Chapter 2 Methods of Proofs

We are now prepared to begin our main topic: mathematical proofs. It is essential to know what methods are available to us if we wish to verify the truth of a mathematical statement.

Section 2.1 Rules for Proving

Using a truth table is one of the methods of proving but it is not convenient when we have to prove the statement that combines many statements together. To solve this problem we use the *Rules for Proving* instead of the truth table.

In proving, we use the following principle.

1. The truth value of the given statement is always true.
2. Use the given statements, laws of tautology, rules for proving, and the statements that have been proved to prove the given conclusion that its truth value is true.

Note that we will only write the statements that truth value is true and we will write only "*p*" when we want to say "*the truth value of p is true.*"

Rules for Proving

2.1.1 Rule of Modus Ponens or Rule of Implication

If $p \to q$ and p, then q

Proof:

2.1.2 Rule of Substitution

There are two types of using Rule of Substitution.

1) Substitute the statement that is tautology (or contradiction) with the other statement
2) Substitute the statement with its equivalent statement.

Proof: By the theorem of tautology, theorem of contradiction, and the theorem of equivalence.

For example,

2.1.3 Definition of Conjunction

There are two types of using Definition of Conjunction.

1) If $p \wedge q$, then p and q.
2) If p and q, then $p \wedge q$

2.1.4 Rule of Modus Tollens

If $p \to q$ and $\sim q$, then $\sim p$

Proof:

2.1.5 Disjunctive Syllogism

There are two types of using Disjunctive Syllogism.

1) If $p \vee q$ and $\sim p$, then q.

2) If $p \vee q$ and $\sim q$, then p.

Proof:

2.1.6 Constructive Dilemma

If $(p \to q) \wedge (r \to s)$ and $p \vee r$, then $q \vee s$

Section 2.2 Direct Proof

The direct proof is the proof that start with what is given then use the rules for proving, axioms, theorems, laws of tautology, or the statements that have been proved to make a conclusion.

Example 1: Prove that if $u \vee (r \to t)$ and $\sim u \wedge \sim t$, then $\sim r$.

Example 2: Prove that if $x \to (y \to z)$, $\sim y \to \sim x$, and x, then z.

Example 3: Prove that if $\sim p \to \sim q$, $\sim u$, $p \to t$, and $q \vee u$, then t.

Example 4: Show $\sim t$ if $\left[r \to (s \to \sim t)\right] \wedge \left[(\sim s \to \sim r) \wedge r\right]$.