case of the binomial option model, as the holding period is divided into progressively smaller subperiods when the interest rate and stock volatility are constant.

5. The Black-Scholes formula applies to options on stocks that pay no dividends. Dividend adjustments may be adequate to price European calls on dividend-paying stocks, but the proper treatment of American calls on dividend-paying stocks requires more complex formulas.
6. Put options may be exercised early, whether the stock pays dividends or not. Therefore, American puts generally are worth more than European puts.
7. European put values can be derived from the call value and the put-call parity relationship. This technique cannot be applied to American puts for which early exercise is a possibility.
8. The implied volatility of an option is the standard deviation of stock returns consistent with an option's market price. It can be backed out of an option-pricing model by finding the stock volatility that makes the option's value equal to its observed price.
9. The hedge ratio is the number of shares of stock required to hedge the price risk involved in writing one option. Hedge ratios are near zero for deep out-of-the-money call options and approach 1.0 for deep in-the-money calls.
10. Although hedge ratios are less than 1.0, call options have elasticities greater than 1.0. The rate of return on a call (as opposed to the dollar return) responds more than one-for-one with stock price movements.
11. Portfolio insurance can be obtained by purchasing a protective put option on an equity position. When the appropriate put is not traded, portfolio insurance entails a dynamic hedge strategy where a fraction of the equity portfolio equal to the desired put option's delta is sold and placed in risk-free securities.
12. The option delta is used to determine the hedge ratio for options positions. Delta-neutral portfolios are independent of price changes in the underlying asset. Even delta-neutral option portfolios are subject to volatility risk, however.
13. Empirically, implied volatilities derived from the Black-Scholes formula tend to be lower on options with higher exercise prices. This may be evidence that the option prices reflect the possibility of a sudden dramatic decline in stock prices. Such "crashes" are inconsistent with the Black-Scholes assumptions.

Related Web sites for this chapter are available at **www.mhhe.com/bkm**

KEY TERMS

intrinsic value	pseudo-American call option value	portfolio insurance
time value	hedge ratio	dynamic hedging
binomial model	delta	gamma
Black-Scholes pricing formula	option elasticity	delta neutral
implied volatility		vega

PROBLEM SETS

i. Basic

1. We showed in the text that the value of a call option increases with the volatility of the stock. Is this also true of put option values? Use the put-call parity theorem as well as a numerical example to prove your answer.
2. Would you expect a $1 increase in a call option's exercise price to lead to a decrease in the option's value of more or less than $1?
3. Is a put option on a high-beta stock worth more than one on a low-beta stock? The stocks have identical firm-specific risk.
4. All else equal, is a call option on a stock with a lot of firm-specific risk worth more than one on a stock with little firm-specific risk? The betas of the two stocks are equal.
5. All else equal, will a call option with a high exercise price have a higher or lower hedge ratio than one with a low exercise price?

ii. Intermediate

6. In each of the following questions, you are asked to compare two options with parameters as given. The risk-free interest rate for *all* cases should be assumed to be 6%. Assume the stocks on which these options are written pay no dividends.

a.

Put	T	X	σ	Price of Option
A	.5	50	.20	$10
B	.5	50	.25	$10

Which put option is written on the stock with the lower price?

i. A.
ii. B.
iii. Not enough information.

b.

Put	T	X	σ	Price of Option
A	.5	50	.2	$10
B	.5	50	.2	$12

Which put option must be written on the stock with the lower price?

i. A.
ii. B.
iii. Not enough information.

c.

Call	S	X	σ	Price of Option
A	50	50	.20	$12
B	55	50	.20	$10

Which call option must have the lower time to expiration?

i. A.
ii. B.
iii. Not enough information.

d.

Call	T	X	S	Price of Option
A	.5	50	55	$10
B	.5	50	55	$12

Which call option is written on the stock with higher volatility?

i. A.
ii. B.
iii. Not enough information.

e.

Call	T	X	S	Price of Option
A	.5	50	55	$10
B	.5	50	55	$ 7

Which call option is written on the stock with higher volatility?

i. A.
ii. B.
iii. Not enough information.

7. Reconsider the determination of the hedge ratio in the two-state model (see page 719), where we showed that one-third share of stock would hedge one option. What would be the hedge ratio for the following exercise prices: 120, 110, 100, 90? What do you conclude about the hedge ratio as the option becomes progressively more in the money?

8. Show that Black-Scholes call option hedge ratios also increase as the stock price increases. Consider a 1-year option with exercise price $50, on a stock with annual standard deviation 20%. The T-bill rate is 3% per year. Find $N(d_1)$ for stock prices $45, $50, and $55.

9. We will derive a two-state put option value in this problem. Data: $S_0 = 100$; $X = 110$; $1 + r = 1.10$. The two possibilities for S_T are 130 and 80.

   a. Show that the range of S is 50, whereas that of P is 30 across the two states. What is the hedge ratio of the put?

   b. Form a portfolio of three shares of stock and five puts. What is the (nonrandom) payoff to this portfolio? What is the present value of the portfolio?

   c. Given that the stock currently is selling at 100, solve for the value of the put.

10. Calculate the value of a call option on the stock in the previous problem with an exercise price of 110. Verify that the put-call parity theorem is satisfied by your answers to Problems 9 and 10. (Do not use continuous compounding to calculate the present value of X in this example because we are using a two-state model here, not a continuous-time Black-Scholes model.)

11. Use the Black-Scholes formula to find the value of a call option on the following stock:

Time to expiration	6 months
Standard deviation	50% per year
Exercise price	\$50
Stock price	\$50
Interest rate	3%

12. Find the Black-Scholes value of a put option on the stock in the previous problem with the same exercise price and expiration as the call option.

13. Recalculate the value of the call option in Problem 11, successively substituting one of the changes below while keeping the other parameters as in Problem 11:

   a. Time to expiration = 3 months.

   b. Standard deviation = 25% per year.

   c. Exercise price = \$55.

   d. Stock price = \$55.

   e. Interest rate = 5%.

   Consider each scenario independently. Confirm that the option value changes in accordance with the prediction of Table 21.1.

14. A call option with $X = \$50$ on a stock currently priced at $S = \$55$ is selling for \$10. Using a volatility estimate of $\sigma = .30$, you find that $N(d_1) = .6$ and $N(d_2) = .5$. The risk-free interest rate is zero. Is the implied volatility based on the option price more or less than .30? Explain.

15. What would be the Excel formula in Spreadsheet 21.1 for the Black-Scholes value of a straddle position?

Use the following case in answering Problems 16–21: Mark Washington, CFA, is an analyst with BIC. One year ago, BIC analysts predicted that the U.S. equity market would most likely experience a slight downturn and suggested delta-hedging the BIC portfolio. As predicted, the U.S. equity markets did indeed experience a downturn of approximately 4% over a 12-month period. However, portfolio performance for BIC was disappointing, lagging its peer group by nearly 10%. Washington has been told to review the options strategy to determine why the hedged portfolio did not perform as expected.

16. Which of the following *best* explains a delta-neutral portfolio? A delta-neutral portfolio is perfectly hedged against:

   a. Small price changes in the underlying asset.

   b. Small price decreases in the underlying asset.

   c. All price changes in the underlying asset.

17. After discussing the concept of a delta-neutral portfolio, Washington determines that he needs to further explain the concept of delta. Washington draws the value of an option as a function of the underlying stock price. Using this diagram, indicate how delta is interpreted. Delta is the:

   a. Slope in the option price diagram.

   b. Curvature of the option price graph.

   c. Level in the option price diagram.

18. Washington considers a put option that has a delta of −0.65. If the price of the underlying asset decreases by $6, then what is the best estimate of the change in option price?
19. BIC owns 51,750 shares of Smith & Oates. The shares are currently priced at $69. A call option on Smith & Oates with a strike price of $70 is selling at $3.50 and has a delta of .69 What is the number of call options necessary to create a delta-neutral hedge?
20. Return to the previous problem. Will the number of call options written for a delta-neutral hedge increase or decrease if the stock price falls?
21. Which of the following statements regarding the goal of a delta-neutral portfolio is *most* accurate? One example of a delta-neutral portfolio is to combine a:
    a. Long position in a stock with a short position in call options so that the value of the portfolio does not change with changes in the value of the stock.
    b. Long position in a stock with a short position in a call option so that the value of the portfolio changes with changes in the value of the stock.
    c. Long position in a stock with a long position in call options so that the value of the portfolio does not change with changes in the value of the stock.
22. Should the rate of return of a call option on a long-term Treasury bond be more or less sensitive to changes in interest rates than is the rate of return of the underlying bond?
23. If the stock price falls and the call price rises, then what has happened to the call option's implied volatility?
24. If the time to expiration falls and the put price rises, then what has happened to the put option's implied volatility?
25. According to the Black-Scholes formula, what will be the value of the hedge ratio of a call option as the stock price becomes infinitely large? Explain briefly.
26. According to the Black-Scholes formula, what will be the value of the hedge ratio of a put option for a very small exercise price?
27. The hedge ratio of an at-the-money call option on IBM is .4. The hedge ratio of an at-the-money put option is −.6. What is the hedge ratio of an at-the-money straddle position on IBM?

eXcel
Please visit us at
www.mhhe.com/bkm

28. Consider a 6-month expiration European call option with exercise price $105. The underlying stock sells for $100 a share and pays no dividends. The risk-free rate is 5%. What is the implied volatility of the option if the option currently sells for $8? Use Spreadsheet 21.1 (available at **www.mhhe.com/bkm**; link to Chapter 21 material) to answer this question.
    a. Go to the Tools menu of the spreadsheet and select Goal Seek. The dialog box will ask you for three pieces of information. In that dialog box, you should *set cell* E6 *to value* 8 *by changing cell* B2. In other words, you ask the spreadsheet to find the value of standard deviation (which appears in cell B2) that forces the value of the option (in cell E6) equal to $8. Then click OK, and you should find that the call is now worth $8, and the entry for standard deviation has been changed to a level consistent with this value. This is the call's implied standard deviation at a price of $8.
    b. What happens to implied volatility if the option is selling at $9? Why has implied volatility increased?
    c. What happens to implied volatility if the option price is unchanged at $8, but option expiration is lower, say, only 4 months? Why?
    d. What happens to implied volatility if the option price is unchanged at $8, but the exercise price is lower, say, only $100? Why?
    e. What happens to implied volatility if the option price is unchanged at $8, but the stock price is lower, say, only $98? Why?
29. A collar is established by buying a share of stock for $50, buying a 6-month put option with exercise price $45, and writing a 6-month call option with exercise price $55. On the basis of the volatility of the stock, you calculate that for a strike price of $45 and expiration of 6 months, $N(d_1) = .60$, whereas for the exercise price of $55, $N(d_1) = .35$.
    a. What will be the gain or loss on the collar if the stock price increases by $1?
    b. What happens to the delta of the portfolio if the stock price becomes very large? Very small?

30. These three put options are all written on the same stock. One has a delta of −.9, one a delta of −.5, and one a delta of −.1. Assign deltas to the three puts by filling in this table.

Put	X	Delta
A	10	
B	20	
C	30	

31. You are *very* bullish (optimistic) on stock EFG, much more so than the rest of the market. In each question, choose the portfolio strategy that will give you the biggest dollar profit if your bullish forecast turns out to be correct. Explain your answer.
    a. *Choice A:* $10,000 invested in calls with $X = 50$.
    Choice B: $10,000 invested in EFG stock.
    b. *Choice A:* 10 call option contracts (for 100 shares each), with $X = 50$.
    Choice B: 1,000 shares of EFG stock.
32. You would like to be holding a protective put position on the stock of XYZ Co. to lock in a guaranteed minimum value of $100 at year-end. XYZ currently sells for $100. Over the next year the stock price will increase by 10% or decrease by 10%. The T-bill rate is 5%. Unfortunately, no put options are traded on XYZ Co.
    a. Suppose the desired put option were traded. How much would it cost to purchase?
    b. What would have been the cost of the protective put portfolio?
    c. What portfolio position in stock and T-bills will ensure you a payoff equal to the payoff that would be provided by a protective put with $X = 100$? Show that the payoff to this portfolio and the cost of establishing the portfolio matches that of the desired protective put.
33. Return to Example 21.1. Use the binomial model to value a 1-year European put option with exercise price $110 on the stock in that example. Does your solution for the put price satisfy put-call parity?
34. Suppose that the risk-free interest rate is zero. Would an American put option ever be exercised early? Explain.
35. Let $p(S, T, X)$ denote the value of a European put on a stock selling at S dollars, with time to maturity T, and with exercise price X, and let $P(S, T, X)$ be the value of an American put.
    a. Evaluate $p(0, T, X)$.
    b. Evaluate $P(0, T, X)$.
    c. Evaluate $p(S, T, 0)$.
    d. Evaluate $P(S, T, 0)$.
    e. What does your answer to (*b*) tell you about the possibility that American puts may be exercised early?
36. You are attempting to value a call option with an exercise price of $100 and 1 year to expiration. The underlying stock pays no dividends, its current price is $100, and you believe it has a 50% chance of increasing to $120 and a 50% chance of decreasing to $80. The risk-free rate of interest is 10%. Calculate the call option's value using the two-state stock price model.
37. Consider an increase in the volatility of the stock in the previous problem. Suppose that if the stock increases in price, it will increase to $130, and that if it falls, it will fall to $70. Show that the value of the call option is now higher than the value derived in the previous problem.
38. Calculate the value of a put option with exercise price $100 using the data in Problem 36. Show that put-call parity is satisfied by your solution.
39. XYZ Corp. will pay a $2 per share dividend in 2 months. Its stock price currently is $60 per share. A call option on XYZ has an exercise price of $55 and 3-month time to expiration. The risk-free interest rate is .5% per month, and the stock's volatility (standard deviation) = 7% per month. Find the pseudo-American option value. (Hint: Try defining one "period" as a month, rather than as a year.)
40. "The beta of a call option on General Electric is greater than the beta of a share of General Electric." True or false?

41. "The beta of a call option on the S&P 500 index with an exercise price of 1,130 is greater than the beta of a call on the index with an exercise price of 1,140." True or false?
42. What will happen to the hedge ratio of a convertible bond as the stock price becomes very large?
43. Goldman Sachs believes that market volatility will be 20% annually for the next 3 years. Three-year at-the-money call and put options on the market index sell at an implied volatility of 22%. What options portfolio can Goldman establish to speculate on its volatility belief without taking a bullish or bearish position on the market? Using Goldman's estimate of volatility, 3-year at-the-money options have $N(d_1) = .6$.
44. You are holding call options on a stock. The stock's beta is .75, and you are concerned that the stock market is about to fall. The stock is currently selling for $5 and you hold 1 million options on the stock (i.e., you hold 10,000 contracts for 100 shares each). The option delta is .8. How much of the market index portfolio must you buy or sell to hedge your market exposure?

iii. Challenge

45. Imagine you are a provider of portfolio insurance. You are establishing a 4-year program. The portfolio you manage is currently worth $100 million, and you hope to provide a minimum return of 0%. The equity portfolio has a standard deviation of 25% per year, and T-bills pay 5% per year. Assume for simplicity that the portfolio pays no dividends (or that all dividends are reinvested).
    a. How much should be placed in bills? How much in equity?
    b. What should the manager do if the stock portfolio falls by 3% on the first day of trading?
46. Suppose that call options on ExxonMobil stock with time to expiration 3 months and strike price $60 are selling at an implied volatility of 30%. ExxonMobil stock currently is $60 per share, and the risk-free rate is 4%. If you believe the true volatility of the stock is 32%, how can you trade on your belief without taking on exposure to the performance of ExxonMobil? How many shares of stock will you hold for each option contract purchased or sold?
47. Using the data in the previous problem, suppose that 3-month put options with a strike price of $60 are selling at an implied volatility of 34%. Construct a delta-neutral portfolio comprising positions in calls and puts that will profit when the option prices come back into alignment.
48. Suppose that JPMorgan Chase sells call options on $1.25 million worth of a stock portfolio with beta = 1.5. The option delta is .8. It wishes to hedge out its resultant exposure to a market advance by buying a market index portfolio.
    a. How many dollars worth of the market index portfolio should it purchase to hedge its position?
    b. What if it instead uses market index puts to hedge its exposure? Should it buy or sell puts? Each put option is on 100 units of the index, and the index at current prices represents $1,000 worth of stock.

CFA® PROBLEMS

1. The board of directors of Abco Company is concerned about the downside risk of a $100 million equity portfolio in its pension plan. The board's consultant has proposed temporarily (for 1 month) hedging the portfolio with either futures or options. Referring to the following table, the consultant states:
    a. "The $100 million equity portfolio can be fully protected on the downside by selling (shorting) 4,000 futures contracts."
    b. "The cost of this protection is that the portfolio's expected rate of return will be zero percent."

Market, Portfolio, and Contract Data

Equity index level	99.00
Equity futures price	100.00
Futures contract multiplier	$250
Portfolio beta	1.20
Contract expiration (months)	3

Critique the accuracy of each of the consultant's two statements.

2. Michael Weber, CFA, is analyzing several aspects of option valuation, including the determinants of the value of an option, the characteristics of various models used to value options, and the potential for divergence of calculated option values from observed market prices.
   a. What is the expected effect on the value of a call option on common stock if the volatility of the underlying stock price decreases? If the time to expiration of the option increases?
   b. Using the Black-Scholes option-pricing model, Weber calculates the price of a 3-month call option and notices the option's calculated value is different from its market price. With respect to Weber's use of the Black-Scholes option-pricing model,
      i. Discuss why the calculated value of an out-of-the-money European option may differ from its market price.
      ii. Discuss why the calculated value of an American option may differ from its market price.
3. Joel Franklin is a portfolio manager responsible for derivatives. Franklin observes an American-style option and a European-style option with the same strike price, expiration, and underlying stock. Franklin believes that the European-style option will have a higher premium than the American-style option.
   a. Critique Franklin's belief that the European-style option will have a higher premium. Franklin is asked to value a 1-year European-style call option for Abaco Ltd. common stock, which last traded at $43.00. He has collected the information in the following table.

Closing stock price	$43.00
Call and put option exercise price	45.00
1-year put option price	4.00
1-year Treasury bill rate	5.50%
Time to expiration	One year

   b. Calculate, using put-call parity and the information provided in the table, the European-style call option value.
   c. State the effect, if any, of each of the following three variables on the value of a call option. (No calculations required.)
      i. An increase in short-term interest rate.
      ii. An increase in stock price volatility.
      iii. A decrease in time to option expiration.
4. A stock index is currently trading at 50. Paul Tripp, CFA, wants to value 2-year index options using the binomial model. The stock will either increase in value by 20% or fall in value by 20%. The annual risk-free interest rate is 6%. No dividends are paid on any of the underlying securities in the index.
   a. Construct a two-period binomial tree for the value of the stock index.
   b. Calculate the value of a European call option on the index with an exercise price of 60.
   c. Calculate the value of a European put option on the index with an exercise price of 60.
   d. Confirm that your solutions for the values of the call and the put satisfy put-call parity.
5. Ken Webster manages a $200 million equity portfolio benchmarked to the S&P 500 index. Webster believes the market is overvalued when measured by several traditional fundamental/economic indicators. He is concerned about potential losses but recognizes that the S&P 500 index could nevertheless move above its current 1136 level.

   Webster is considering the following *option collar* strategy:
   - Protection for the portfolio can be attained by purchasing an S&P 500 index put with a strike price of 1130 (just out of the money).
   - The put can be financed by selling two 1150 calls (farther out-of-the-money) for every put purchased.
   - Because the combined delta of the two calls (see following table) is less than 1 (that is, $2 \times .36 = .72$), the options will not lose more than the underlying portfolio will gain if the market advances.

The information in the following table describes the two options used to create the collar.

Characteristics	1150 Call	1130 Put
Option price	$8.60	$16.10
Option implied volatility	22%	24%
Option's delta	0.36	−0.44
Contracts needed for collar	602	301

Notes:
- Ignore transaction costs.
- S&P 500 historical 30-day volatility = 23%.
- Time to option expiration = 30 days.

a. Describe the potential returns of the combined portfolio (the underlying portfolio plus the option collar) if after 30 days the S&P 500 index has:

i. risen approximately 5% to 1193.
ii. remained at 1136 (no change).
iii. declined by approximately 5% to 1080.
(No calculations are necessary.)

b. Discuss the effect on the hedge ratio (delta) of *each* option as the S&P 500 approaches the level for *each* of the potential outcomes listed in part (*a*).

c. Evaluate the pricing of *each* of the following in relation to the volatility data provided:

i. the put
ii. the call

E-INVESTMENTS EXERCISES

Option Price Differences

Select a stock for which options are listed on the CBOE Web site (**www.cboe.com**). The price data for captions can be found on the "delayed quotes" menu option. Enter a ticker symbol for a stock of your choice and pull up its option price data.

Using daily price data from **finance.yahoo.com** calculate the annualized standard deviation of the daily percentage change in the stock price. Create a Black-Scholes option-pricing model in a spreadsheet, or use our Spreadsheet 21.1, available at **www.mhhe.com/bkm** with Chapter 21 material. Using the standard deviation and a risk-free rate found at **www.bloomberg.com/markets/rates/index.html**, calculate the value of the call options.

How do the calculated values compare to the market prices of the options? On the basis of the difference between the price you calculated using historical volatility and the actual price of the option, what do you conclude about expected trends in market volatility?