0}{0.002}) + E(\overset{\Rightarrow 0}{a_t})$

$E(r_t) = 0$

Question1.2 (10 points)

$$r_t = 0.002 + a_t$$

$$a_t = \sigma_t \epsilon_t$$

Your score.....................

Find out the unconditional variance of a_t :$Var(a_t)$ and the conditional variance of a_t :$Var(a_t|F_{t-1})$

unconditional

$E[a_t] = 0$

$var(a_t) = \text{constant}$

i.e. $var(a_t) = \frac{\alpha_0}{1-\alpha_1}$

$= \frac{0.002}{1-1}$

$=$

conditional

$E[a_t | F_{t-1}] = 0$

$var[a_t | F_{t-1}] = \sigma_t^2$ ← not constant

i.e. $\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2$

if we apply ARCH(1)

$\Rightarrow \sigma_t^2 = 0.002 + 1\, a_{t-1}^2$

Question1.3 (10 points)

$$r_t = 0.002 + a_t$$

$$a_t = \sigma_t \epsilon_t$$

Your score.....................

Let h = 100 be the forecast origin with $a_h = 0.015$ and $\sigma_h = 0.2$. Calculate the 1-step ahead prediction $r_h(1)$ and 1-step ahead volatility forecast.

$r_{h+1} = c_0 - \theta_1 a_h + a_{h+1}$

$= 0.002 + \theta_1 a_h$

$= 0.002 + \theta a_h$

$= 0.002 + 0.2(0.015)$

$= 5 \times 10^{-3} = 0.005$

$\hat{r}_h(1) + 1.96\sqrt{\sigma_a^2}$

$\hat{r}_h(1) = c_0 - \theta_1 a_h$

$e_h(1) = r_h - \hat{r}_h(1) = a_{h+1}$

$$\sigma_t^2 = 0.01 + 0.1 a_{t-1}^2$$

$var(e_h(1)) = \sigma_a^2$

Question1.4 (10 points) Your score.....................

Calculate the ∞-step ahead prediction $r_h(\infty)$ and the ∞-step ahead volatility forecast at the forecast origin h.

$$r_{h+1} = c_0 - \theta_1 a_h + a_{h+1}$$

$$\hat{r}_h(\infty) = c_0 - \theta_1 a_h$$

$$\lim_{\ell \to \infty} \hat{r}_h(\ell) = (\text{unconditional mean of } r) = 0$$

$$\lim_{\ell \to \infty} Var(e_h(\ell)) = \text{unconditional variance} = \Rightarrow \sigma_t^2 = 0.002 + 1\, a_{t-1}^2$$