Quiz 1: Date: April 19, 2022 from 11.00-12.30

Question 1 (10 Points) Score.....................

Consider the one-period model of consumption and portfolio choice. Let an individual in this economy has the utility function as follow:

$$U(C) = ln(C)$$

Also, let $\frac{C_1}{C_0}$ is distributed as log-normal with mean equals μ_c and its variance is σ_c.
Please read and answer the following questions carefully and completely.

Score.....................

Question 1.1 (10 marks) Calculate the risk free rate R_f in terms of the individual's consumption, C_0 and C_1. Then,explain the relationship between the level of consumption and the risk free rate in this economy.

From $\frac{1}{R_f} = \delta E\left[\frac{u'(c_1)}{u'(c_0)}\right]$ $u(c) = \ln(c)$

$u'(c) = \frac{1}{c}$

$= \delta E\left[\frac{c_0}{c_1}\right] = \delta \frac{c_0}{c_1}$

$R_f = \frac{c_1}{\delta c_0}$

When R_f is high, period 1 consumption is high.

When R_f is high, period 0 consumption is low.

Reflecting that the (expected) growth in consumption is high when R_f is high.

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Score.....................

Question 1.2 (10 marks) Calculate the elasticity of intertemporal substitution in this setting. If in the next year, the interest rate is falling, Will the individual's consumption level increase or decrease? Why? To support your answer, use the concepts of income effect and substitution effect.

$$\varepsilon = \frac{\partial\left(\frac{c_1}{c_0}\right)}{\partial R_f} \cdot \frac{R_f}{\frac{c_1}{c_0}}$$

$$= \delta \cdot \frac{R_f}{\frac{c_1}{c_0}}$$

$$= \delta \cdot \frac{\frac{1}{\delta}\left(\frac{c_1}{c_0}\right)}{\frac{c_1}{c_0}} = 1$$

$$R_f = \frac{1}{\delta}\left(\frac{c_1}{c_0}\right)$$

$$\frac{\partial R_f}{\partial\left(\frac{c_1}{c_0}\right)} = \frac{1}{\delta} \rightarrow \frac{\partial\left(\frac{c_1}{c_0}\right)}{\partial R_f} = \delta$$

As the elasticity of intertemporal substitution equals to 1, change in interest rate will result in one-for-one change in the consumption where income effect and substitution effect offset each other. The result of incentive to save more for next period consumption offsets the result of raising consumption for both periods when the interest rate increases.

Score.....................

Question 1.3 (10 marks) Solve for the pricing kernel P_i of any risky asset i in this economy. Then explain the meaning of this pricing kernel.

$$P_i = E\left[\delta \frac{u'(c_1)}{u'(c_0)} X_i\right]$$

$$= E\left[\delta \left[\frac{c_0}{c_1}\right] X_i\right]$$

$$P_i = \delta \left[\frac{c_0}{c_1}\right] X_i$$

In the state where $u'(c_1)$ is high (or low c_1) and $u'(c_0)$ is low (or high c_0), the price of risky asset will be high.

Score.....................

Question 1.4 (10 marks) Calculate Hansen-Jaganathan Bound and explain the meaning.

$u(c) = \ln(c)$, $m_{01} = \delta \frac{c_0}{c_1} = \delta e^{\ln(c_0/c_1)}$

$$\frac{\sigma_{m_{01}}}{E(m_{01})} = \frac{\sqrt{var(e^{\ln(c_0/c_1)})}}{E(e^{\ln(c_0/c_1)})}$$

$var(x) = E(x^2) - E(x)^2$

$$= \frac{\sqrt{E[(e^{\ln(c_0/c_1)})^2] - E(e^{\ln(c_0/c_1)})^2}}{E(e^{\ln(c_0/c_1)})}$$

$$= \sqrt{\frac{E[(e^{2\ln(c_0/c_1)})]}{E(e^{\ln(c_0/c_1)})^2} - 1}$$

$$= \sqrt{\frac{e^{2\mu_c + 2\sigma_c^2}}{e^{2\mu_c + \sigma_c^2}} - 1}$$

$$= \sqrt{e^{\sigma_c^2} - 1} \simeq \sigma_c$$

The sharp ratio will not exceed the bound, σ_c, from the theory.