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***Quiz 3**: Two questions given. Attempt only one of them.*

Question 1 A market demand function is given by

$$y^{-2}(p_x + 1)\sqrt{q_x + 3} = 1000p_z^{-1}$$

where p_x is price per unit, q the amount of quantity, and y is the level of income. Consider the following problems.

1.1 (2 point) Rewrite the equation in the implicit-form function.

$$F(*) = y^{-2}(p_x + 1)\sqrt{q_x + 3} - 1000p_z^{-1} = 0$$

1.2 (4 points) Use the implicit function theorem to state the relationship between good x and good z. Are they substitute/complementary product?

$$\frac{\partial q_x}{\partial p_z} = -\frac{F_z}{F_{q_x}} = -\frac{1000p_z^2}{y^{-2}(p_x + 1)\left(\frac{1}{2}\right)(q_x + 3)^{-\frac{1}{2}}} < 0$$

That is, an increase in price of z causes a decrease in quantity demanded for x. That's because both x and z are complementary good. Intuitively, rising price of good z would be associated with a decrease in consumption of good z. But in this exercise, we see that it also leads to a decrease in consumption of good x as well. That means both x and y must be used together; otherwise, consumption on both goods wouldn't have dropped together at the same time.

1.3 (4 points) Apply the implicit function theorem, and calculate income elasticity when $p_x = 24,\ y = 1\ and\ p_z = 1.$

If you plug everything into the demand equation, you will find that $q_x = 1597$

$$\frac{\partial q_x}{\partial y} * \frac{y}{q_x} = -\frac{F_y}{F_{q_x}} * \frac{y}{q_x} = -\frac{-2y^{-3}(p_x + 1)\sqrt{q_x + 3}}{y^{-2}(p_x + 1)\left(\frac{1}{2}\right)(q_x + 3)^{-\frac{1}{2}}} * \frac{1}{1597}$$

$$= 4y^{-1}(q_x + 3) * \frac{1}{1597} = \frac{4(1600)}{1596} = \frac{6400}{1597} > 1$$

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Quiz 3: *Two questions given. Attempt only one of them.*

<u>Question 2</u> Suppose that demand for capital goods can be given the following equation: $K^* = \left(\frac{w}{r}\right)^{\alpha} Y^{\beta}$, where K^* is the optimal level of capital installation, w is wage, r is capital price, and Y is the targeted level of output. Assume further that all the parameters are positive, and that $\alpha + \beta = 1$. Consider the following question

2.1 (2 points) Is the given function a homogenous function. If yes, state the degree of homogeneity.

HM of degree β. Here is why. *Rescale everything by t times yield us*

$$\left(\frac{tw}{tr}\right)^{\alpha} (tY)^{\beta} = t^{\beta} \left(\frac{w}{r}\right)^{\alpha} (Y)^{\beta} = t^{\beta} K^*$$

2.2 (4 points) Apply the partial derivative to state the relationship between K^* and w. Explain your result by providing some economic intuitions.

$$\frac{\partial K^*}{\partial w} = \alpha \left(\frac{w}{r}\right)^{\alpha-1} \left(\frac{1}{r}\right) Y^{\beta} > 0$$

This means, firms will be using more capital when labor wage increases. Basically, this is factor input substitution process that firm substitutes away from the expensive factor input, and use a relatively cheaper factor input. In this case, capital becomes cheaper after price of labor increases.

2.3 (4 points) Calculate the elasticity of capital installation with respect to price of capital goods(r).

Recall that elasticity of y with respect to x is $\frac{dy}{dx} * \frac{x}{y} = \frac{\frac{dy}{y}}{\frac{dx}{x}} = \frac{dln(y)}{dln(x)}$

Apply natural log to the demand function, we yield that.

$$\ln(K^*) = ln\left(\frac{w}{r}\right)^{\alpha} + \ln\left(Y^{\beta}\right) = \alpha \ln(w) - \alpha \ln(r) + \beta \ln(Y)$$

Thus, the elasticity of capital with respect to r is $-\alpha$.