Generalized Autoregressive Conditional Heteroscedastic (GARCH) Models

Example

$$SET_t = \beta_0 + \beta_1 IBR + \beta_2 GOLDB + \beta_3 USDS + \varepsilon_t$$

where: SET_t = Return on Stock Exchange of Thailand (SET).

IBR_t = Inter-bank rate.

$GOLDB_t$ = Gold price.

$USDS_t$ = Exchange rate ($US/฿Baht).

ε_t = Residual which has GARCH(p,q) process:

$$\sigma_t^2 = \alpha_0 + \delta_1\sigma_{t-1}^2 + \delta_2\sigma_{t-2}^2 + \cdots + \delta_p\sigma_{t-p}^2 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + \cdots + \alpha_q\varepsilon_{t-q}^2$$

Testing GARCH Effect

Firstly, estimate the model using OLS without ARCH(p) process.

```
. regress set ibr usds goldb

      Source |       SS       df       MS              Number of obs =     118
-------------+------------------------------           F(  3,   114) =  103.43
       Model |  215609.598     3  71869.8659           Prob > F      =  0.0000
    Residual |  79215.8193   114  694.875608           R-squared     =  0.7313
-------------+------------------------------           Adj R-squared =  0.7242
       Total |  294825.417   117  2519.87536           Root MSE      =   26.36

------------------------------------------------------------------------------
         set |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         ibr |  -142.0644   68.73617    -2.07   0.041    -278.2302   -5.898551
        usds |  -78.06484    5.30484   -14.72   0.000    -88.57369     -67.556
       goldb |    .029042   .0192431     1.51   0.134    -.0090784    .0671624
       _cons |   3698.659   332.8835    11.11   0.000     3039.219    4358.099
------------------------------------------------------------------------------

. estat archlm
LM test for autoregressive conditional heteroskedasticity (ARCH)
-----------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+---------------------------------------------------------------
       1     |         63.052               1                   0.0000
-----------------------------------------------------------------------------
         H0: no ARCH effects      vs.  H1: ARCH(p) disturbance
```

In this case, since p-value of the ARCH effect test (F-statistic or Chi-Square (Obs*R-squared = (117*0.538907) = 63.05211)) is less than level of significance 0.05, thus, null hypothesis that there is no ARCH effect is rejected, thus, there exists significant ARCH effect in this model with 0.05 significant level.

Identify Order (p,q) and Estimation

The next step is to identify order of GARCH(p,q) by estimating GARCH models in several orders and choose the model with the lowest AIC or SIC.

Estimate GARCH(p,q) using MLE

```
. arch set ibr usds goldb, arch(1) nolog
BFGS stepping has contracted, resetting BFGS Hessian (0)
     ...
BFGS stepping has contracted, resetting BFGS Hessian (4)

ARCH family regression

Sample:  02dec2003 to 28mar2004                 Number of obs      =       118
                                                Wald chi2(3)       =   1122.15
Log likelihood = -517.3025                      Prob > chi2        =    0.0000
------------------------------------------------------------------------------
             |                 OPG
         set |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
set          |
         ibr |  -64.13751   41.80188    -1.53   0.125    -146.0677    17.79268
        usds |  -72.48378    2.63951   -27.46   0.000    -77.65712   -67.31043
       goldb |   .0590191   .0138355     4.27   0.000      .031902    .0861362
       _cons |   3178.165    197.789    16.07   0.000     2790.506    3565.825
-------------+----------------------------------------------------------------
ARCH         |
        arch |
         L1. |   .9245129   .3233346     2.86   0.004     .2907888    1.558237
       _cons |   93.59632   38.20728     2.45   0.014     18.71142    168.4812
------------------------------------------------------------------------------

. estat ic
-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
           . |    118           .   -517.3025      6     1046.605    1063.229
-----------------------------------------------------------------------------

. arch set ibr usds goldb, arch(1/2) nolog
BFGS stepping has contracted, resetting BFGS Hessian (17)

ARCH family regression

Sample:  02dec2003 to 28mar2004                 Number of obs      =       118
                                                Wald chi2(3)       =   1158.49
Log likelihood = -516.9879                      Prob > chi2        =    0.0000
------------------------------------------------------------------------------
             |                 OPG
         set |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
set          |
         ibr |  -61.07753   33.81901    -1.81   0.071    -127.3616    5.206507
        usds |  -72.49558   2.831238   -25.61   0.000     -78.0447   -66.94645
       goldb |   .0599955   .0140527     4.27   0.000     .0324528    .0875382
       _cons |    3167.73   207.2899    15.28   0.000      2761.45    3574.011
-------------+----------------------------------------------------------------
ARCH         |
        arch |
         L1. |   .9952712   .3383216     2.94   0.003      .332173    1.658369
         L2. |  -.1255426   .1038731    -1.21   0.227    -.3291301     .078045
       _cons |    117.519   40.07261     2.93   0.003     38.97815    196.0599
------------------------------------------------------------------------------

. estat ic
-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
           . |    118           .   -516.9879      7     1047.976    1067.371
-----------------------------------------------------------------------------
```

Estimate GARCH(1,1)

```
. arch set usds, arch(1) garch(1) nolog

ARCH family regression

Sample:  02dec2003 to 28mar2004                 Number of obs      =       118
                                                Wald chi2(1)       =    882.55
Log likelihood = -518.5556                      Prob > chi2        =    0.0000

------------------------------------------------------------------------------
             |                 OPG
         set |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
set          |
        usds |  -74.04275   2.492366   -29.71   0.000    -78.92769    -69.1578
       _cons |   3607.771   97.79026    36.89   0.000     3416.106    3799.437
-------------+----------------------------------------------------------------
ARCH         |
        arch |
         L1. |   1.063916    .297246     3.58   0.000     .4813242    1.646507
       garch |
         L1. |  -.1115611   .1054421    -1.06   0.290    -.3182239    .0951017
       _cons |   112.9776   46.69068     2.42   0.016     21.46551    204.4896
------------------------------------------------------------------------------
```